Gravitational wave sources in the frequency range of Advanced LIGO and Virgo detectors

Collaborators: Mario Spera, Brunetto Ziosi, Marica Branchesi, Alessandro Trani, Sandro Bressan, Luca Zampieri

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#### Michela Mapelli

INAF-Osservatorio Astronomico di Padova

#### OUTLINE:

**1. gravitational waves for dummies** 

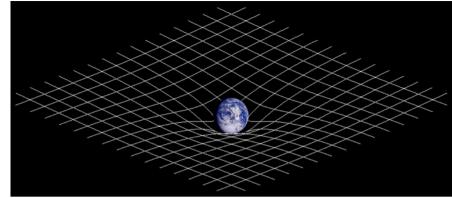
**2.** a couple of words about detectors

**3. gravitational wave sources (for LIGO/Virgo)** 

4. how to estimate the merger rate

5. impact of environment on merger rate

Browsing wikipedia `gravitational waves are ripples in the curvature of spacetime which propagate as waves, travelling outward from the source'



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

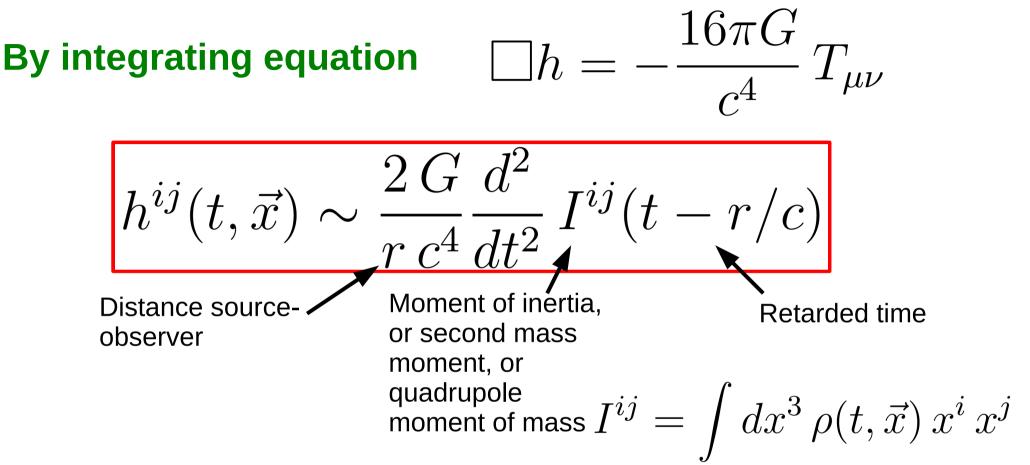
Weak field (far from source)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad with |h_{\mu\nu}| \ll 1$$

Using gauge invariance and assuming vacuum (*T*=0 no mass no energy)

$$\Box h = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$$

#### **Equation of WAVES!!**

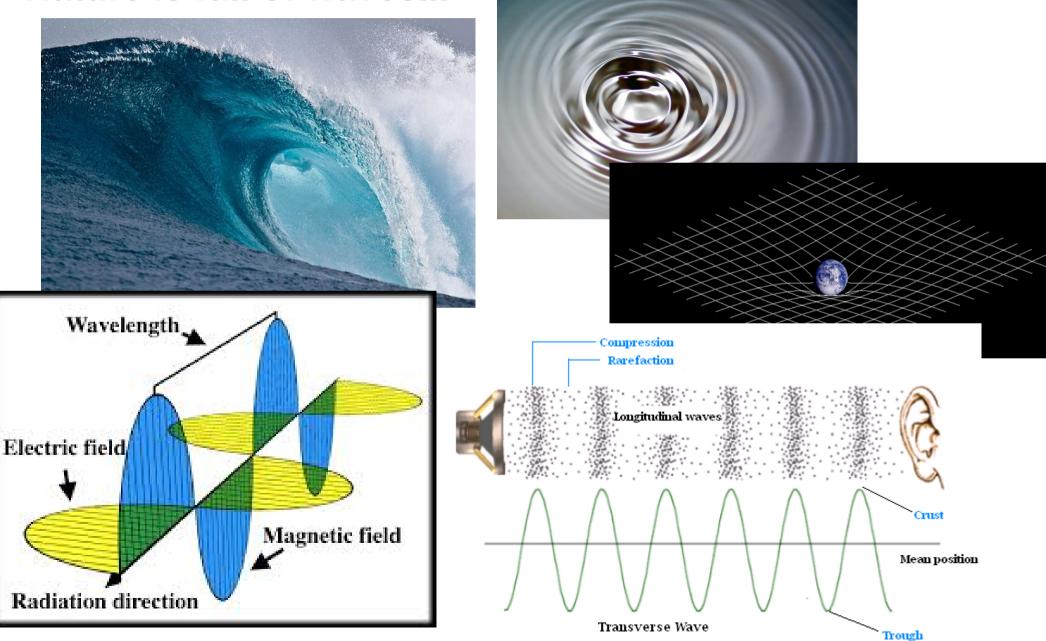


 $\rightarrow$  not all accelerating masses do this job but only those with  $\underline{QUADRUPOLE}$ 

