



**How to be ready for  
multi-messenger astronomy:  
Gravitational wave sources in  
the frequency range of  
Advanced LIGO and VIRGO**

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Sandro Bressan, Monica Colpi, Luca Zampieri*

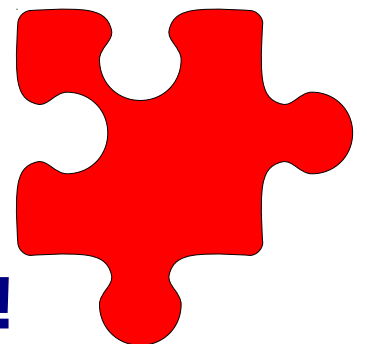
***Michela Mapelli***  
INAF-Osservatorio  
Astronomico di Padova

Jena, September 1st 2015

## OUTLINE:

1. **gravitational wave (GW) sources for LIGO/Virgo: merging black hole (BH) and neutron star (NS) binaries**
2. **possible electromagnetic counterparts**
3. **how to estimate the merger rate of BH and NS binaries**
4. **impact of environment on merger rate**

**NOTE: MIND THE SLIDES WITH THIS FLAG  
THEY ARE SIMPLE SUGGESTIONS FOR  
TEACHING TO HIGH-SCHOOL STUDENTS!!!**



# 1. gravitational wave sources for LIGO/Virgo

Virgo (Santo Stefano a Macerata, Cascina, Italy)

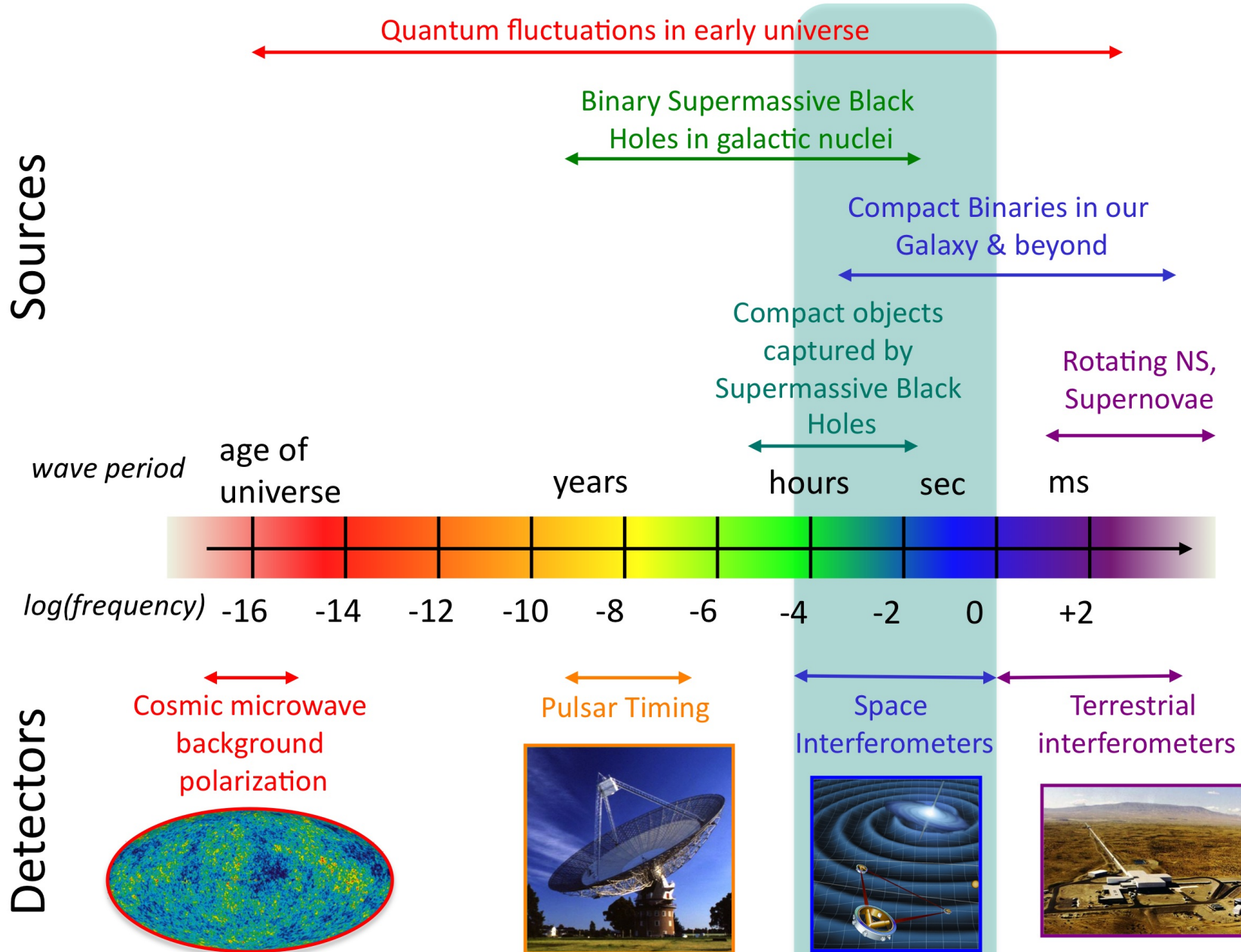
**FREQUENCY RANGE ~ 10 – 10 000 Hz**





# 1. gravitational wave sources for LIGO/Virgo

## The Gravitational Wave Spectrum

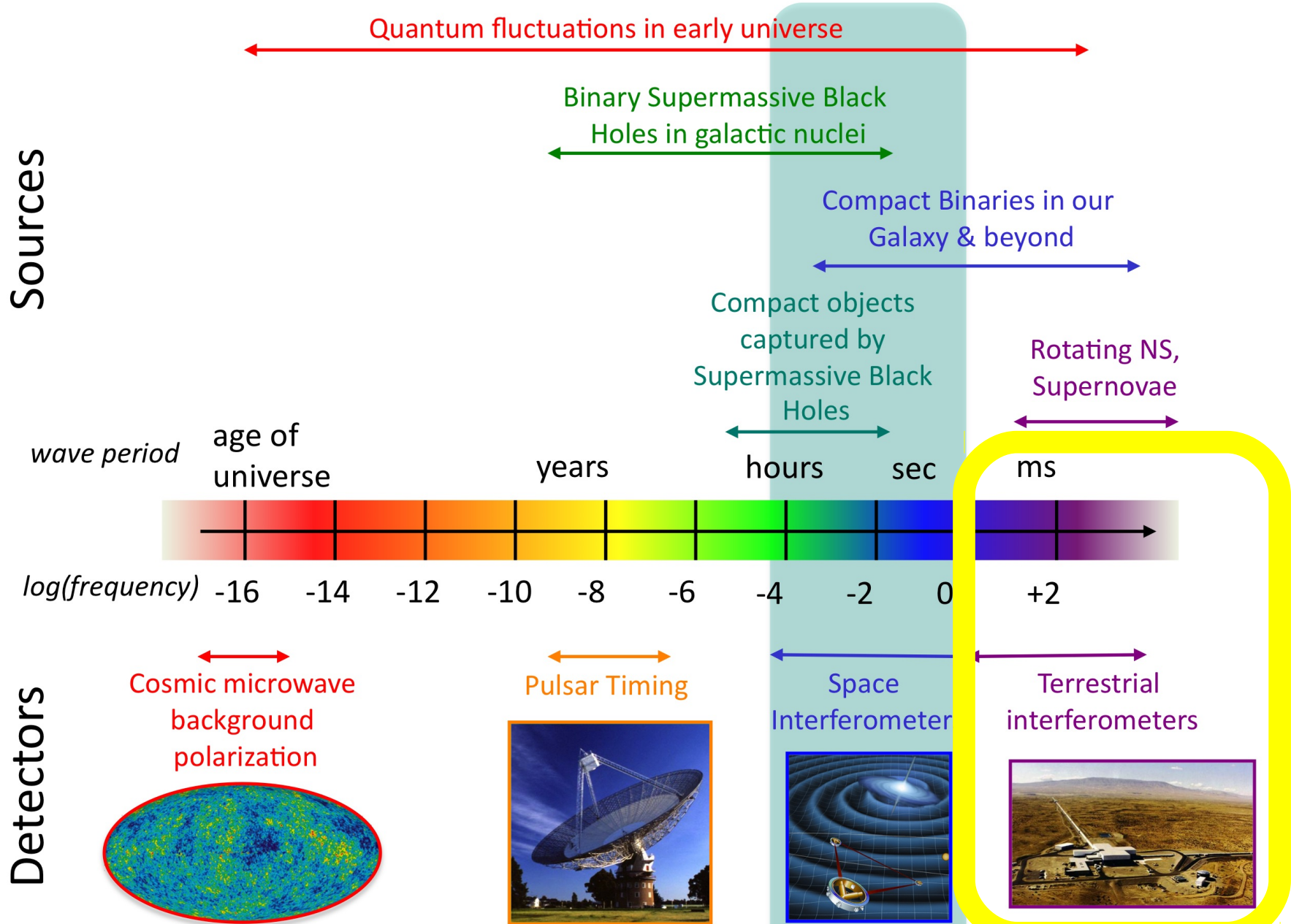


Credits: NASA



# 1. gravitational wave sources for LIGO/Virgo

## The Gravitational Wave Spectrum



# 1. gravitational wave sources for LIGO/Virgo

**Focus on MERGING double compact-object binaries because LIGO-VIRGO sources**

**black hole – black hole (BH-BH) binaries**

**neutron star-neutron star (NS-NS) binaries**

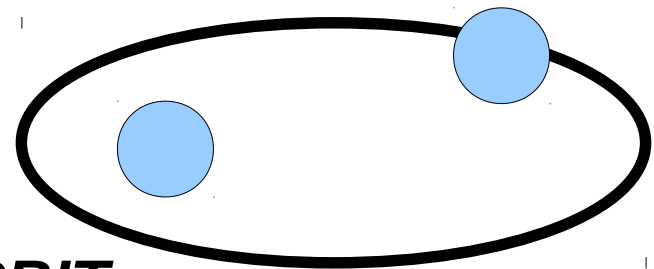
**black hole – neutron star (BH-NS) binaries**

*For BH-BH, BH-NS  
and NS-NS binary*

*I mean KEPLERIAN binary:*

*BOUND SYSTEM on ELLIPTICAL ORBIT*

*composed of 2 BHs, or 2 NSs or 1 BH + 1 NS*



# 1. gravitational wave sources for LIGO/Virgo

## QUESTIONS TO ADDRESS IN THIS LECTURE FROM A SIMPLE POINT OF VIEW

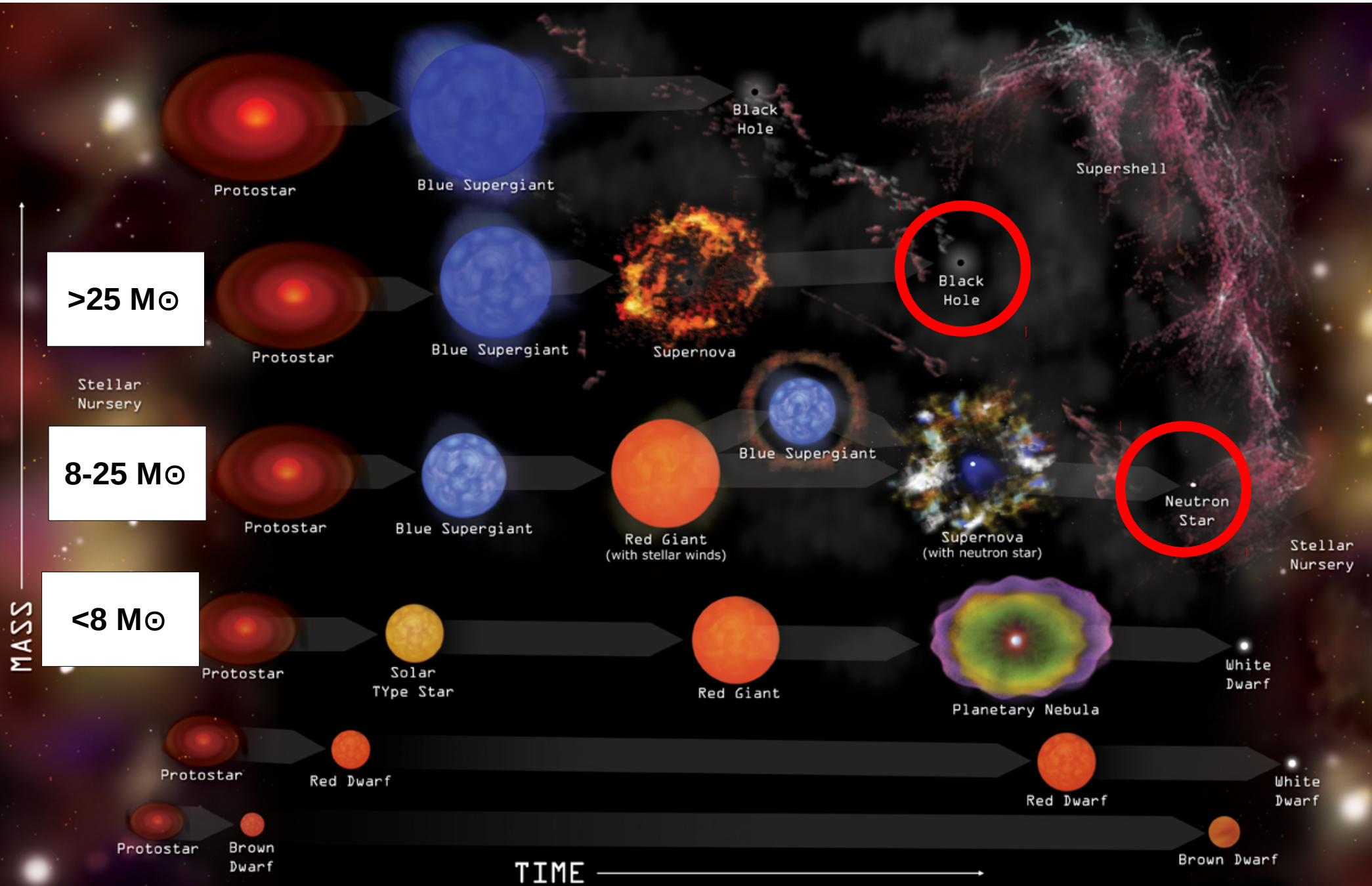
- 1 → HOW DO BHs and NSs form?
- 2 → WHY DO GWs cause BH- and NS-binaries to merge (GW frequencies)?
- 3 → Are there electromagnetic (EM) counterparts (if any)?
- 4 → WHAT IS THE RATE of MERGERS?
- 5 → WHAT IS THE ENVIRONMENT EFFECT ON THE RATE?



# 1. gravitational wave sources

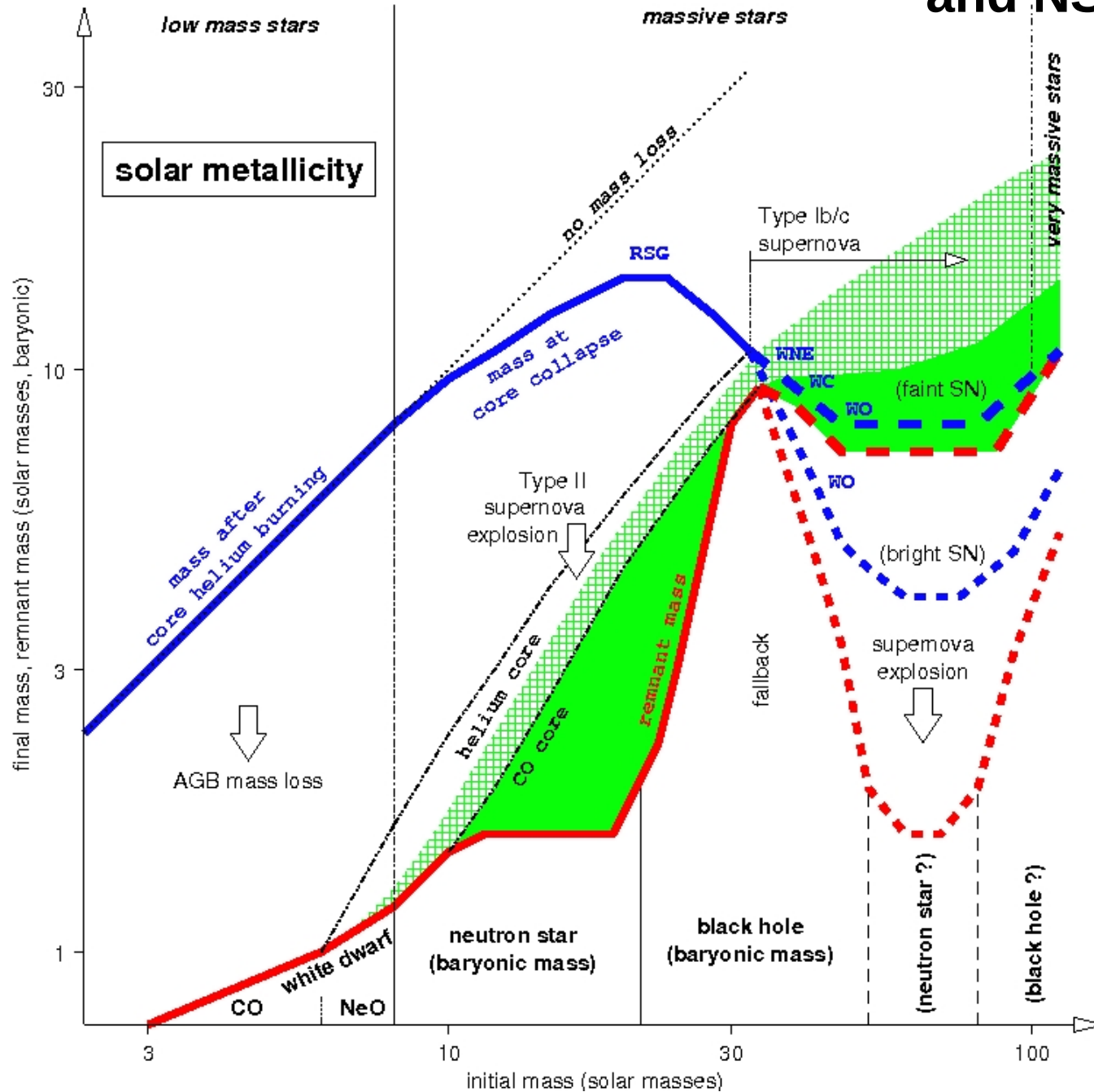
## 1 → HOW DO BHs and NSs form?

Credits: Chandra



# 1. gravitational wave sources

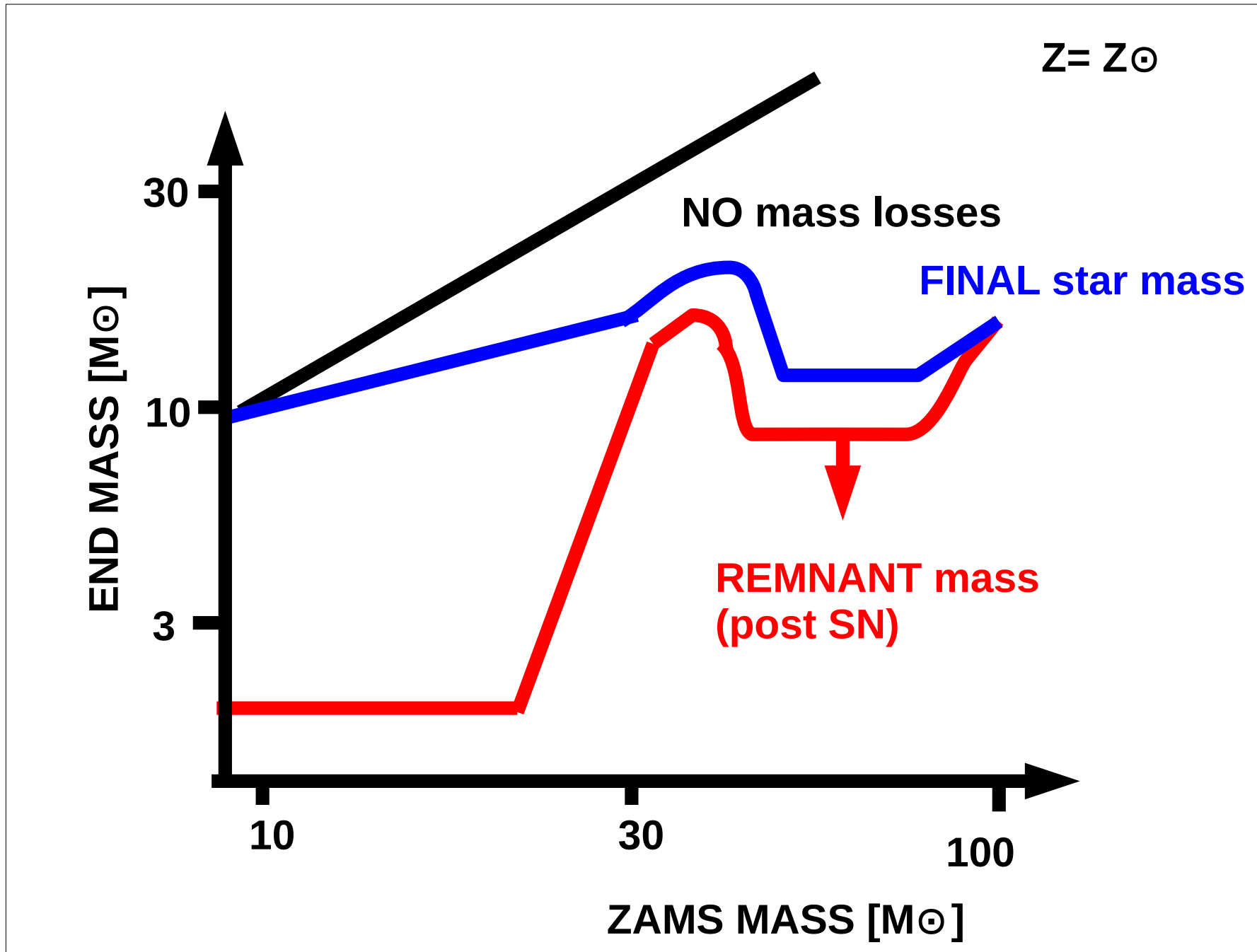
1 → HOW DO BHs and NSs form?