If you do calculation, monopole and dipole disappear

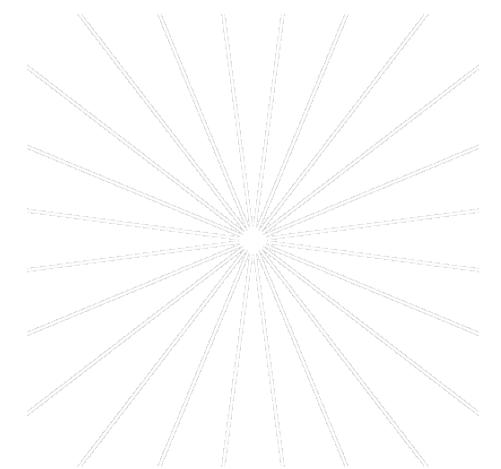
 $\rightarrow$  for a gravitational wave to form, there must be an ASYMMETRY IN MASS DISTRIBUTION

# 1. Gravitational waves for dummies TO EXPLAIN TO HIGH-SCHOOL STUDENTS Nature is full of waves...



#### **TO EXPLAIN TO HIGH-SCHOOL STUDENTS** Analogy with electromagnetic field

- an accelerating charge produces a perturbation in electromagnetic field that propagates as wave



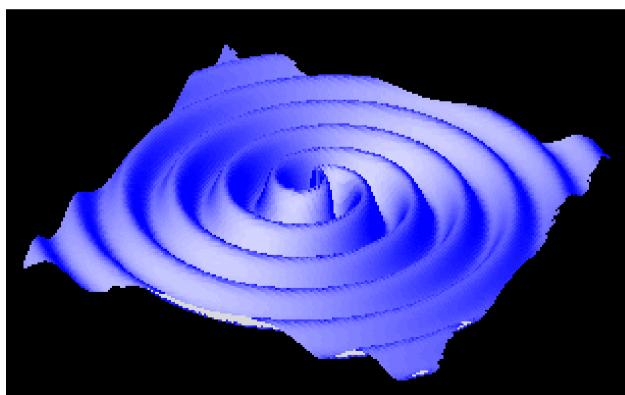
#### **TO EXPLAIN TO HIGH-SCHOOL STUDENTS** Analogy with electromagnetic field

a mass is source of gravitational field as a charge is source of electromagnetic field

→ an accelerating mass should produce perturbations in gravitational field,

i.e. intrinsic perturbations of space-time that propagate as waves:

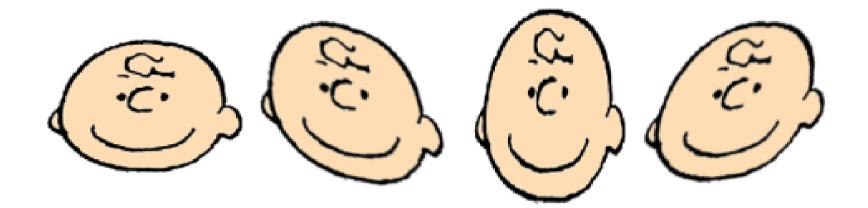
do not move in space-time but MOVE SPACE-TIME at speed of light (i.e. lead deformation in space time – squeeze stretch)



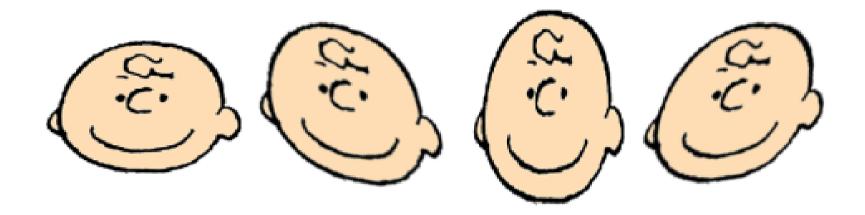
When GW passes through space deforms it



When GW passes through space deforms it

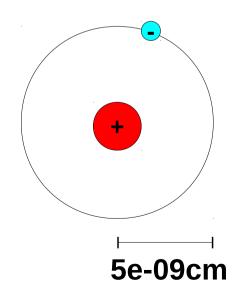


When GW passes through space deforms it



But deformations are very small: strain=relative deformation  $h \sim 1e-21$ 

For I<sub>Sun-Earth</sub>~ 1.5e13 cm *h I<sub>Sun-Earth</sub>* ~ 1e-21 x 1.5e13 ~ 1.5e-08cm < size of H atom at distance Sun-Earth