Heger et al. (2003)

# 1. gravitational wave sources

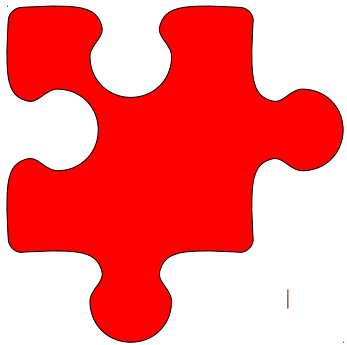
1 → HOW DO BHs and NSs form?





# 1. gravitational wave sources

## CARTOON OF NS-NS MERGER



Movies by NASA are a perfect reservoir for high-school students

<https://www.youtube.com/watch?v=g8s81MzzJ5c>

Credits: NASA

**2 → WHY DO GWs cause BH- and NS-binaries to merge?**

# 1. gravitational wave sources

2 → WHY DO GWs cause BH- and NS-binaries to merge?

From yesterday talks:

- GW emission only from QUADRUPOLE  
A binary has a non-zero mass QUADRUPOLE  
Thus emits GW
- GW emission implies energy loss

$$\text{Binary's energy} = E_{\text{int}} = -\frac{G m_1 m_2}{2a}$$

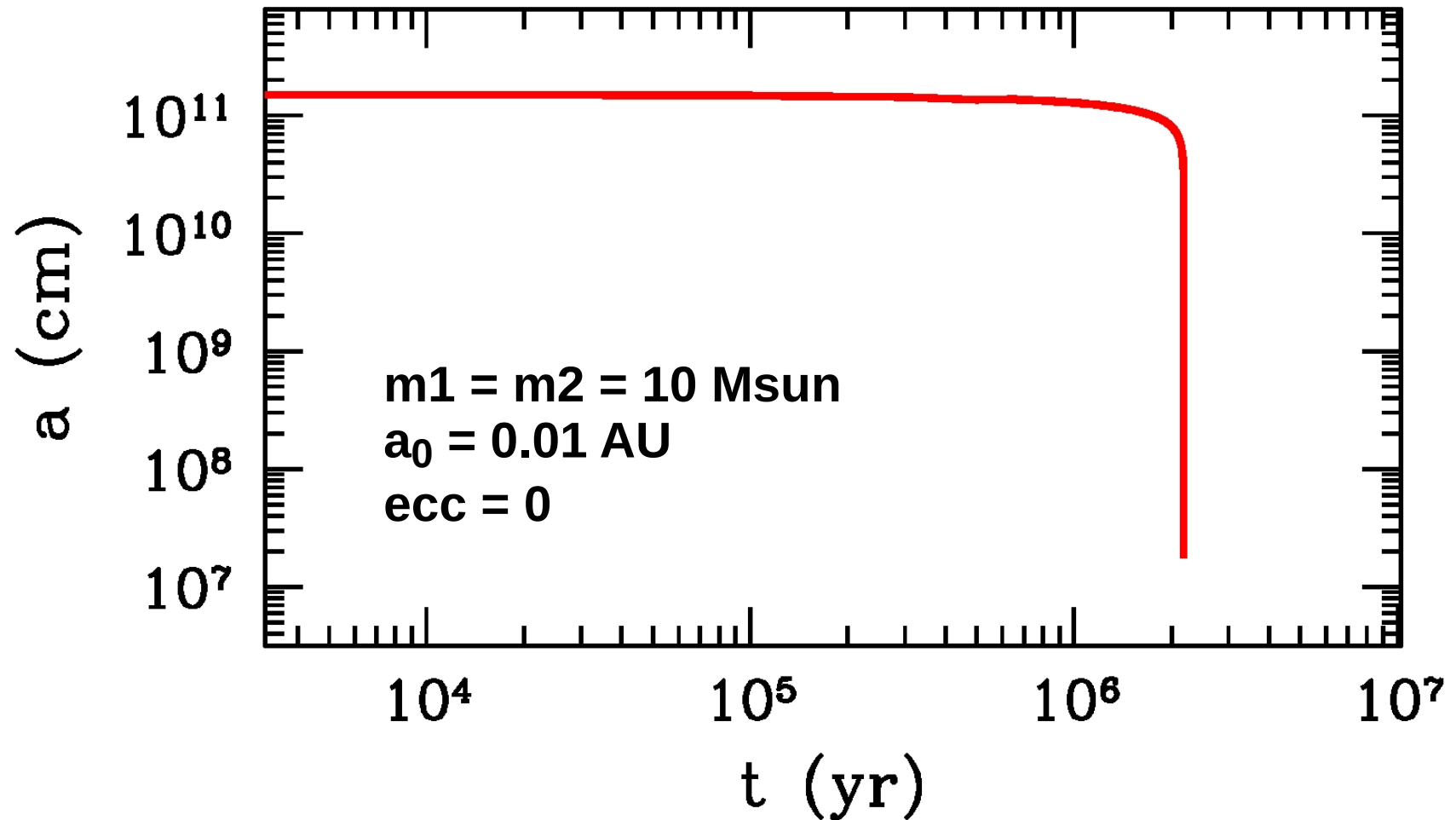
Semi-major axis of Kepler binary changes as

$$\frac{da}{dt} \sim -\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 a^3 (1 - e^2)^{7/2}}$$

Peters (1964)

# 1. gravitational wave sources

$$a(t) = a_0 \left[ 1 - \frac{256/5 G^3 m_1 m_2 (m_1 + m_2) t}{c^5 (1 - e^2)^{7/2} a_0^4} \right]^{1/4}$$





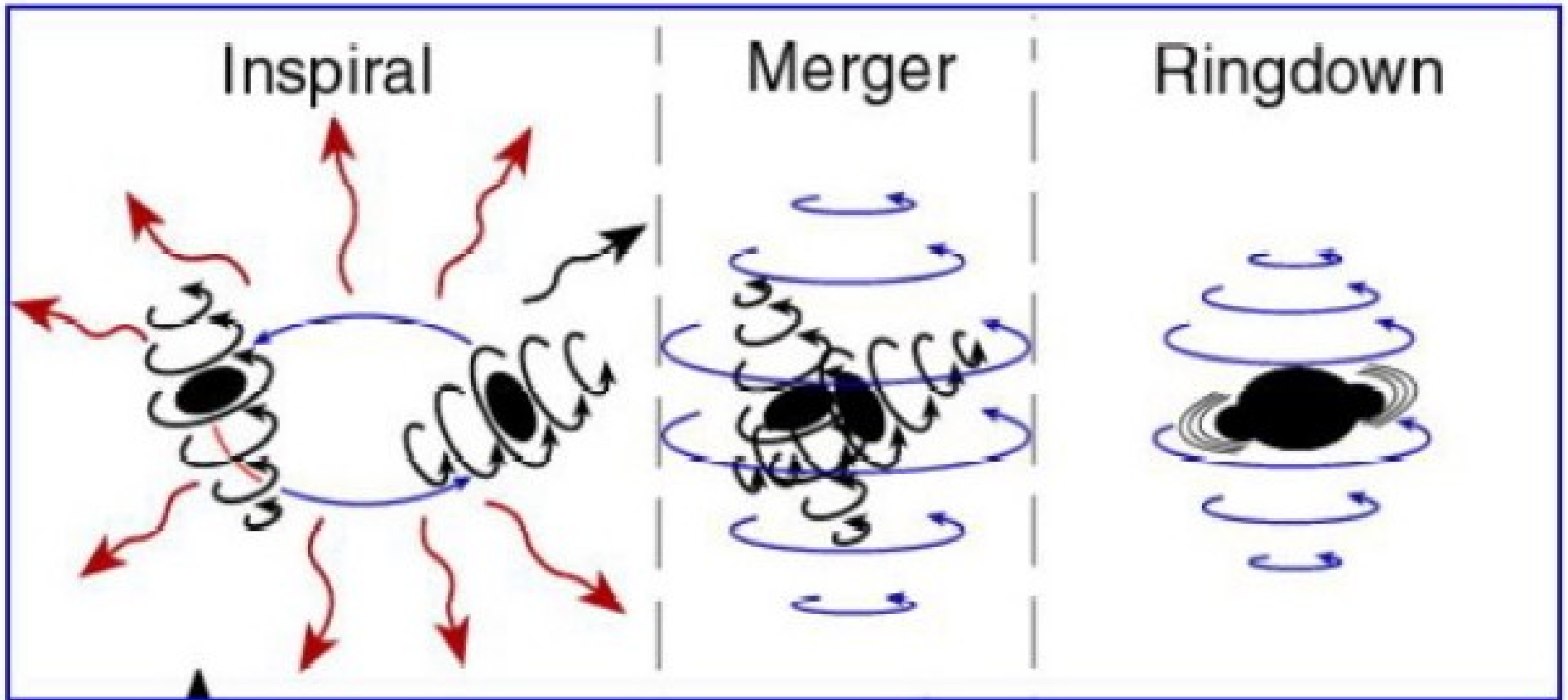
# 1. gravitational wave sources

$$a(t) = a_0 \left[ 1 - \frac{256/5 G^3 m_1 m_2 (m_1 + m_2) t}{c^5 (1 - e^2)^{7/2} a_0^4} \right]^{1/4}$$

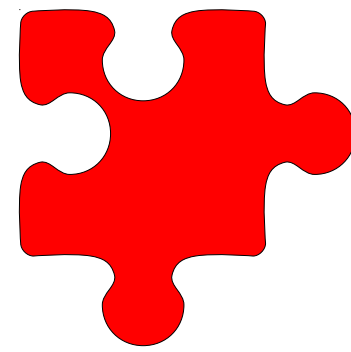
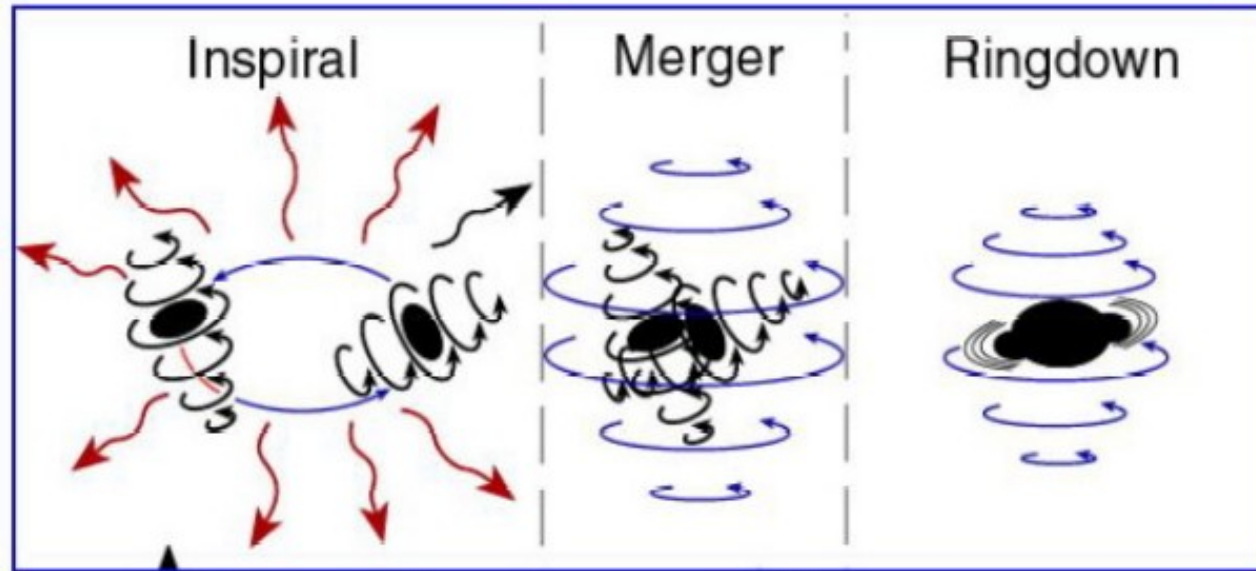
➔ **MERGER TIMESCALE: time for the semi-major axis to shrink to zero due to GW emission**

$$t_{\text{GW}} = \frac{5}{256} \frac{c^5 (1 - e^2)^{7/2} a_0^4}{G^3 m_1 m_2 (m_1 + m_2)}$$

# 1. gravitational wave sources



# 1. gravitational wave sources



***Some back of the envelope calculations:***

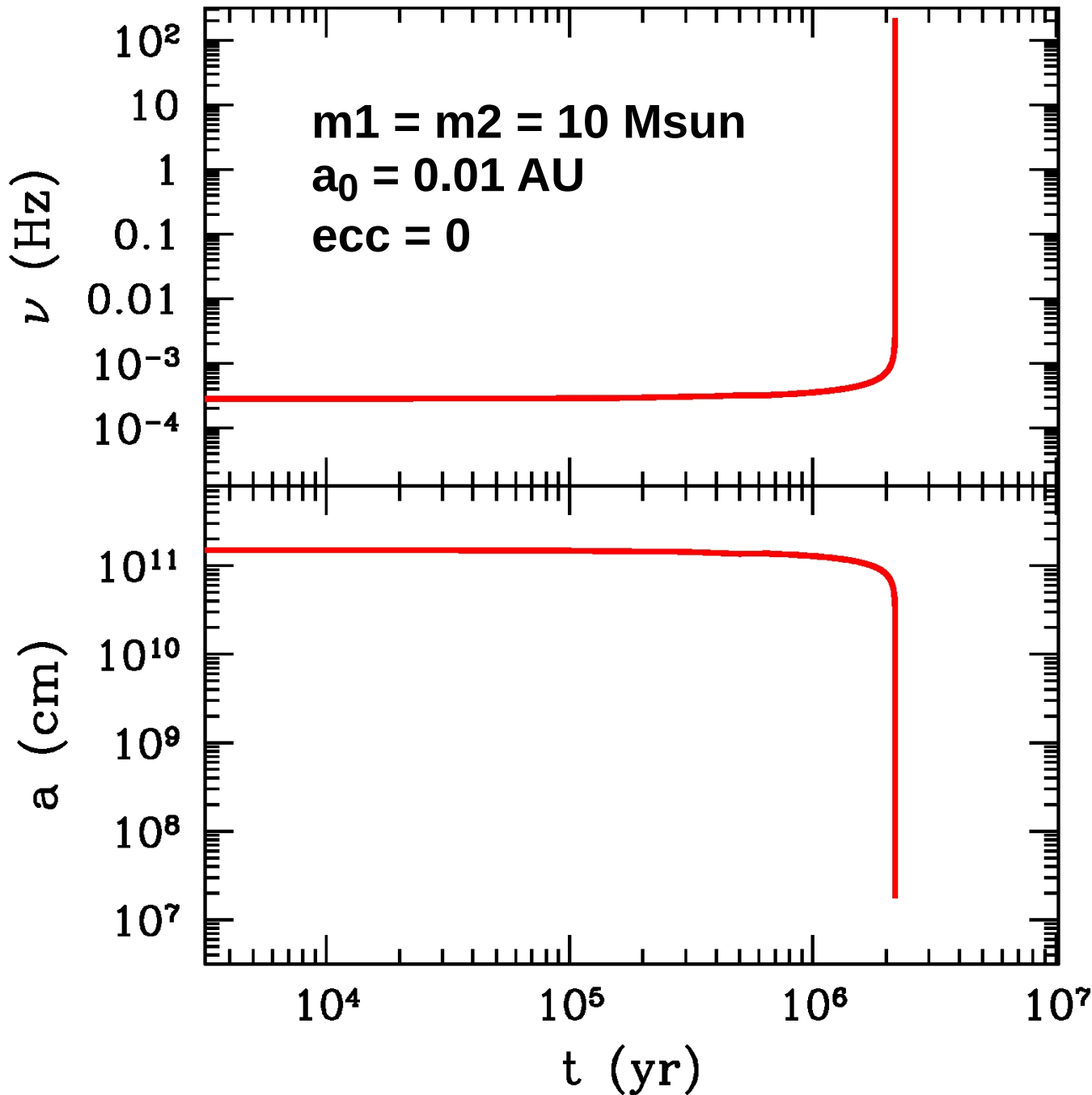
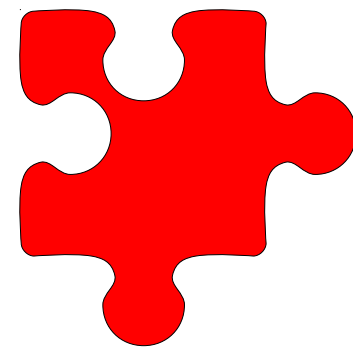
- frequency of gravitational waves

$$\omega_{\text{GW}} = 2 \omega_{\text{orb}}$$

**HIGH-SCHOOL STUDENTS CAN CALCULATE KEPLERIAN  
FREQUENCY!!!**

$$\omega_{\text{orb}} = \sqrt{\frac{G m_{\text{TOT}}}{a^3}}$$

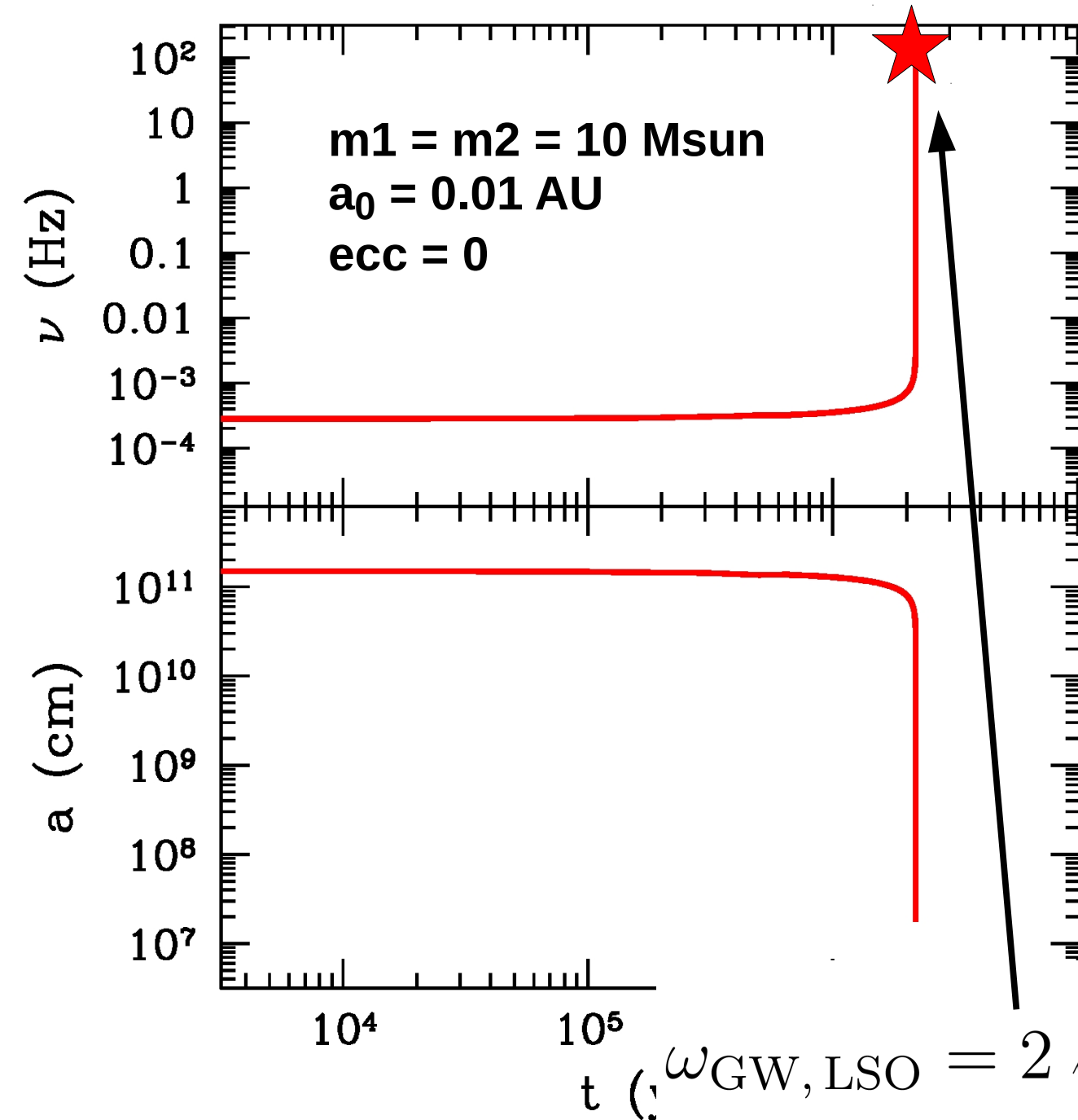
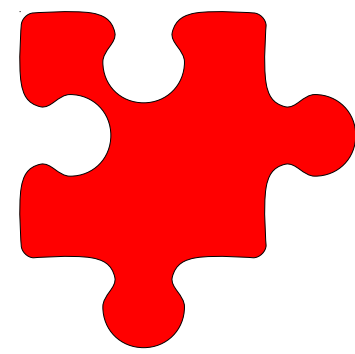
# 1. gravitational wave sources



$$\omega_{\text{GW}} = 2\omega_{\text{orb}}$$

means that when two  
objects  
are FAR away  
GW frequency  
is LOWER

# 1. gravitational wave sources



→ higher frequency just before merger

- last stable orbit

$$r_{\text{LSO}} = 6 GM/c^2$$

- GW frequency at last stable orbit (=end of inspiral)



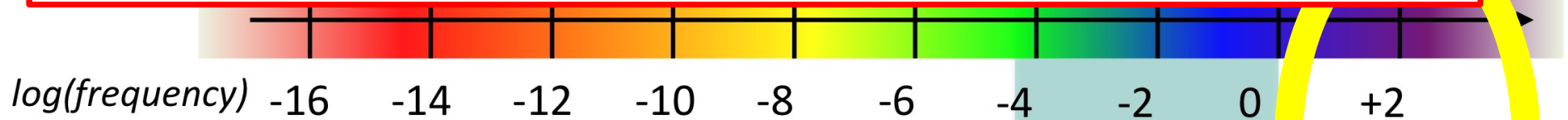
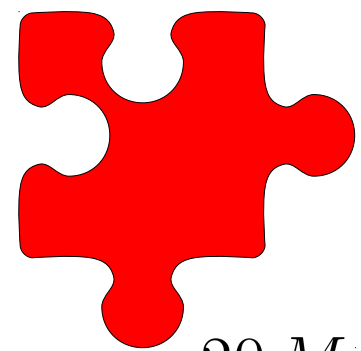
# 1. gravitational wave sources

*Some back of the envelope calculations:*

- GW frequency at last stable orbit (=end of inspiral)

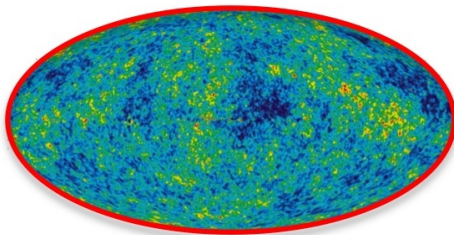
$$\omega_{\text{GW,LSO}} = 2 \sqrt{\frac{G(m_1 + m_2)}{r_{\text{LSO}}^3}} = \frac{2c^3}{6^{3/2} G(m_1 + m_2)} \sim 1385 \text{ s}^{-1} \frac{20 M_\odot}{m_1 + m_2}$$

$$\nu_{\text{GW,LSO}} \sim 220 \text{ Hz} \frac{20 M_\odot}{m_1 + m_2}$$



Detectors

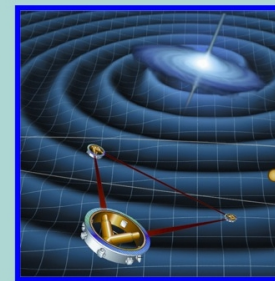
Cosmic microwave background polarization



Pulsar Timing



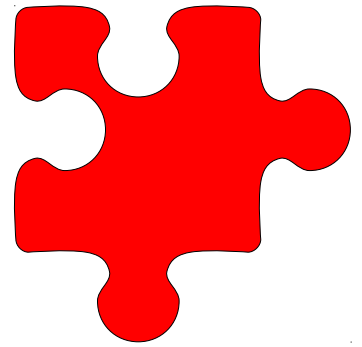
Space Interferometers



Terrestrial interferometers



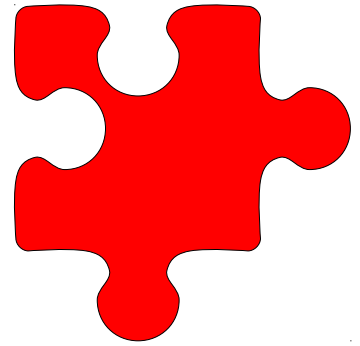
# 1. gravitational wave sources



**WHY 'normal' stars in binaries are not sources of gravitational waves?**

# 1. gravitational wave sources

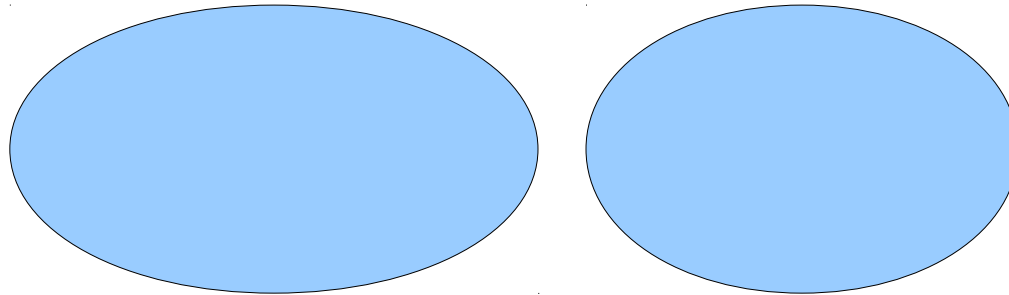
**WHY 'normal' stars in binaries are not sources of GWs?**



**Stars have a much larger radius than NSs and BHs**

→

**When 2 stars are too close, tidal forces between them  
SQUEEZE and DISRUPT THEM**

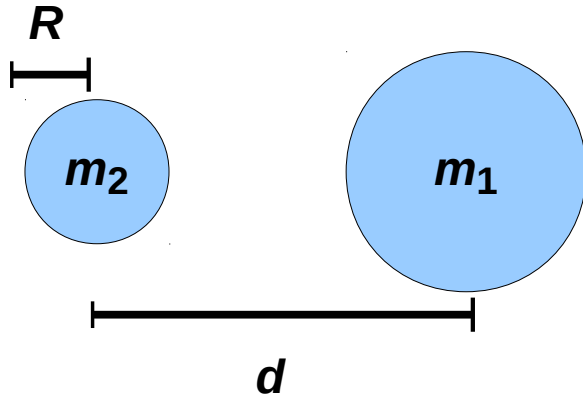


**The 2 stars merge before they emit GWs in the  
LIGO/VIRGO range**

# 1. gravitational wave sources

WITH THE PROPER FORMALISM

Mind the definition of tidal radius



$$r_t \sim d \left( \frac{m_2}{3 m_1} \right)^{1/3}$$

If  $R \geq r_t$  the star is tidally disrupted

$$\omega_{\text{GW}} = 2 \omega_{\text{orb}} = 2 \left[ \frac{G (m_1 + m_2)}{d^3} \right]^{1/2} \sim 2 \left[ \frac{G (m_1 + m_2) m_2}{3 m_1 r_t^3} \right]^{1/2}$$

If  $r_t = R = 1 R_{\text{sun}} = 6.96 \times 10^8 \text{ cm}$ ,  $m_1 = m_2 = 1 M_{\text{sun}}$ ,  
the maximum GW frequency that can be emitted by 2 sun-like stars  
(before tidal disruption) is

$$\omega_{\text{GW}} \sim 3 \times 10^{-4} \text{ Hz}$$

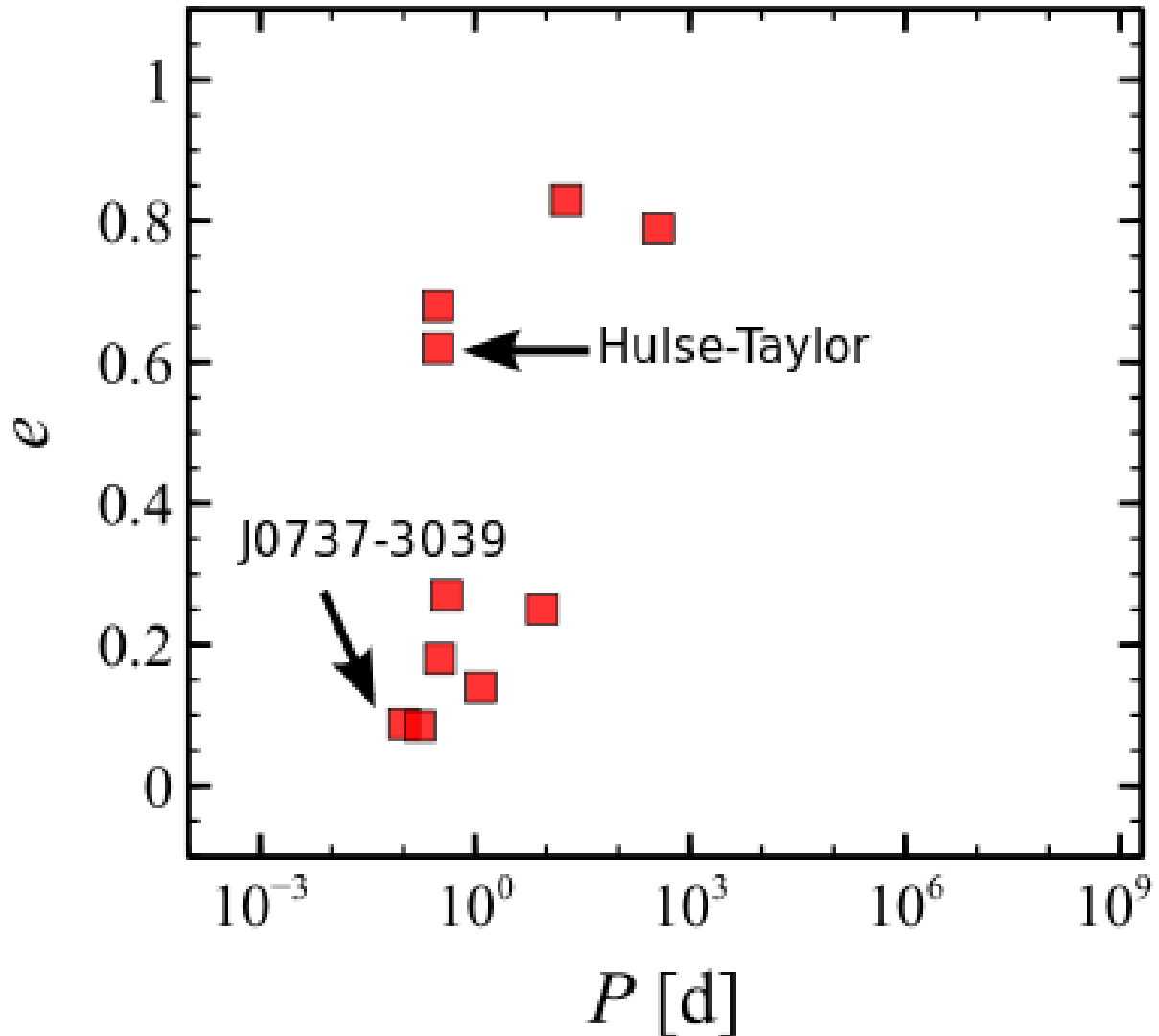
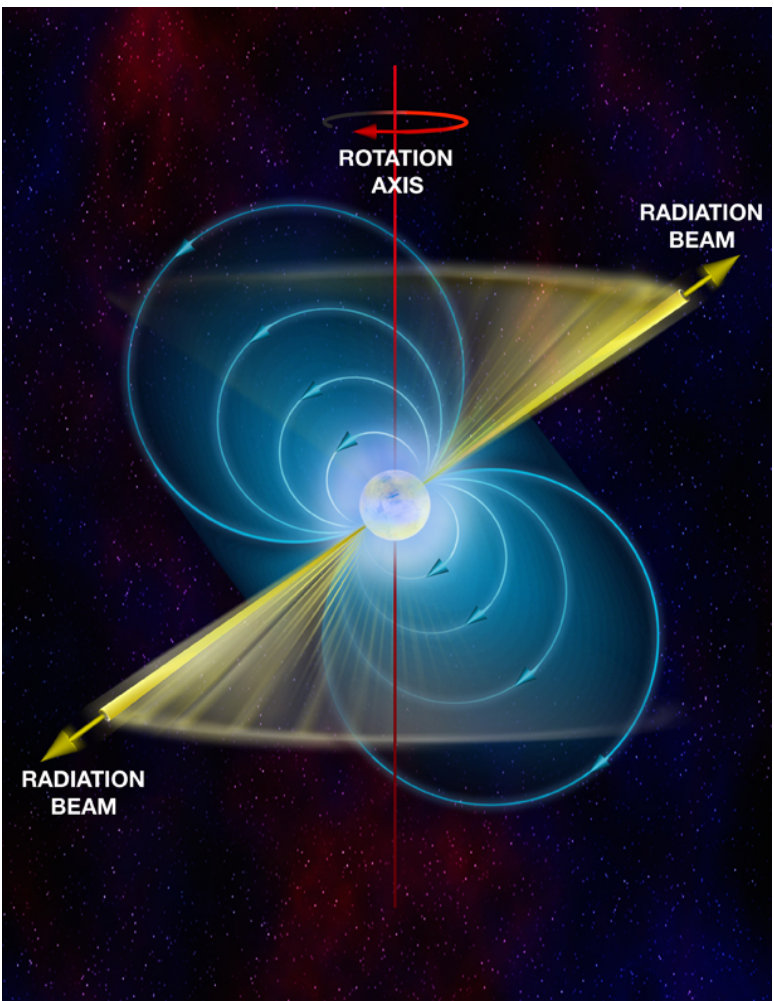
**MUCH LOWER THAN LIGO-VIRGO range!!!**

# 2. possible electromagnetic counterparts precursors

- OBSERVATION OF ~10 NS-NS systems

e.g. D. R. Lorimer 2008,  
Binary and Millisecond Pulsars,  
[arxiv.org/pdf/0811.0762v1.pdf](http://arxiv.org/pdf/0811.0762v1.pdf)

See Niccolò's talk!



CREDIT: Bill Saxton, NRAO/AUI/NSF

From Ziosi, PhD thesis



## 2. possible electromagnetic counterparts precursors

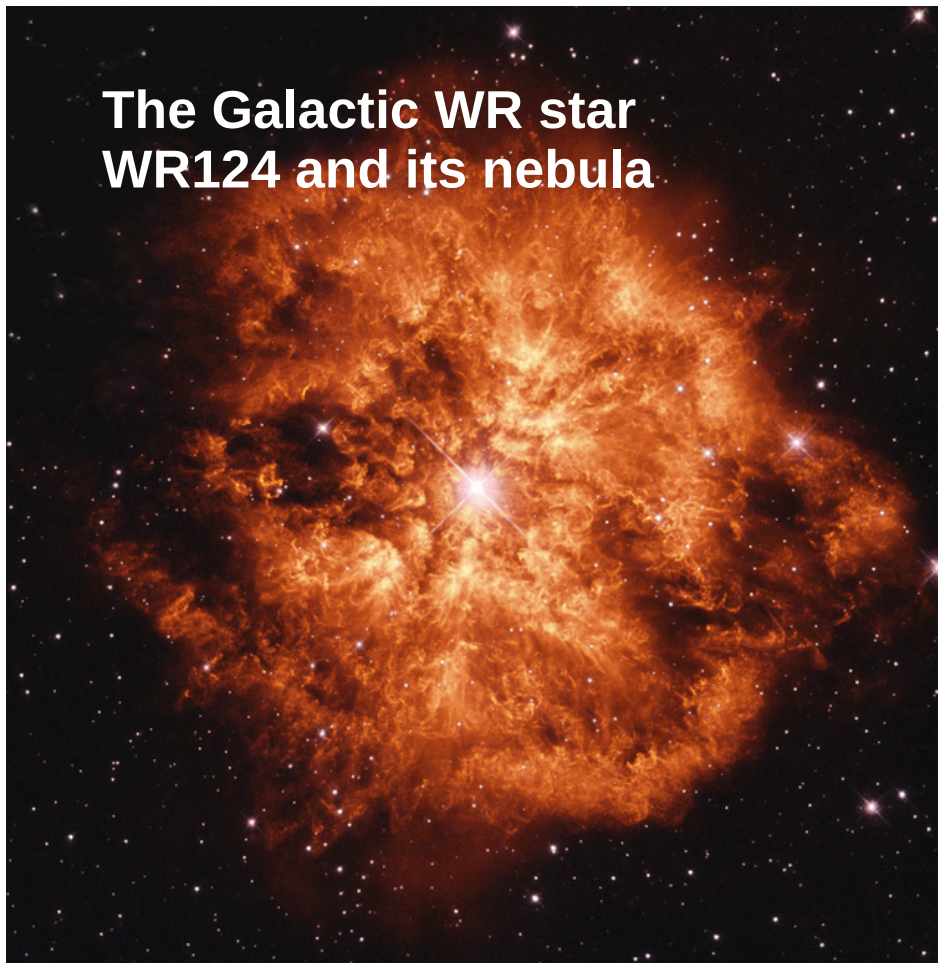
- NO OBSERVATIONS OF STELLAR BH-BH BINARIES

in principle possible only from dynamics signatures (eg a pulsar in a triple BH-BH-NS system)

- BH-WR binaries:

WR stars are naked Helium stars that will end as BH or NS

→ BH-WR are precursor of BH-BH (or possibly BH-NS)



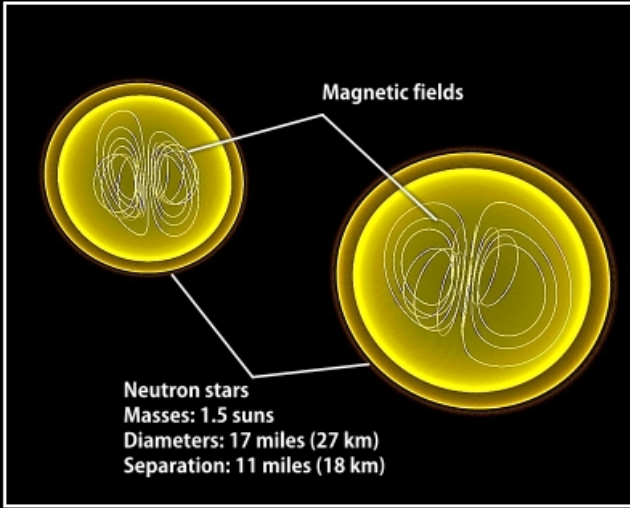
Cartoon of a BH-WR



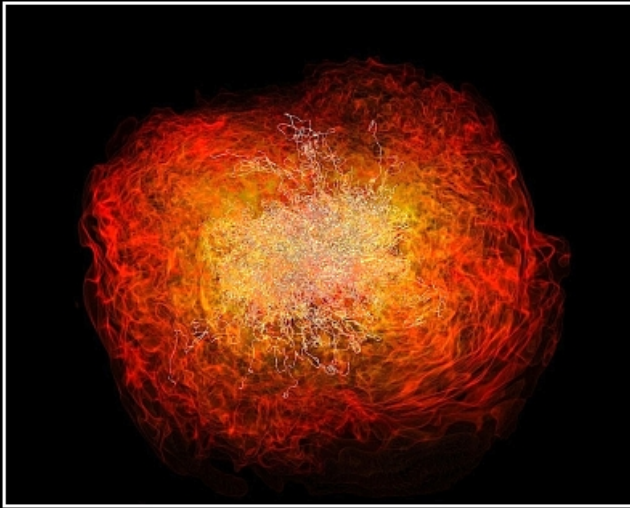
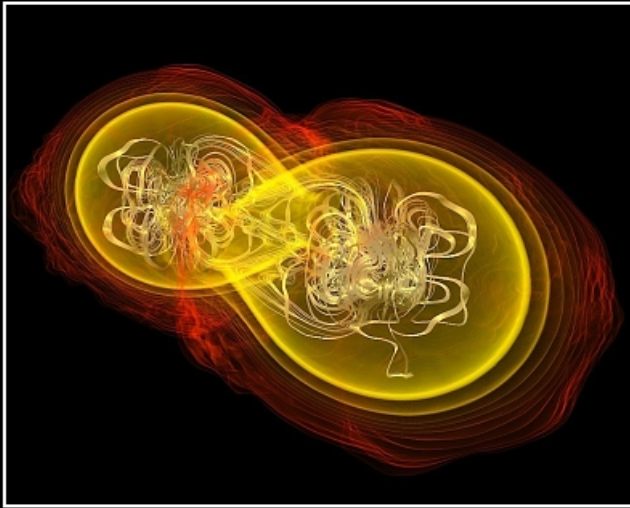
# 2. possible electromagnetic counterparts

The 'Genuine' EM counterparts of NS-NS and NS-BH are SHORT GRBs..

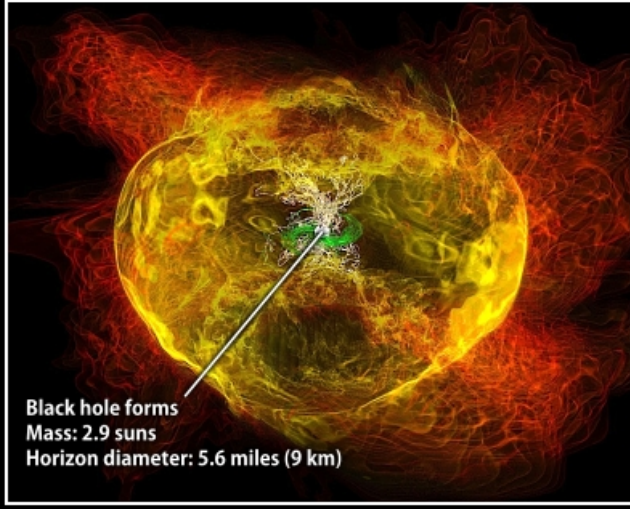
## Crashing neutron stars can make gamma-ray burst jets



Simulation begins



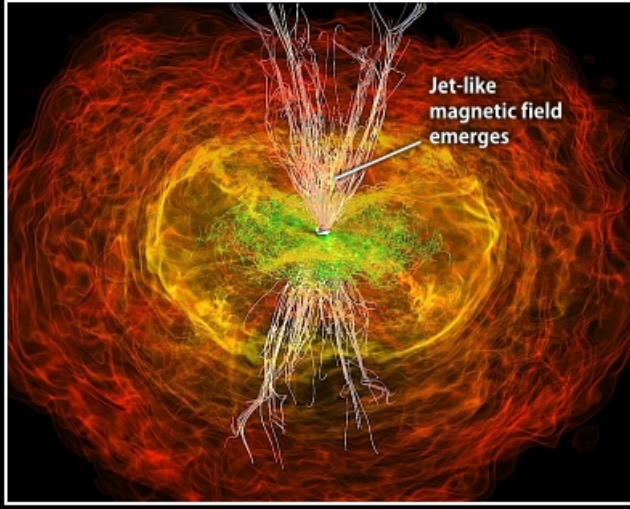
13.8 milliseconds



15.3 milliseconds



21.2 milliseconds



26.5 milliseconds

Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

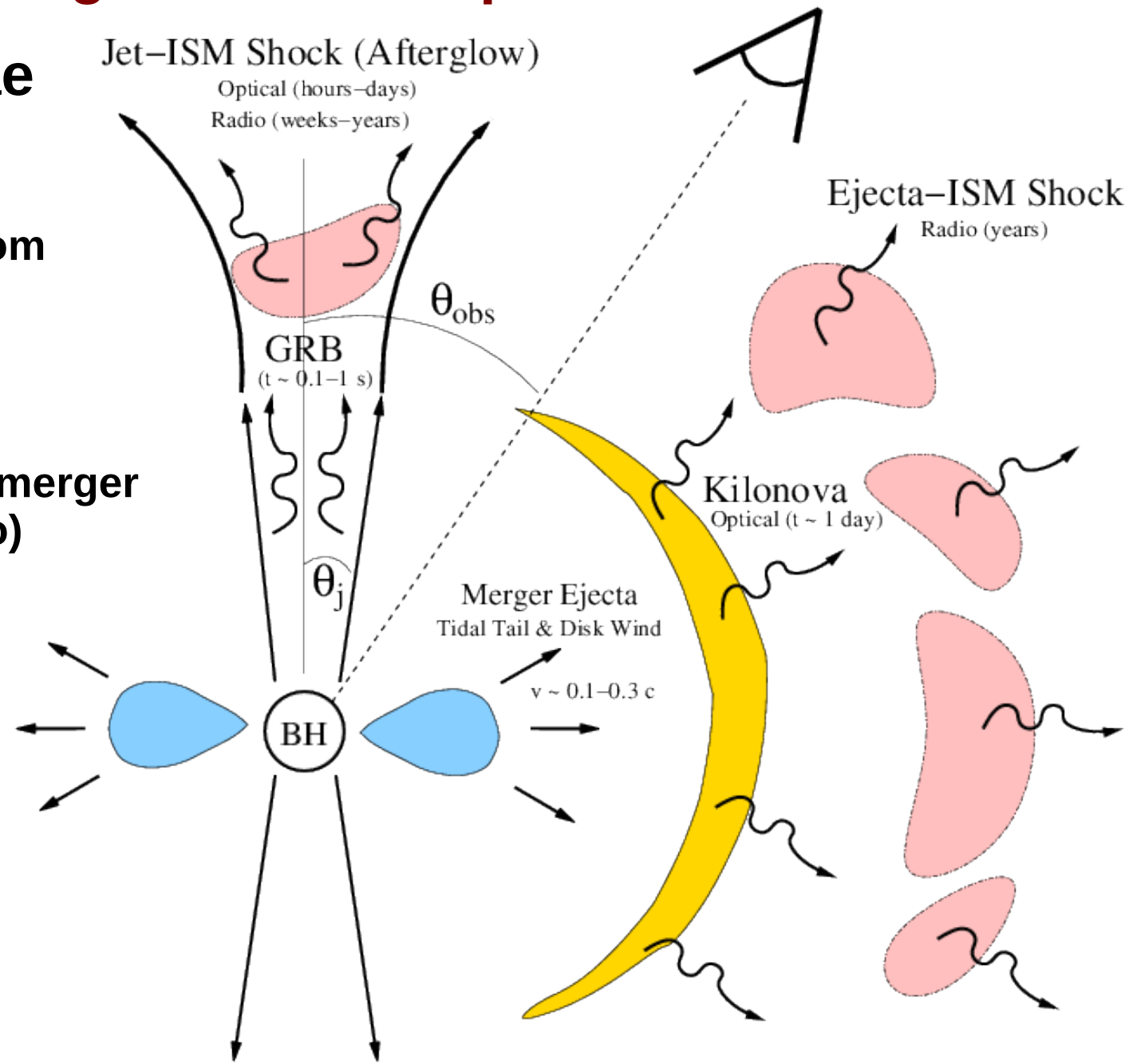


# 2. possible electromagnetic counterparts

...and/or kilonovae

GRB afterglow comes from shocks with ISM

Kilonova comes from radioactive decay of merger ejecta (optical - radio)



## 2. possible electromagnetic counterparts

...and/or kilonovae

### Short GRB:

- gamma-rays (<2 sec) come from relativistic jet (BEAMED!)
- GRB (X-ray to optical) afterglow comes from shocks with ISM
- optical (isotropic) luminosity up to  $\sim 10^{43}$  erg/s for hours-days
- radio emission lasts for weeks-yrs
- $\sim 50$  short GRBs with afterglows observed since 2005

### Kilonova:

- comes from radioactive decay of merger ejecta (optical – radio)
- expected high speed ( $\sim 10^4$  km/s, Kasen+ 2015)
- optical – NIR luminosity up to  $\sim 10^{41}$  erg/s lasts for  $\sim 1$  day
- radio emission lasts for years
- maybe associated with NIR excess observed in some GRB (Tanvir+ 2013, Berger+ 2013)

## **2. possible electromagnetic counterparts**

### **IMPORTANCE of COUNTERPARTS:**

**1. COLLECT MORE INFORMATION ON OBJECTS  
THAT EMIT GWs (e.g. space location )**

**→ see other talks**

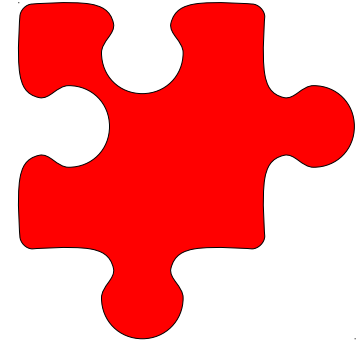
**2. HELP ESTIMATING THE MERGER RATE**

**→ next slides**



### 3. how to estimate the merger rate (and detection rate)

**CALCULATE RATES IS IMPORTANT  
FOR EVERYDAY LIFE  
(even of high-school students)**



**RATE of RAINY DAYS**

e.g. number of rainy days per year in Italy

**A limited time: I have 1 week to do my measurements**

**M**

**T**

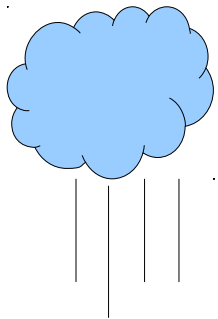
**W**

**T**

**F**

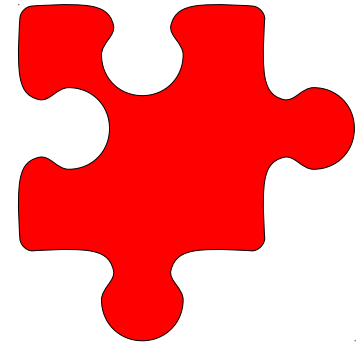
**S**

**S**



### 3. how to estimate the merger rate (and detection rate)

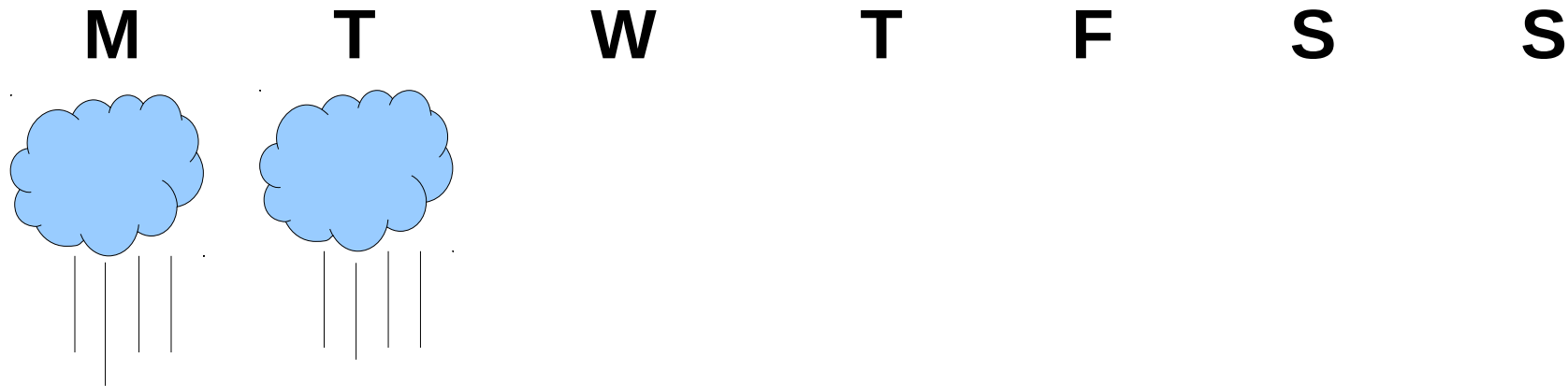
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**RATE of RAINY DAYS**

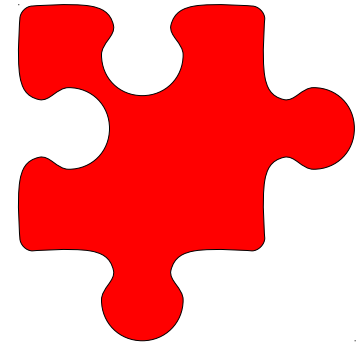
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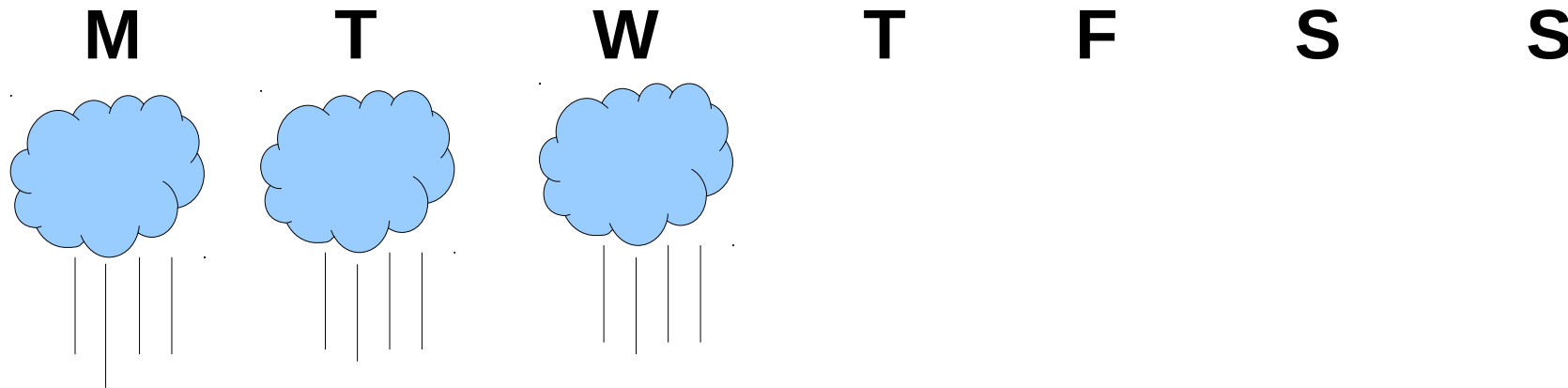
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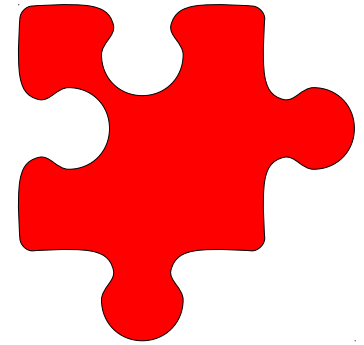
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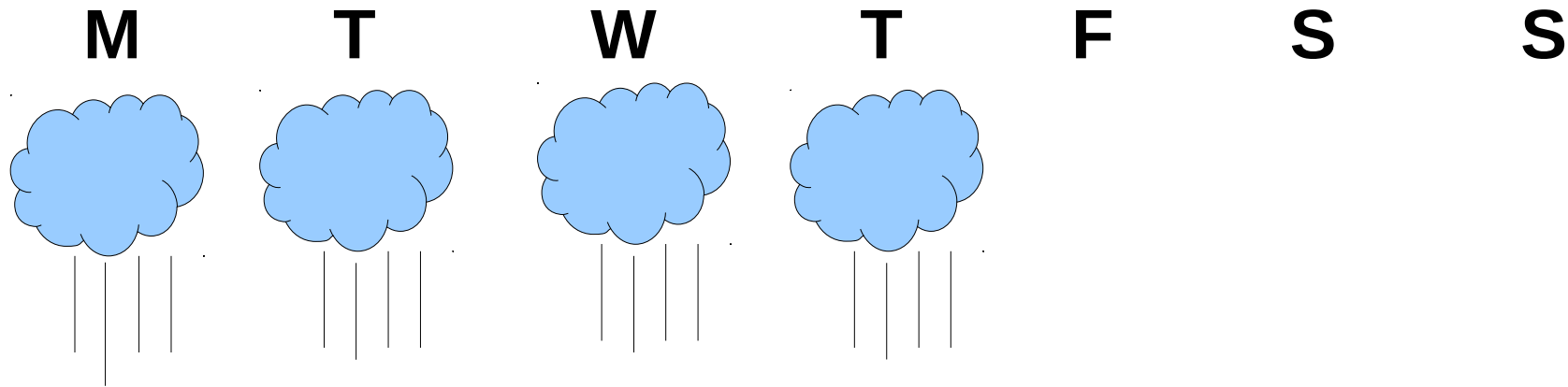
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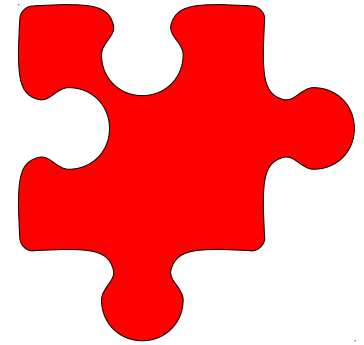
e.g. number of rainy days per year in Italy

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### 3. how to estimate the merger rate (and detection rate)

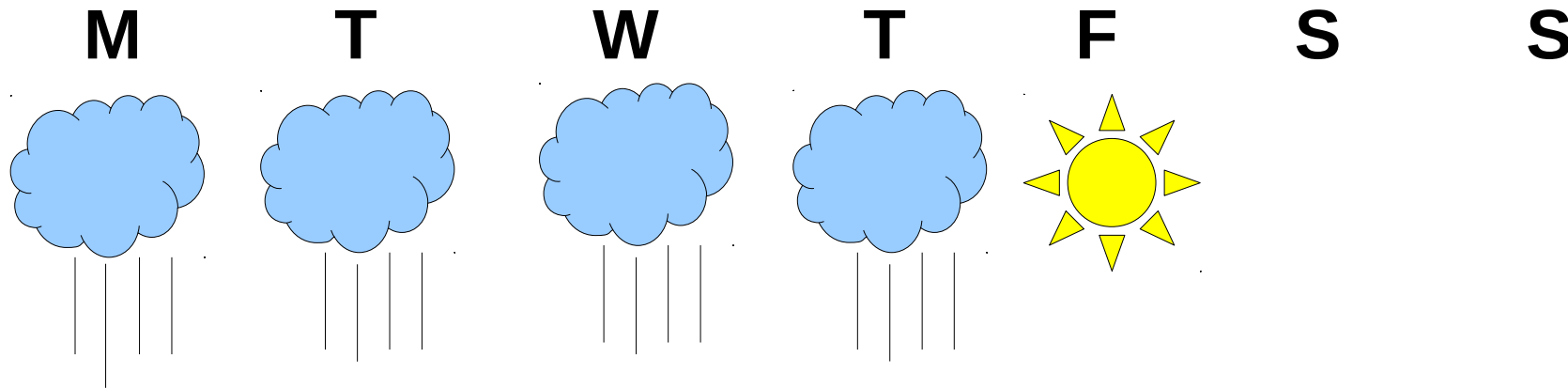
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**RATE of RAINY DAYS**

e.g. number of rainy days per year in Italy

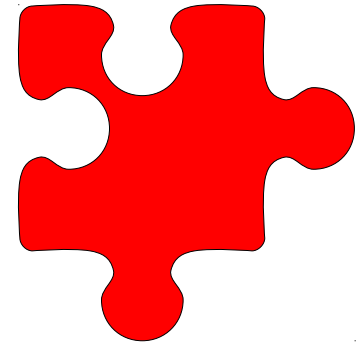
**A limited time: I have 1 week to do my measurements**





### 3. how to estimate the merger rate (and detection rate)

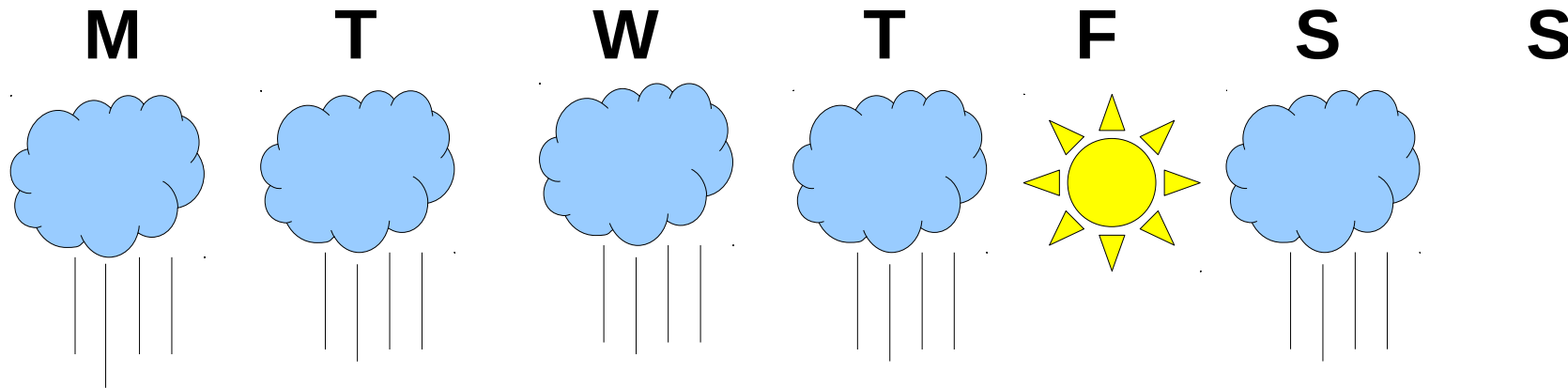
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**RATE of RAINY DAYS**

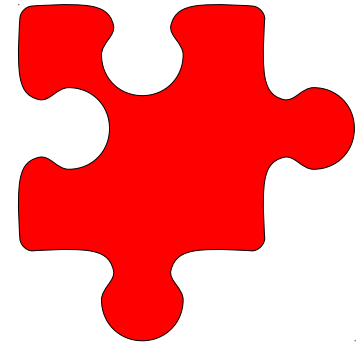
e.g. number of rainy days per year in Italy

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### 3. how to estimate the merger rate (and detection rate)

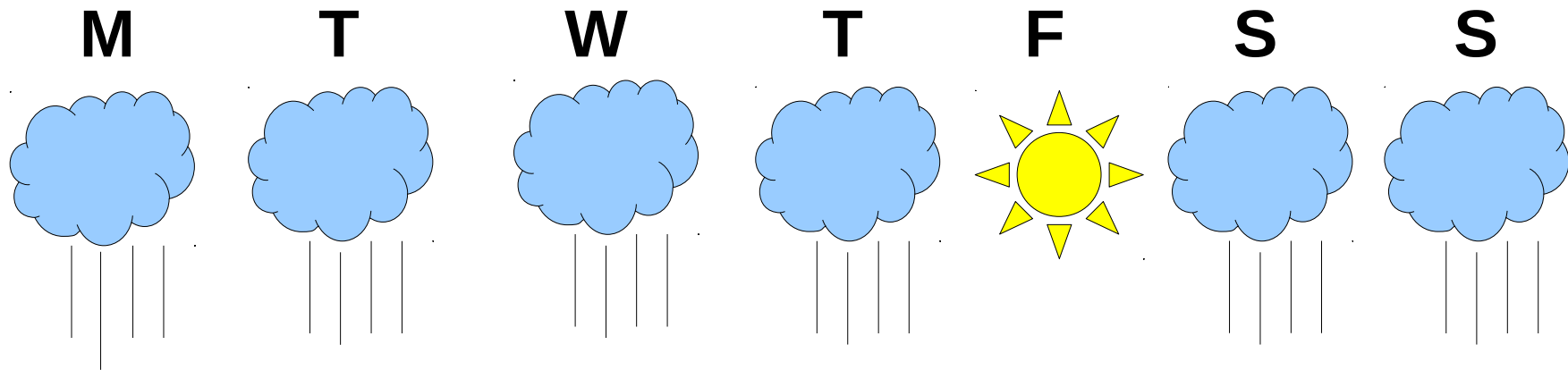
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**RATE of RAINY DAYS**

e.g. number of rainy days per year in Italy

**A limited time: I have 1 week to do my measurements**



**Rate : 6 over 7 days**

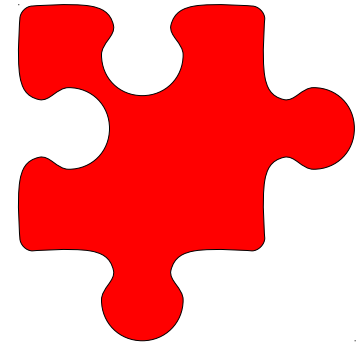
**I assume my week is representative of entire year**

→ extrapolated rate = 312 raining days yr<sup>-1</sup>

**BIAS #1 : my week is in autumn (I can try to correct..)**

### 3. how to estimate the merger rate (and detection rate)

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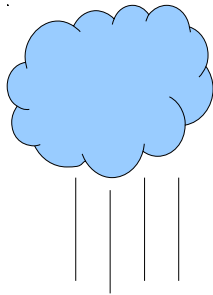


**RATE of RAINY DAYS**

e.g. number of rainy days per year in Italy

**A limited volume: I see only rain in my town, PADOVA**

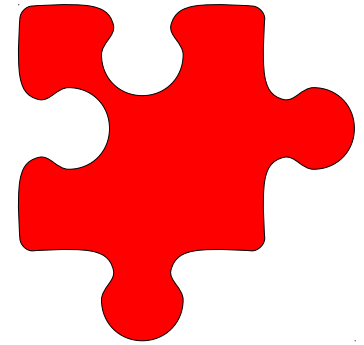
**Padova**



**I assume Padova is representative of entire Italy  
extrapolated rate = 312 raining days yr<sup>-1</sup> in Italy**

### 3. how to estimate the merger rate (and detection rate)

**CALCULATE RATES IS IMPORTANT  
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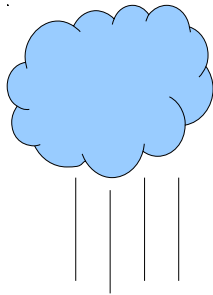


**RATE of RAINY DAYS**

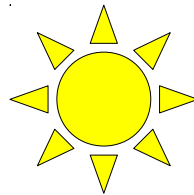
e.g. number of rainy days per year in Italy

**A limited volume: I see only rain in my town, PADOVA**

**Padova**



**Palermo**

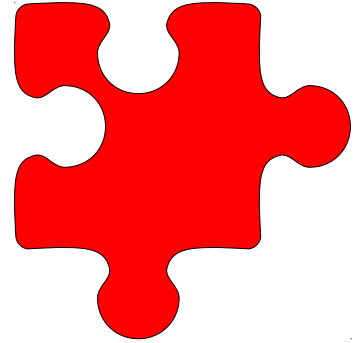


**I assume Padova is representative of entire Italy  
extrapolated rate = 312 raining days yr<sup>-1</sup> in Italy**

**BIAS 2: my town is not representative of entire Italy (I might correct..)**

### 3. how to estimate the merger rate (and detection rate)

**Whenever I have to estimate a RATE**



**The problem is I can collect information only for**

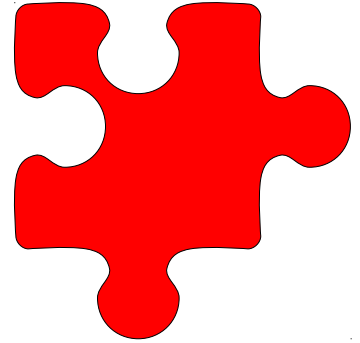
**BIAS 1 - a limited time**

**BIAS 2 - a limited volume**

**→ I need to make some extrapolation / model**

### 3. how to estimate the merger rate (and detection rate)

**EXPECTED RATE of NS-NS MERGERS:**  
Number of NS-NS I expect to merge  
per unit time per unit volume



The problem is I can collect information only for

**BIAS 1 - a limited time**

**BIAS 2 - a limited volume**

**BIAS 3 - a proxy of the quantity I want to measure  
(I cannot measure the quantity I am interested in, but  
something close to)**

→ I need to make some extrapolation / model



### 3. how to estimate the merger rate (and detection rate)

#### FROM OBSERVATIONS

observable quantity	NS-NS merger	BH-NS merger	BH-BH merger
1- number of observed NS-NS	YES	NO	NO
2- short gamma-ray burst rate	YES	MAYBE	NO
3- number of observed BH-WR	NO	MAYBE	YES

### 3. how to estimate the merger rate (and detection rate)

#### From observations

#### 1- number of observed NS-NS binaries:

1. take the PROXY of NS-NS mergers:  
properties of observed NS-NS (semi-major axis, mass, eccentric.)
2. estimate GW merger timescale  $t_{\text{GW}}$  for each of them
3. sum  $1/t_{\text{GW}}$  over all NS NS binaries in Milky Way (MW)

$$R = \sum_i \frac{1}{t_{\text{GW},i}}$$

Units of Time<sup>-1</sup>

Based on observations in a limited TIME (now) and VOLUME (MW)

How can I correct for larger Universe?

- I must do assumptions
- I can choose different approximations (differently affect results)

### 3. how to estimate the merger rate (and detection rate)

#### From observations

#### 1- number of observed NS-NS binaries:

4. I assume number of NS-NS in a region of the Universe depends on the STAR FORMATION RATE (how many stars form in a place in a time)

→ normalize to MW star formation rate ( $\text{SFR}_{\text{MW}} \sim 0.25 \text{ Msun yr}^{-1}$ ) and multiply by density of star formation rate in the local Universe ( $\rho_{\text{SFR}} \sim 0.015 \text{ Msun yr}^{-1} \text{ Mpc}^{-3}$ , Hopkins & Beacom 2006)

$$R = \sum_i \frac{1}{t_{\text{GW},i}} \frac{\rho_{\text{SFR}}}{\text{SFR}_{\text{MW}}}$$

**BUT I might have assumed the number of NS-NS scales with the TOTAL STAR MASS!!!!**

→ Which differences do you expect?

### 3. how to estimate the merger rate (and detection rate)

#### From observations

#### 1- number of observed NS-NS binaries:

4. I assume number of NS-NS in a region of the Universe depends on the STAR FORMATION RATE (how many stars form in a place in a time)

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$$R = \sum_i \frac{1}{t_{\text{GW},i}} \frac{\rho_{\text{SFR}}}{\text{SFR}_{\text{MW}}}$$

Units of  $\text{Time}^{-1} \text{ Volume}^{-1}$

**THIS IS A MERGER RATE PER UNIT VOLUME**

**HOW CAN I DERIVE THE DETECTION RATE (= how many of these mergers can be observed by LIGO/Virgo)?**

### 3. how to estimate the merger rate (and detection rate)

From observations

1- number of observed NS-NS binaries:

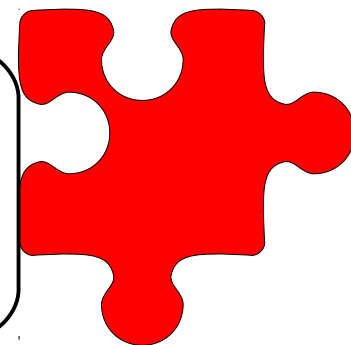
5. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS

$V \sim \frac{4}{3} \pi L^3$  (with  $L=200$  Mpc)

YOU GET THE DETECTION RATE:

$$R = \sum_i \frac{1}{t_{\text{GW},i}} \frac{\rho_{\text{SFR}}}{\text{SFR}_{\text{MW}}} \frac{4}{3} \pi L^3$$

**CALCULATE  $R$  – EXERCISE FOR SCHOOL STUDENTS**  
(give them the properties of observed NS-NS to derive  $t_{\text{GW}}$ ,  
 $\rho_{\text{SFR}}$ ,  $\text{SFR}_{\text{MW}}$  and  $L$  are known values.)



### 3. how to estimate the merger rate (and detection rate)

From observations

1- number of observed NS-NS binaries:

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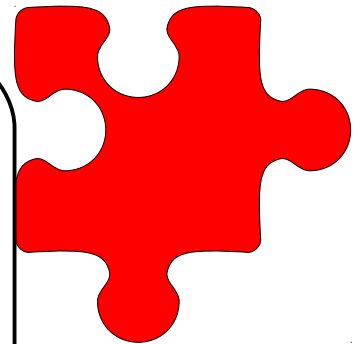
YOU GET THE DETECTION RATE:

$$R = \sum_i \frac{1}{t_{\text{GW},i}} \frac{\rho_{\text{SFR}}}{\text{SFR}_{\text{MW}}} \frac{4}{3} \pi L^3$$

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AND THE ANSWER IS ...

0.4 yr<sup>-1</sup> with ADVANCED LIGO/VIRGO!





### 3. how to estimate the merger rate (and detection rate)

From observations

1- number of observed NS-NS binaries:

5. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS

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YOU GET THE DETECTION RATE:

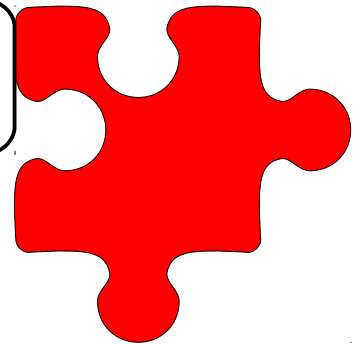
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**0.4 yr<sup>-1</sup> with ADVANCED LIGO/VIRGO!**

**CLOSE TO PESSIMISTIC ESTIMATE FOR ADV. LIGO/VIRGO  
(0.4 – 400 yr<sup>-1</sup> – uncertainty of 3 orders of magnitude!!!!)**

**BUT THERE ARE MORE BIASES TO ACCOUNT FOR**

**AND THERE ARE SEVERAL PROCEDURES SIMILAR TO THIS ONE!!!**



### 3. how to estimate the merger rate (and detection rate)

#### From observations

#### 2- short gamma ray burst rate

gamma ray burst <2 s thought to be produced by NS-NS or NS-BH merger

1. take observed short gamma-ray burst rate  $R_{\text{GRB}}$
2. correct for beaming (we see only short  $\gamma$ -ray bursts pointing toward us)  
 $\sim (1 - \cos\theta)^{-1}$
3. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS  
 $V \sim \frac{4}{3} \pi L^3$  (with  $L=200$  Mpc)

**YOU GET THE DETECTION RATE:**

$$R = R_{\text{GRB}} (1 - \cos\theta)^{-1} \frac{4}{3} \pi L^3$$

**EASY BUT ASSUMPTION that gamma-ray burst means merger**

(e.g. Coward+2012; Siellez+ 2014)

### 3. how to estimate the merger rate (and detection rate)

#### From observations

#### 3- BH-WR binaries:

WR stars are naked Helium stars that will end as BH or NS

→ BH-WR are precursor of BH-BH (or possibly BH-NS)

Calculation similar to NS-NS binaries:

1. take properties of observed BH-WR (semi-major axis, mass, eccentric.)
2. estimate GW merger timescale  $t_{\text{GW},i}$  for each of them

$$R = \sum_i \frac{1}{t_{\text{GW},i}}$$

Units of Time<sup>-1</sup>

### 3. how to estimate the merger rate (and detection rate)

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3. normalize to star formation rate of their host galaxy ( $\text{SFR}_i$ )

& sum  $1/(\text{SFR}_i t_{\text{GW},i})$  over all BH-WR in local Universe

$$R = \sum_i \frac{1}{t_{\text{GW},i} \text{SFR}_i}$$

Units of  $\text{Time}^{-1} \text{Mass}^{-1} \text{Time}$

# 3. how to estimate the merger rate (and detection rate)

## From observations

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$$R = \sum_i \frac{1}{t_{\text{GW},i} \text{SFR}_i} \rho_{\text{SFR}}$$

Units of  $\text{Time}^{-1} \text{ Volume}^{-1}$

**THIS IS A MERGER RATE PER UNIT VOLUME**

**HOW CAN I DERIVE THE DETECTION RATE ?**

# 3. how to estimate the merger rate (and detection rate)

## From observations

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$$R = \sum_i \frac{1}{t_{\text{GW},i} \text{SFR}_i} \rho_{\text{SFR}} \frac{4}{3} \pi L^3$$



### 3. how to estimate the merger rate (and detection rate)

From observations

3- BH-WR binaries:

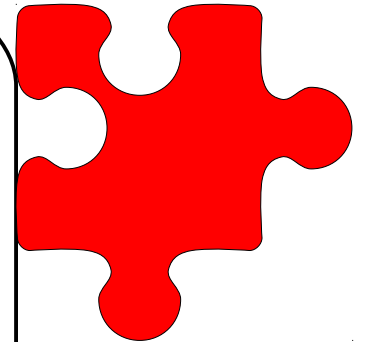
$$R = \sum_i \frac{1}{t_{\text{GW},i} \text{SFR}_i} \rho_{\text{SFR}} \frac{4}{3} \pi L^3$$

CALCULATE  $R$  – EXERCISE FOR SCHOOL STUDENTS  
(give them the properties of observed BH-WR to derive  $t_{\text{GW}}$ ,  
 $\rho_{\text{SFR}}$ ,  $\text{SFR}_{\text{MW}}$  and  $L$  are known values.)

AND THE ANSWER IS....  $R < 10 \text{ yr}^{-1}$

Upper limit because  
we assume SNe do not  
unbind binaries

See Esposito+ 2015 for details



### 3. how to estimate the merger rate (and detection rate)

From theory

Coming back to the rain metaphor:

I decide not to look at my windows,  
sit down to my computer,  
and develop a climate model of Italy!!!!!!

1. no time limited

2. no volume limited

3. I can model what I really  
want to measure..



### 3. how to estimate the merger rate (and detection rate)

From theory

Method	NS-NS merger	BH-NS merger	BH-BH merger
1- population synthesis simulations	YES	YES	YES
2- dynamical simulations (Monte Carlo or N-body)	YES	YES	YES

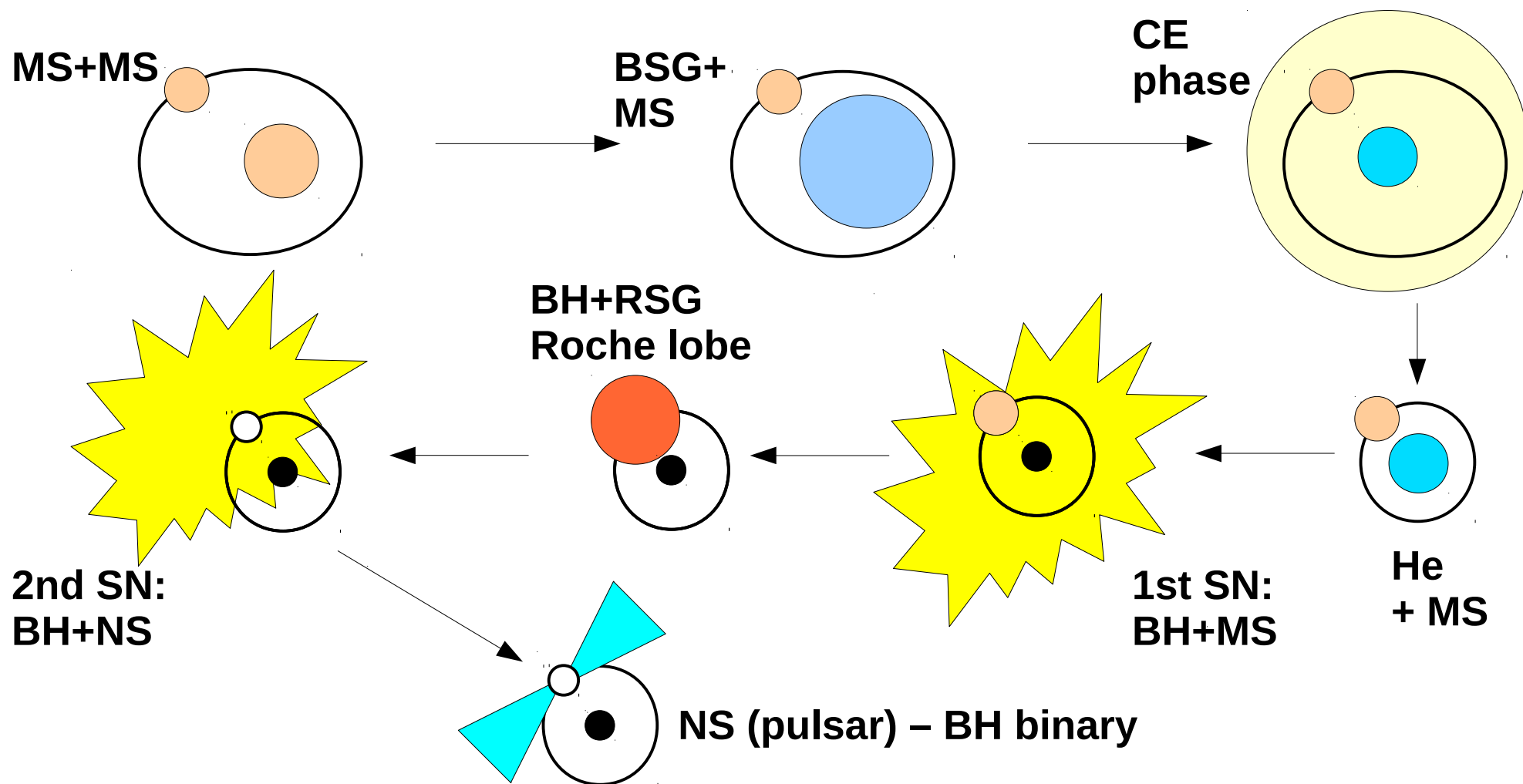
Theory seems better than observations, but.....

### 3. how to estimate the merger rate (and detection rate)

From theory

#### 1- population synthesis simulations

Codes that evolve a binary of massive stars (stellar evolution + orbital evolution) until it forms a NS-NS or NS-BH or BH-BH



### 3. how to estimate the merger rate (and detection rate)

From theory

#### 1- population synthesis simulations

Codes that evolve a binary of massive stars (stellar evolution + orbital evolution) until it forms a NS-NS or NS-BH or BH-BH



### **3. how to estimate the merger rate (and detection rate)**

#### **From theory**

#### **1- population synthesis simulations**

**Codes that evolve a binary of massive stars (stellar evolution + orbital evolution) until it forms a NS-NS or NS-BH or BH-BH**

- 1. simulate a large grid of ISOLATED binaries (e.g. equal to total stellar mass of the Milky Way)**
- 2. extract the number of mergers of NS-NS, NS-BH or BH-BH in the simulations in a Hubble time:  
this gives the merger rate of the Milky Way**
- 3. either normalize to the SFR of the MW and multiply by the SFR density (see description about NS-NS observations)  
or normalize to the MW mass and multiply by the mass density of galaxies in the local Universe**
- 4. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS, NS-BH, BH-BH**



### 3. how to estimate the merger rate (and detection rate)

From theory

2- dynamical simulations (Monte Carlo or N-body)

**WHY**

**DYNAMICS????????**

## 4. impact of environment on merger rate

**WHY DYNAMICS???????**

**COLLISIONAL/COLLISIONLESS**

- **Collisional systems** are systems where interactions between stars are EFFICIENT with respect to the lifetime of the system

- **Collisionless systems** are systems where interactions are negligible

**When is a stellar system collisional/collisionless?**

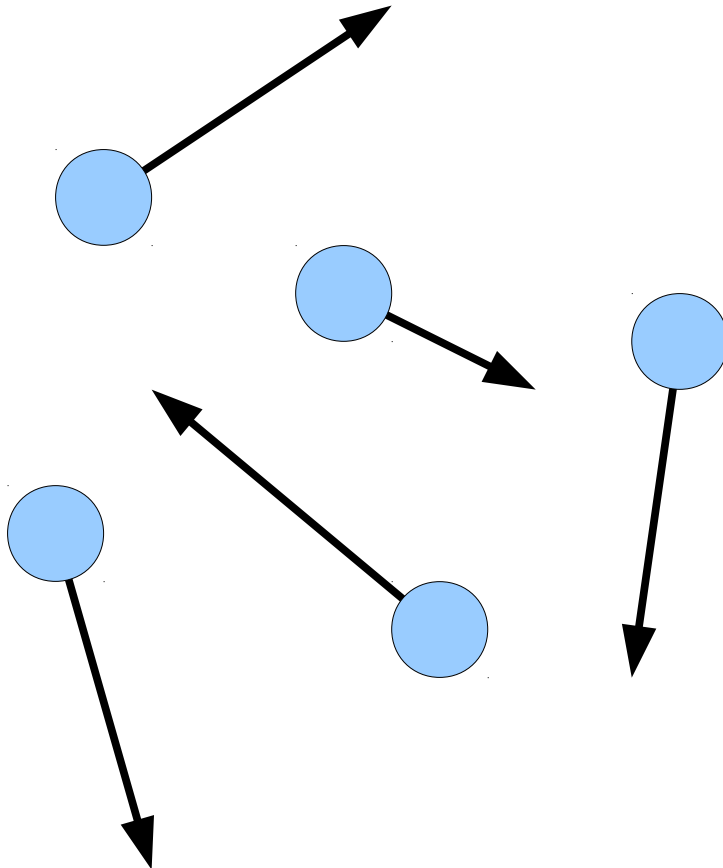
## 4. impact of environment on merger rate

### COLLISIONAL/COLLISIONLESS

#### COLLISIONLESS systems:

astrophysical systems where the **stellar density is low**

→ gravitational interactions between stars are weak and rare, and do not affect the evolution of the system



Interaction  
Rate  
scales as

density / vel<sup>3</sup>

## 4. impact of environment on merger rate

### COLLISIONAL/COLLISIONLESS

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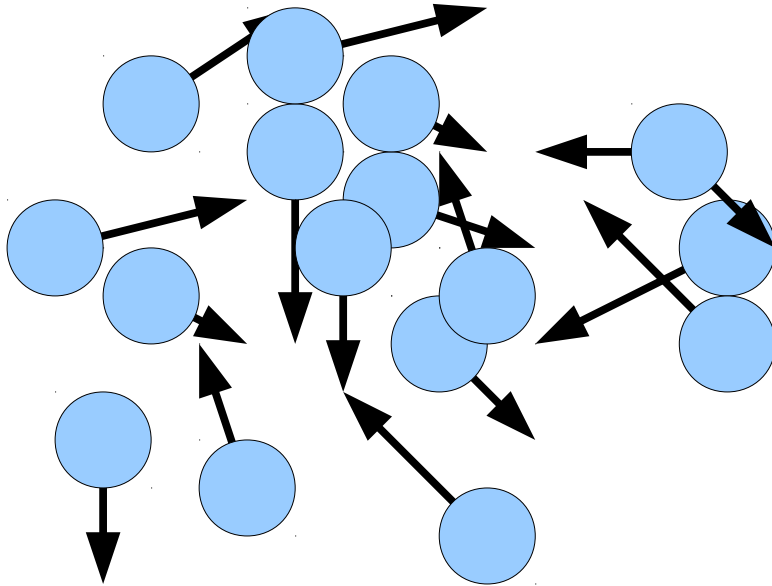
e.g. GALAXIES are COLLISIONLESS SYSTEMS



## 4. impact of environment on merger rate

### COLLISIONAL/COLLISIONLESS

**COLLISIONAL systems:** SYSTEMS WHERE the stellar **DENSITY** is so high that single gravitational interactions between particles are frequent, strong and affect the overall evolution of system



Interaction  
Rate  
scales as

density /  $vel^3$

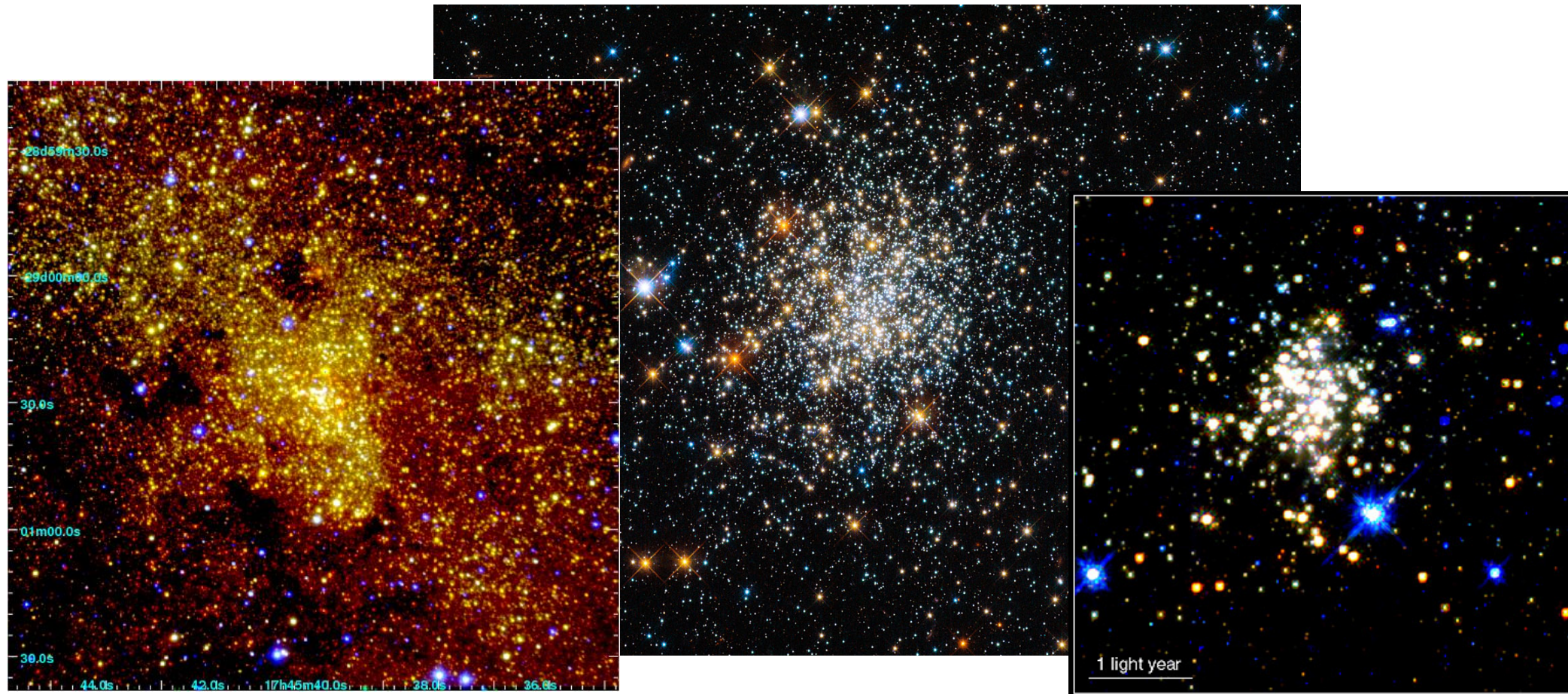


## 4. impact of environment on merger rate

### COLLISIONAL/COLLISIONLESS

**COLLISIONAL systems:** SYSTEMS WHERE the stellar **DENSITY** is so high that single gravitational interactions between particles are frequent, strong and affect the overall evolution of system

### THE DENSEST STELLAR SYSTEMS: STAR CLUSTERS and GALACTIC NUCLEI



#### 4. impact of environment on merger rate

## **BINARIES as ENERGY RESERVOIR**

Binaries have a energy reservoir (their internal energy) that can be exchanged with stars.

INTERNAL ENERGY: 
$$E_{int} = -\frac{G m_1 m_2}{2 a} = -E_b$$

where  $E_b$  is the **BINDING ENERGY** of the binary.

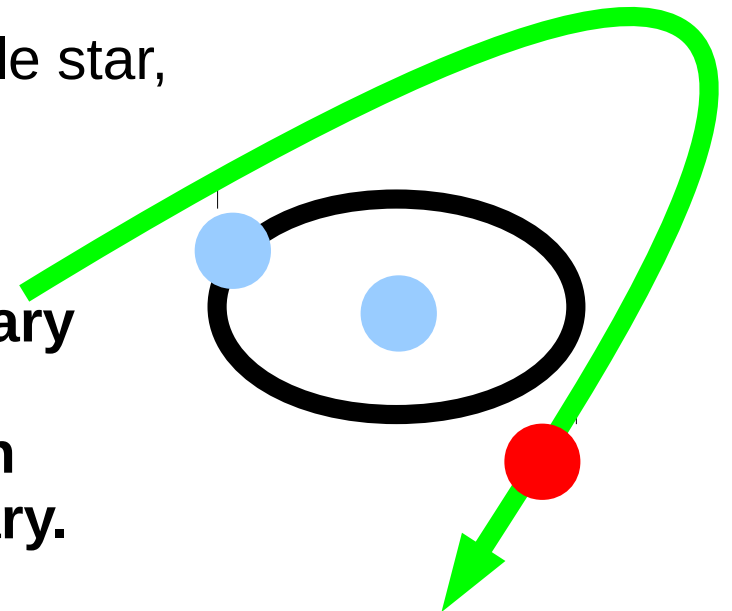
***THE ENERGY RESERVOIR of BINARIES can be EXCHANGED with stars:***

during a **3-BODY INTERACTION**,  
i.e. an interaction between a binary and a single star,

the single star can either

**EXTRACT INTERNAL ENERGY** from the binary

or lose a fraction of its kinetic energy, which  
is converted into internal energy of the binary.



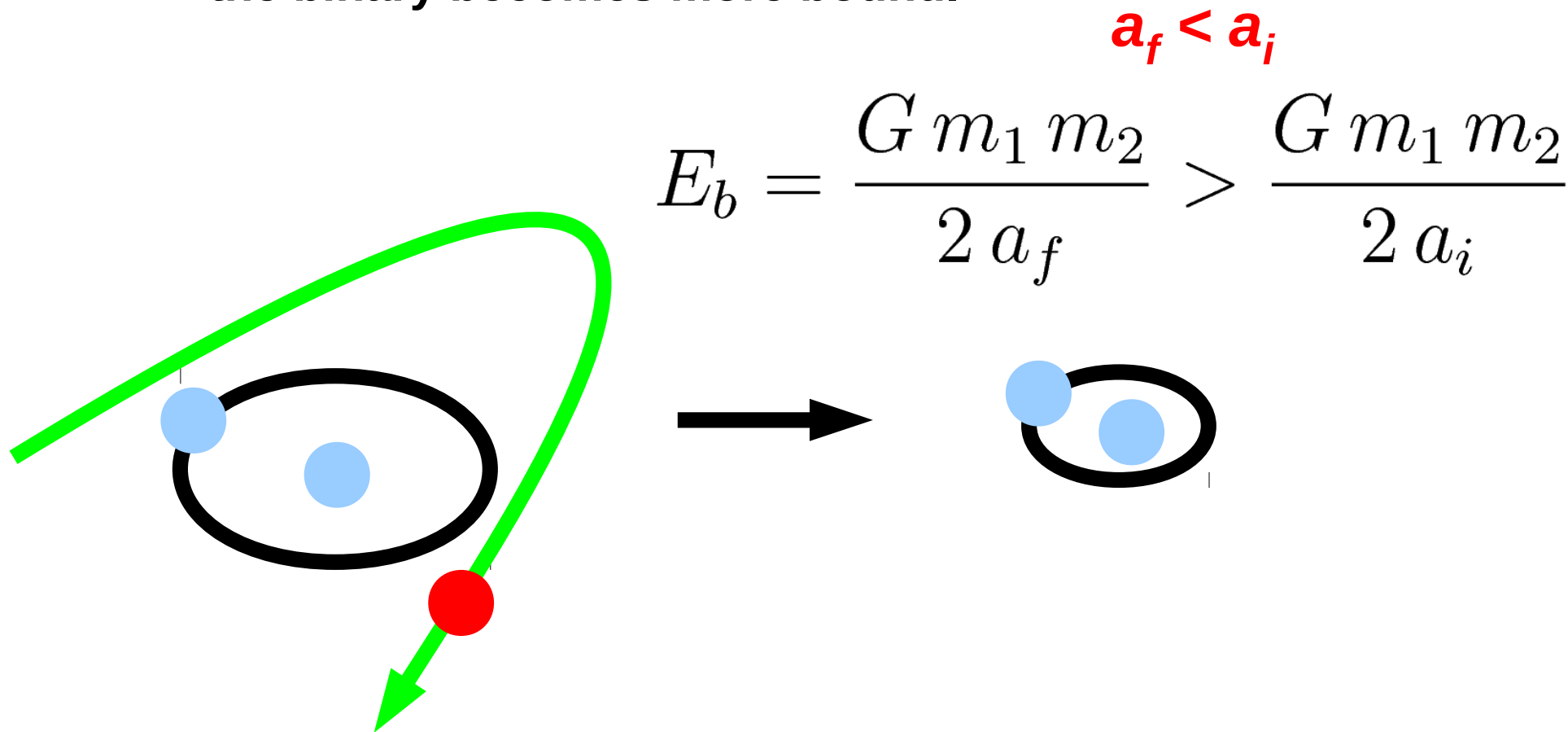
#### 4. impact of environment on merger rate

## BINARIES as ENERGY RESERVOIR

If the star extracts  $E_{int}$  from the binary,

1) final kinetic energy of star  $>$  initial kinetic energy.  
STAR and BINARY acquire **RECOIL VELOCITY**

2)  $E_{int}$  becomes more negative, i.e.  $E_b$  higher:  
the binary becomes more bound.



CARTOON of a FLYBY ENCOUNTER where  $a_f < a_i \rightarrow E_b$  increases



#### 4. impact of environment on merger rate

## BINARIES as ENERGY RESERVOIR

If the star transfer kinetic energy to the binary,

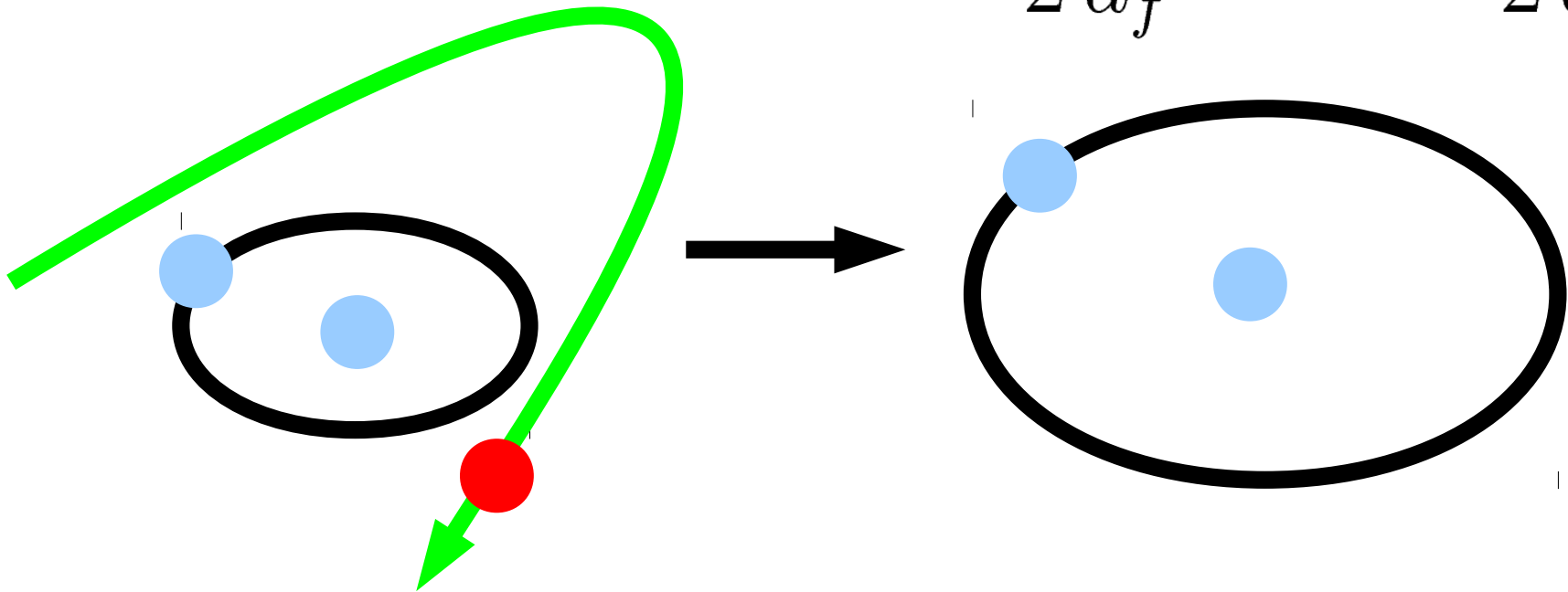
1) final kinetic energy of star < initial kinetic energy.

2)  $E_{int}$  becomes less negative, i.e.  $E_b$  smaller:

the binary becomes less bound

or is even **IONIZED** (:= becomes UNBOUND).

$$E_b = \frac{G m_1 m_2}{2 a_f} < \frac{G m_1 m_2}{2 a_i} \quad a_f > a_i$$



CARTOON of a FLYBY ENCOUNTER where  $a_f > a_i \rightarrow E_b$  decreases

#### 4. impact of environment on merger rate

### **BINARIES as ENERGY RESERVOIR**

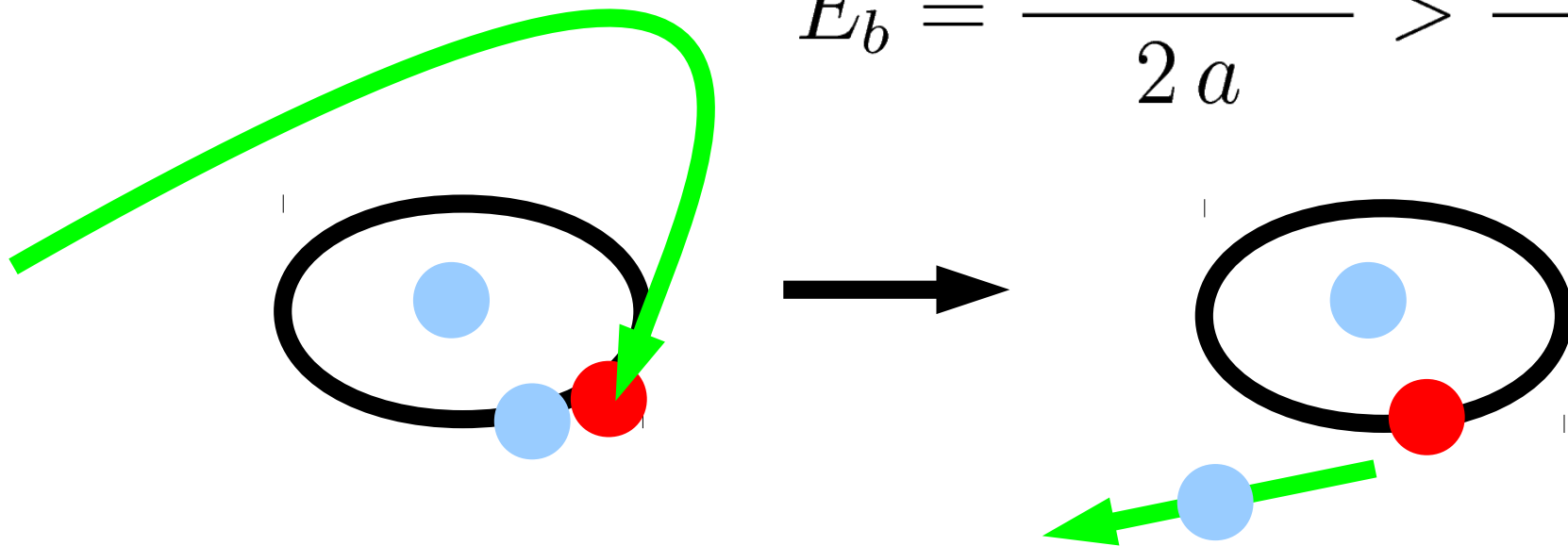
Alternative way for a binary to transfer internal energy to field stars:

### **EXCHANGE**

**the single star replaces one of the former members of the binary.**

An exchange interaction is favoured when the mass of the single star  $m_3$  is HIGHER than the mass of one of the members of the binary so that the new  $E_b$  of the binary is higher than the former:

$$E_b = \frac{G m_1 m_3}{2 a} > \frac{G m_1 m_2}{2 a} \quad m_3 > m_2$$

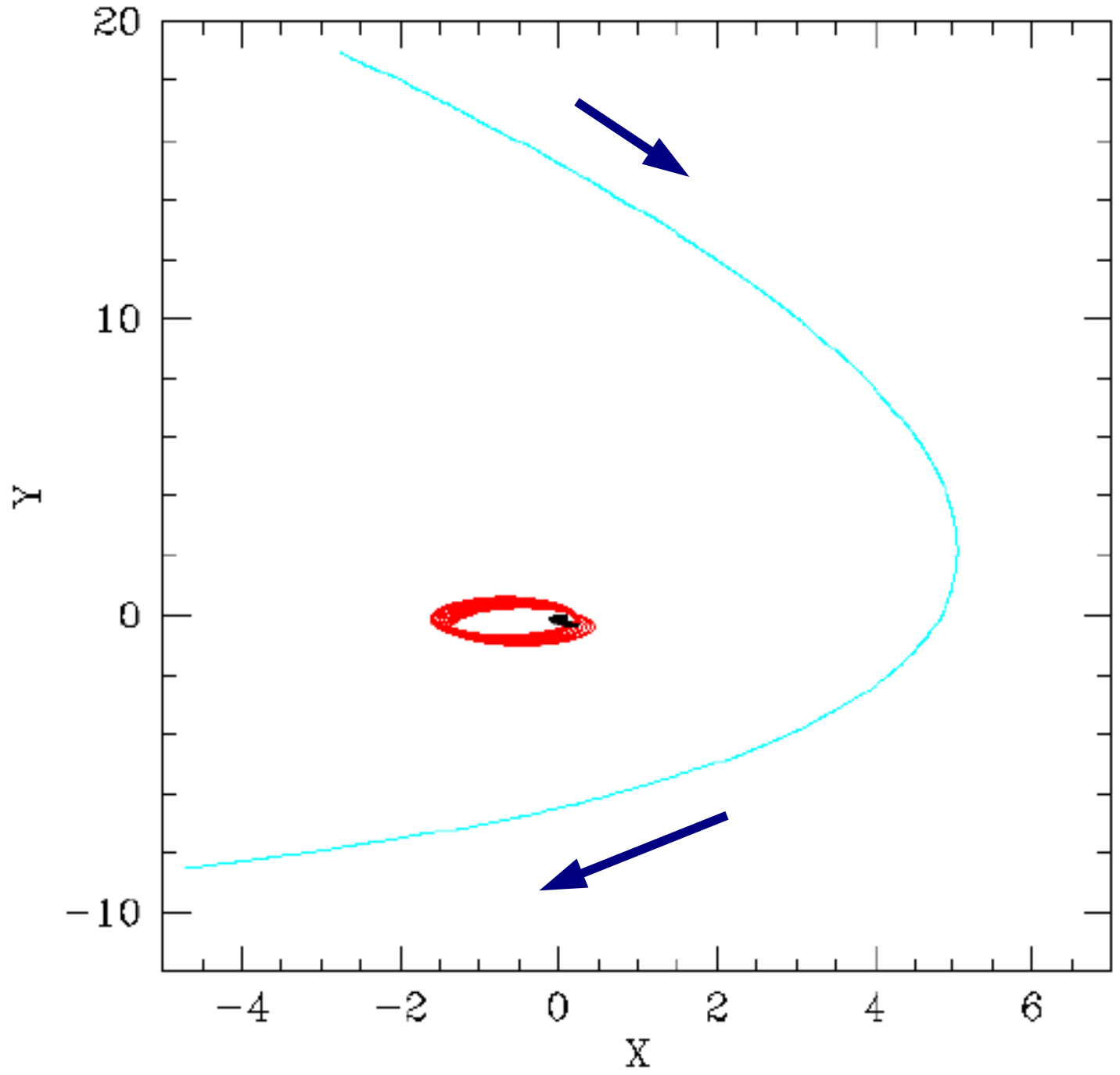


CARTOON of a EXCHANGE ENCOUNTER where  $m_3 > m_2 \rightarrow E_b$  increases

4. impact of environment on merger rate

# EXAMPLES of SIMULATED 3-BODY ENCOUNTERS

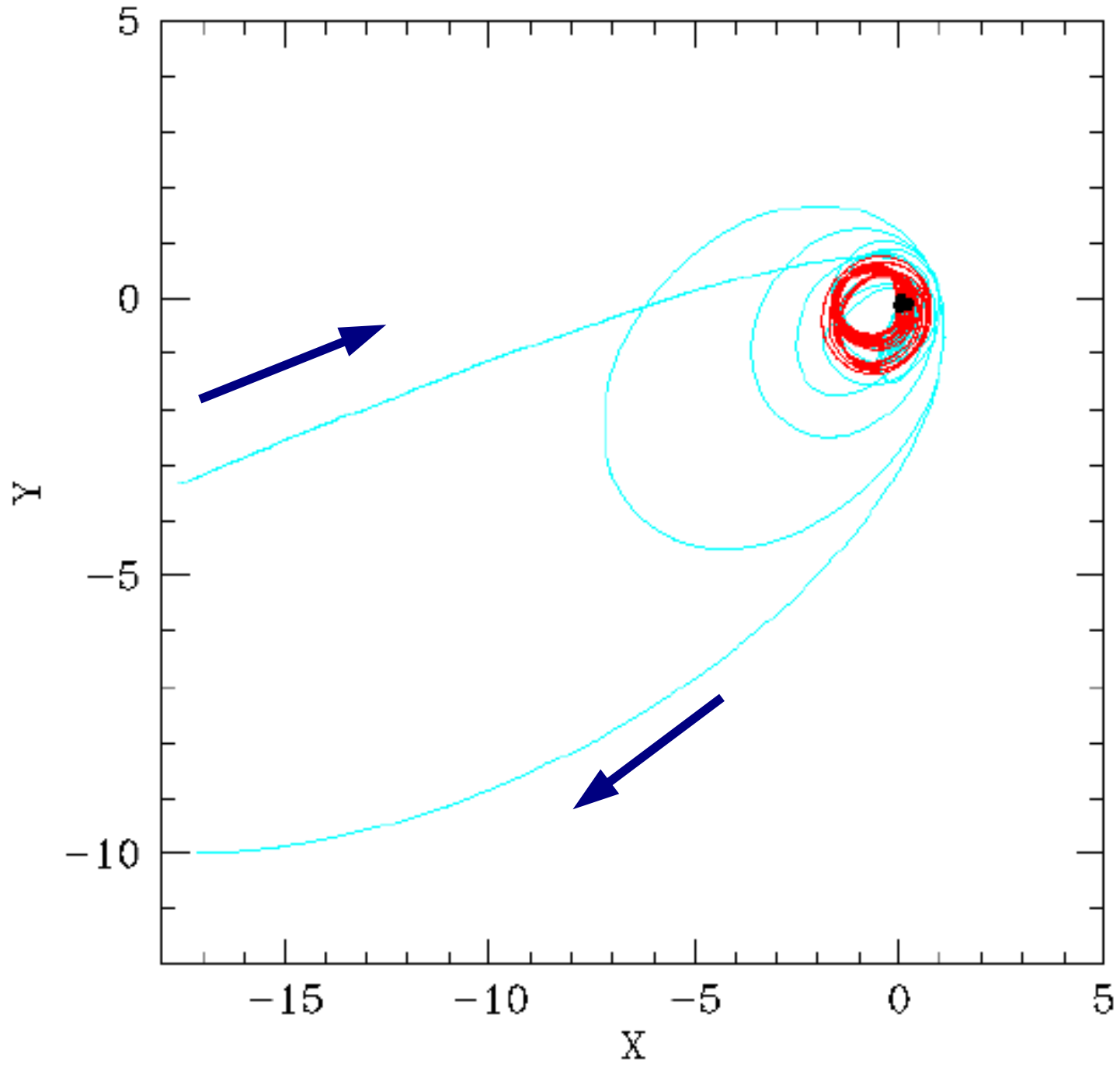
PROMPT  
FLYBY:



4. impact of environment on merger rate

# EXAMPLES of SIMULATED 3-BODY ENCOUNTERS

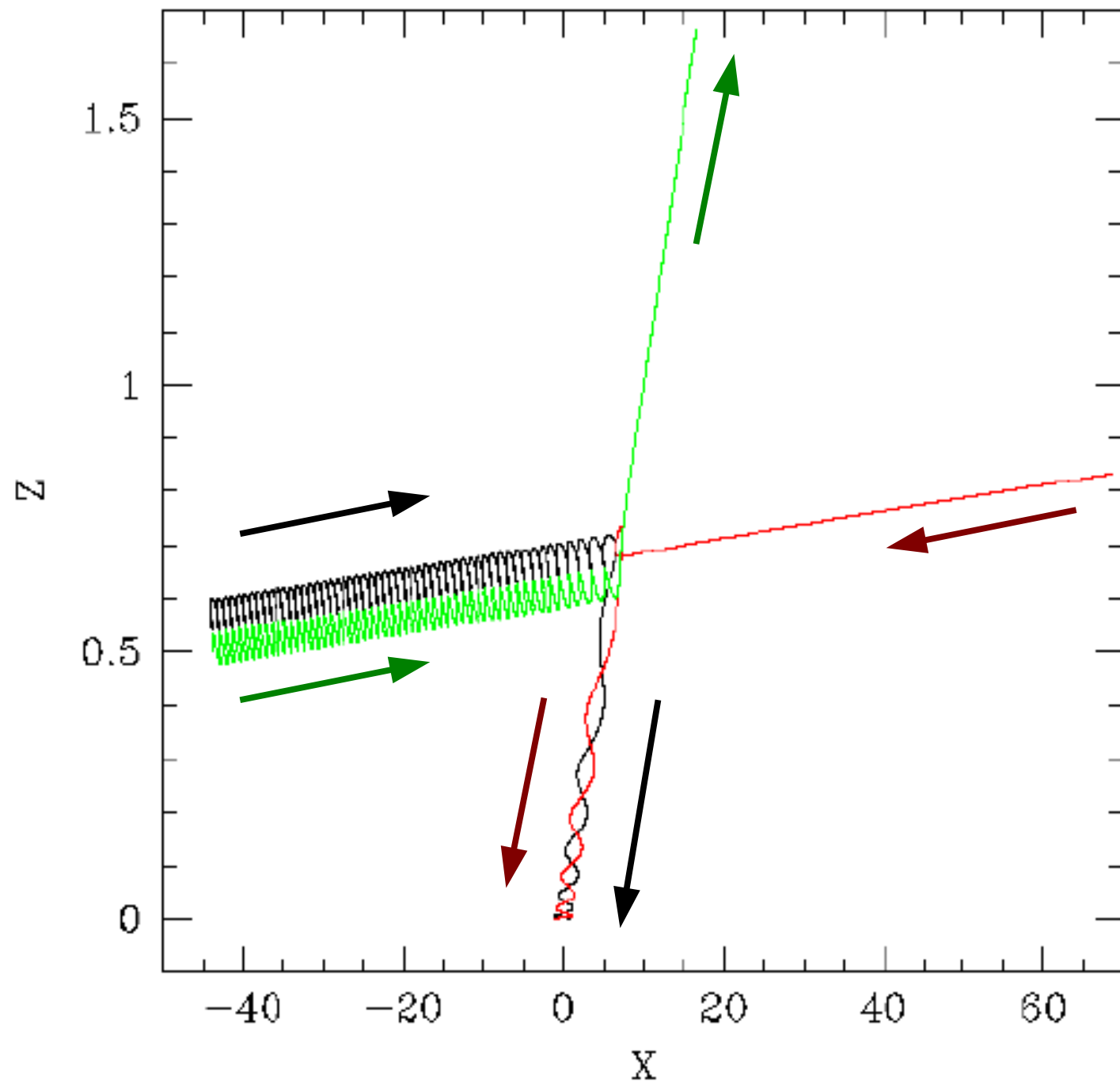
RESONANT  
FLYBY:



#### 4. impact of environment on merger rate

## EXAMPLES of SIMULATED 3-BODY ENCOUNTERS

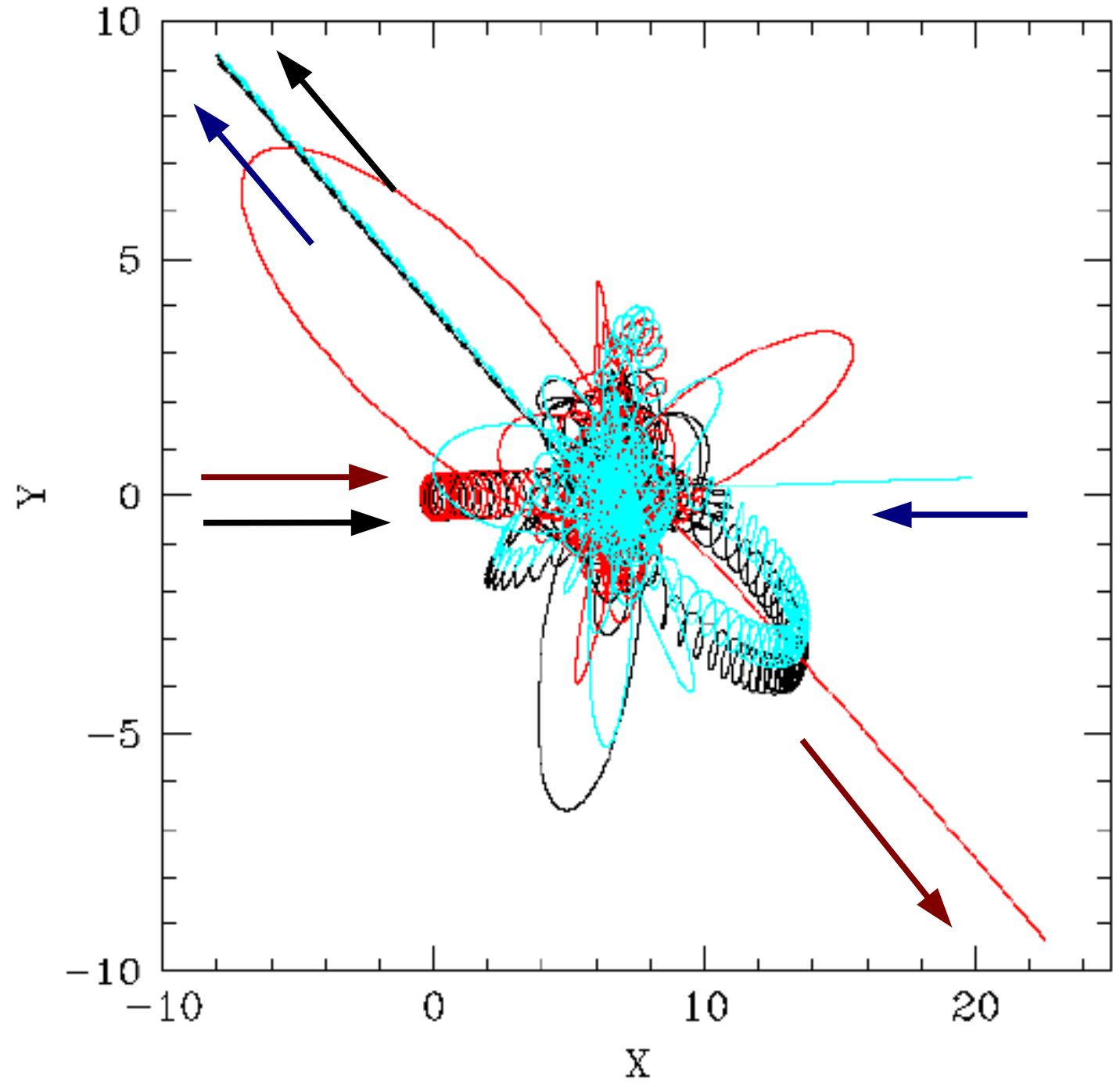
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4. impact of environment on merger rate

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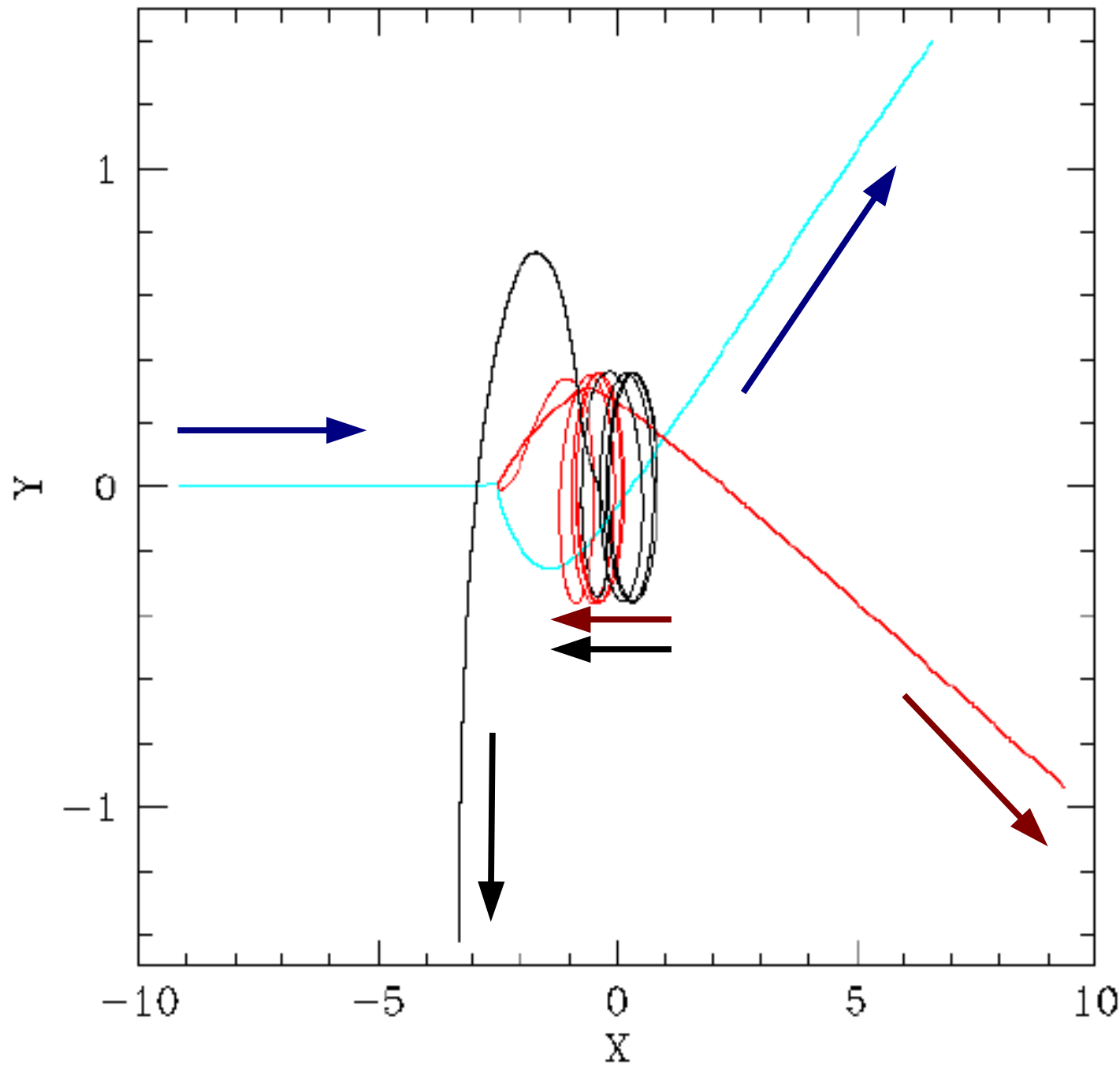
RESONANT  
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4. impact of environment on merger rate

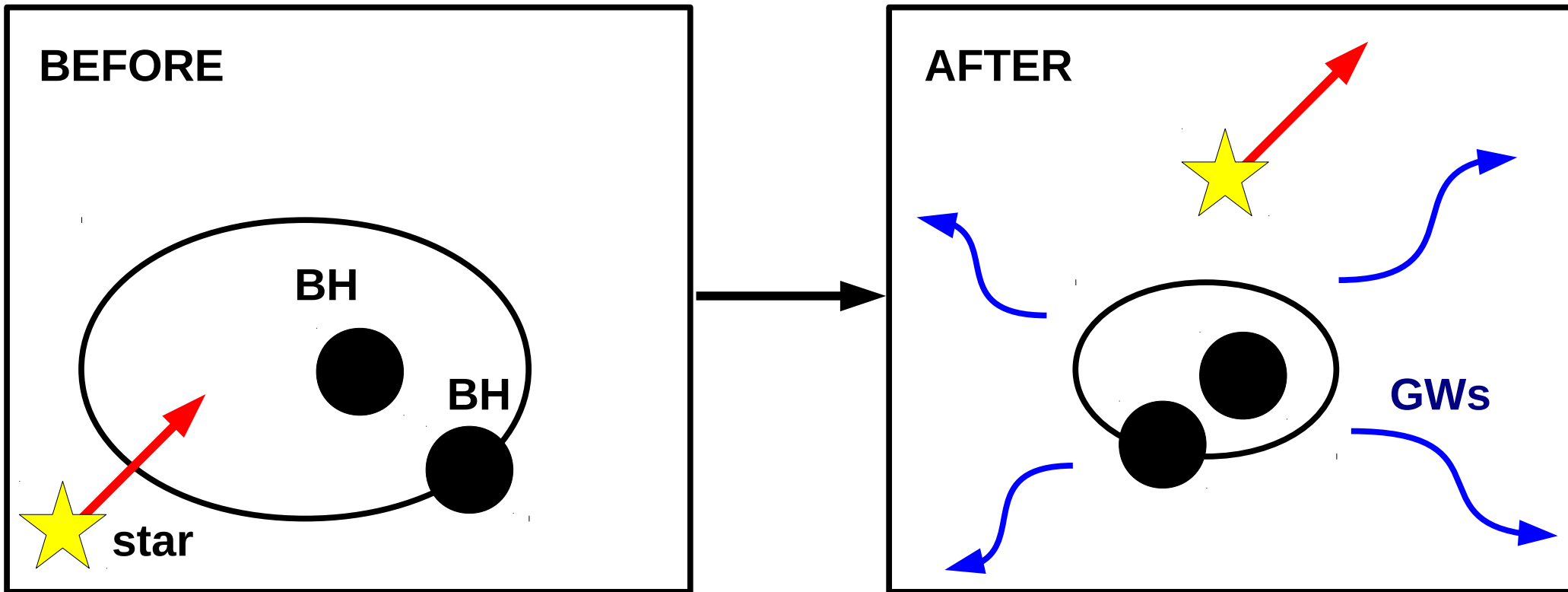
# EXAMPLES of SIMULATED 3-BODY ENCOUNTERS

IONIZATION:



#### 4. impact of environment on merger rate

*Which is the effect of 3-body encounters on BH-BH binaries?*

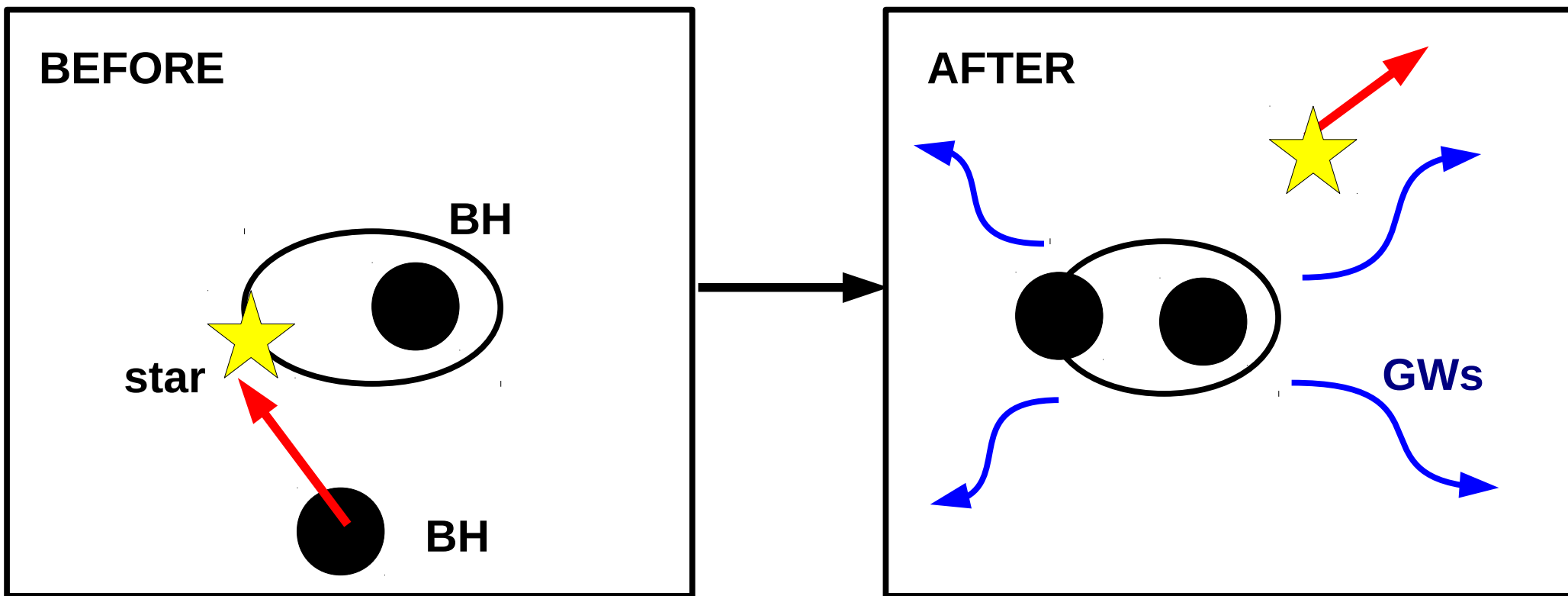


*After 3-body encounters, the semi-major axis shrinks and the BH-BH (or BH-NS or NS-NS) binary becomes important as gravitational wave (GW) source*



#### 4. impact of environment on merger rate

*Which is the effect of 3-body encounters on BH-BH binaries?*



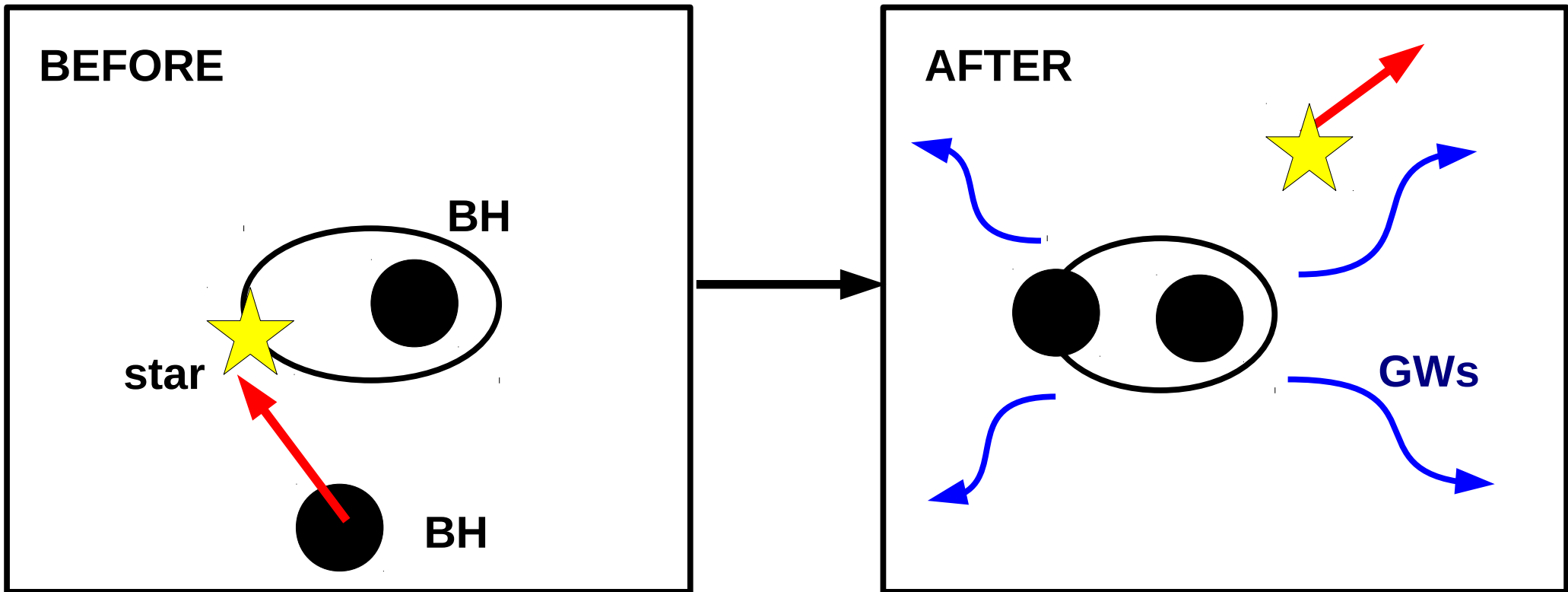
***Exchanges are very important: bring BHs in binaries***

***BHs are FAVOURED BY EXCHANGES BECAUSE THEY ARE MASSIVE!***

*BH BORN FROM SINGLE STAR IN THE FIELD NEVER ACQUIRES A COMPANION  
BH BORN FROM SINGLE STAR IN A SC LIKELY ACQUIRES COMPANION FROM  
DYNAMICS*

#### 4. impact of environment on merger rate

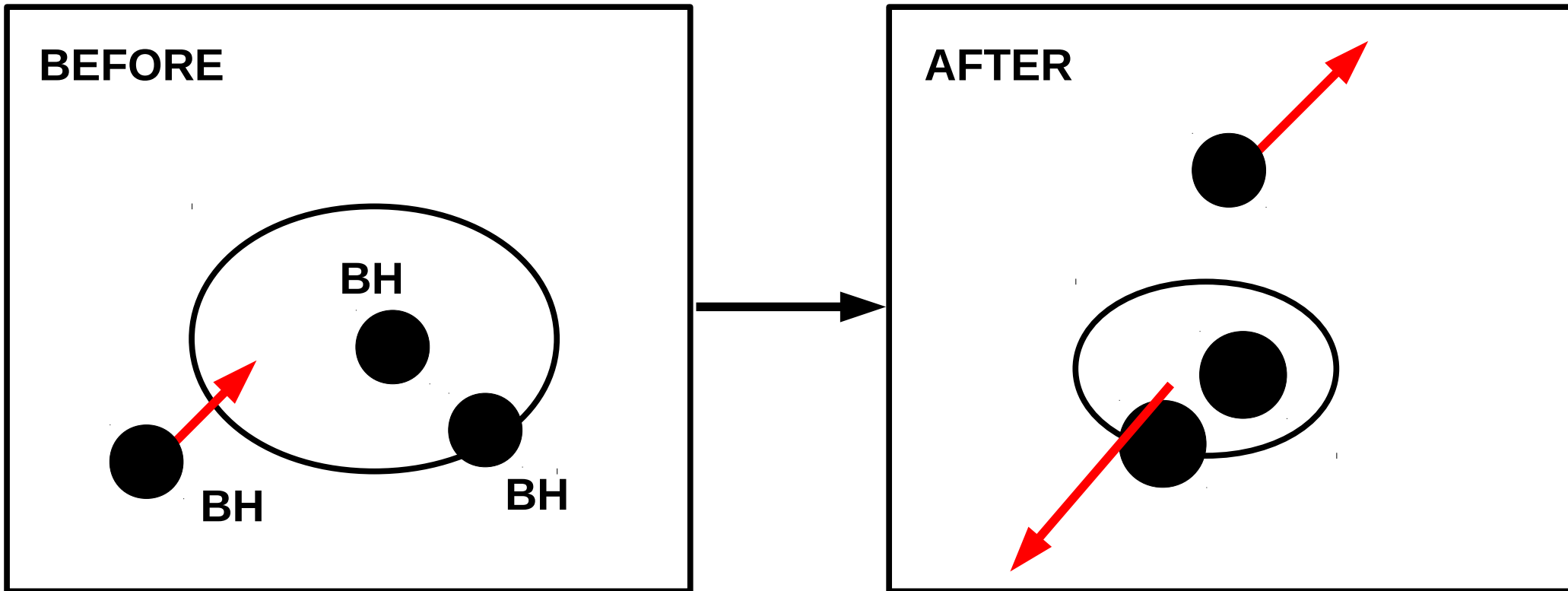
*Which is the effect of 3-body encounters on BH-BH binaries?*



***EXCHANGES FAVOUR THE FORMATION of BH-BH BINARIES WITH THE MOST MASSIVE BHs !!***

#### 4. impact of environment on merger rate

*Which is the effect of 3-body encounters on BH-BH binaries?*



**Internal energy is extracted from the binary**

➔ **converted into KINETIC ENERGY of the INTRUDER AND of the CM of the BINARY**

➔ **BOTH RECOIL and can be ejected from SC**

#### 4. impact of environment on merger rate

### *What do we learn from dynamics?*

If a BH or NS forms in a **DENSE STELLAR SYSTEM**  
(collisional system),  
**DYNAMICS CAN AFFECT**

**1. THE FORMATION OF BH-BH, NS-NS and BH-NS BINARIES**

**2. THEIR MERGER RATE**



**DYNAMICAL SIMULATIONS  
ARE NEEDED to investigate  
MERGER RATE in STAR  
CLUSTERS!**

#### 4. impact of environment on merger rate

### *What do we learn from dynamics?*

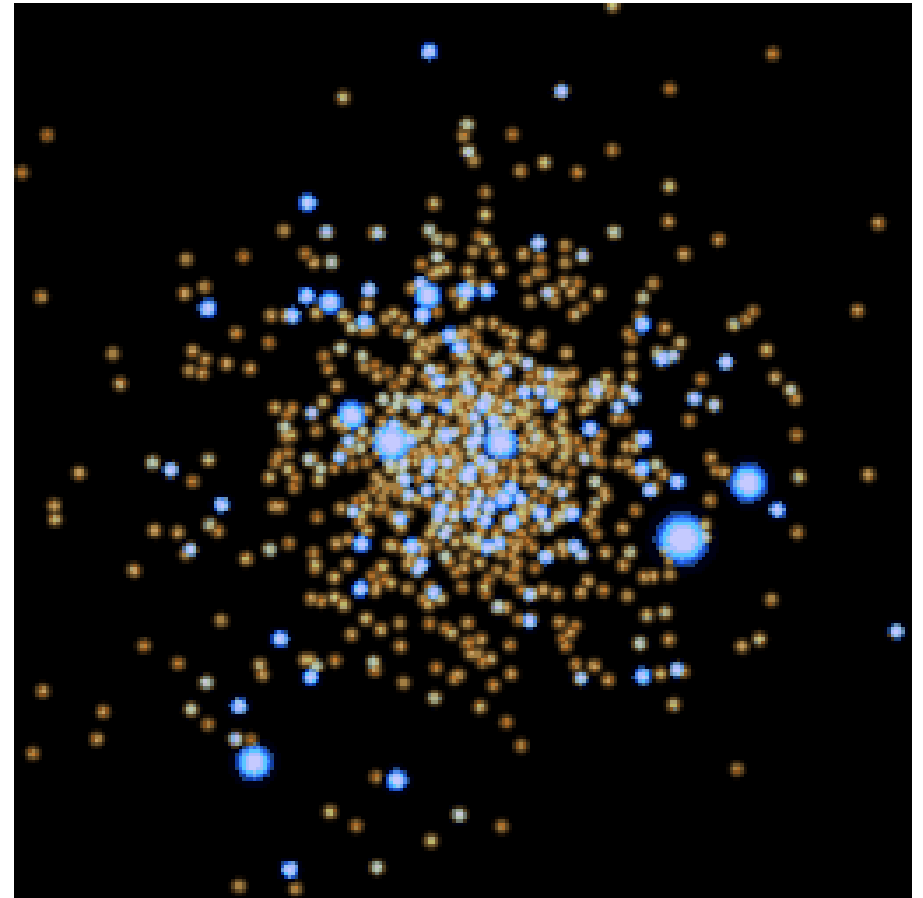
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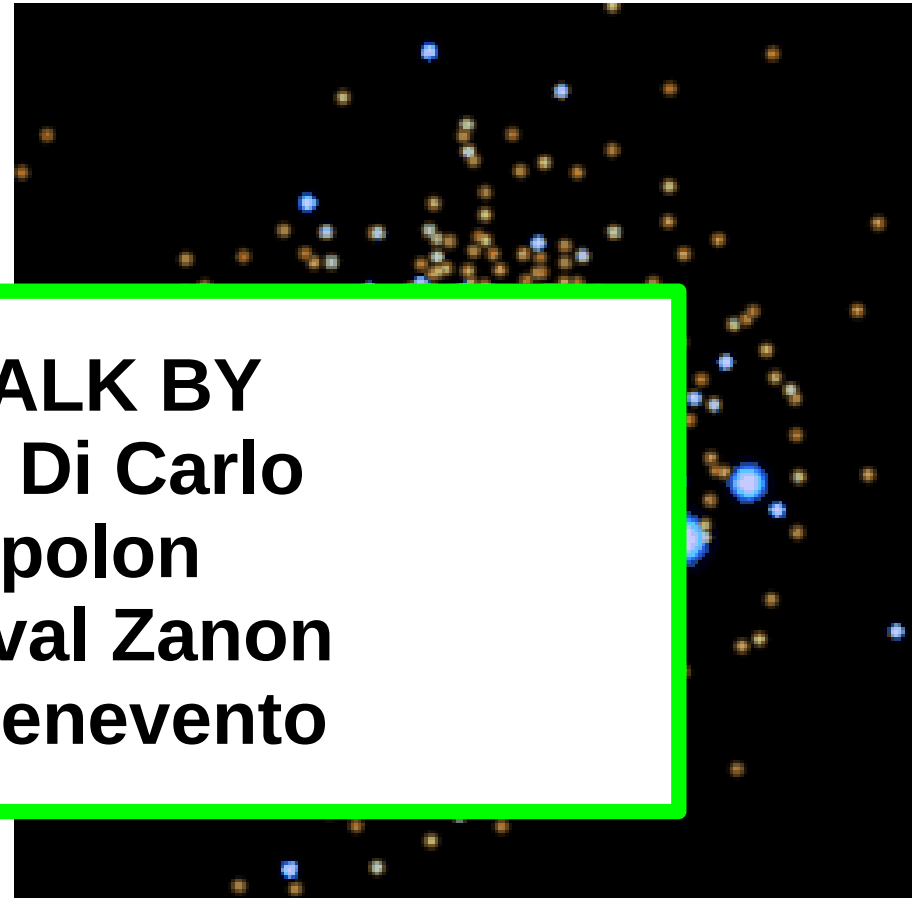
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- 1. THE FORMATION OF BH-BH, NS-NS and BH-NS BINARIES**
- 2. THEIR MERGER RATE**



DYNAMICS  
ARE NEARLY  
MERGER  
CLUSTERS

**SEE THE TALK BY**  
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## **CONCLUSIONS:**

- **Advanced LIGO-Virgo detectors are about to start operating (2016) – frequency 10-10 000 Hz**
- **Double compact object binaries are one of the best GW sources in that range**
- **High-school students can estimate binary inspiral (semi-major axis shrinking, GW frequency)**
- **Main counterparts are short GRBs and kilonovae**
- **Estimate merger and detection rate is very important task, and a very dangerous/uncertain one**
- **The environment can affect the merger rate dramatically**



**THANK YOU**