#### **Michelson interferometers**



#### **Michelson interferometers**

#### 2 LIGO detectors in the US

#### 1 Virgo (Italy)



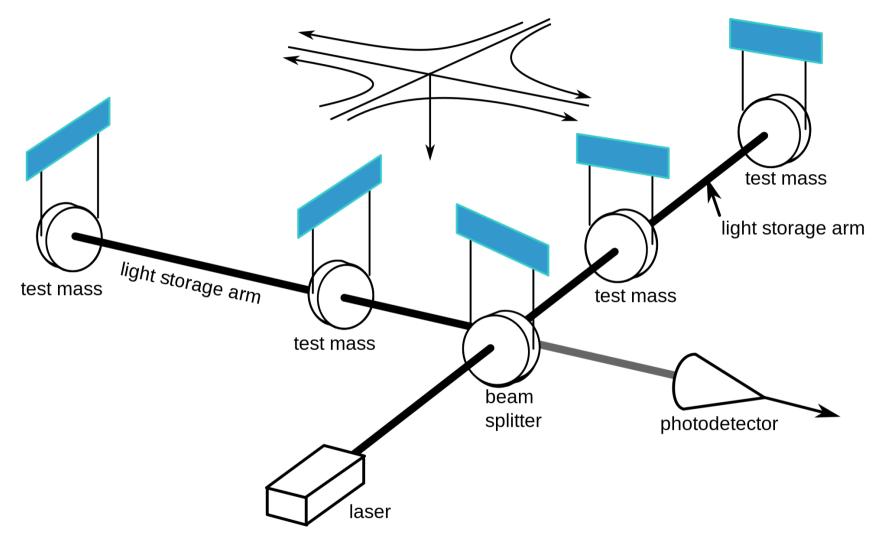
**Design started in the '90s** 

First science runs ~ 2007 (no detection)

Now being upgraded

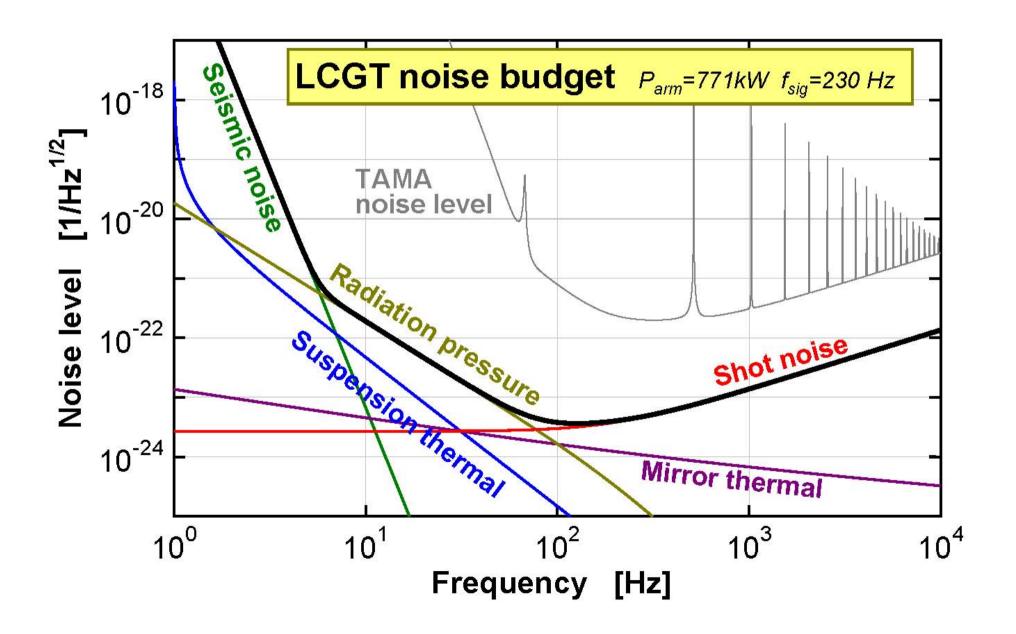
Next runs ~2016

#### **Michelson interferometers**



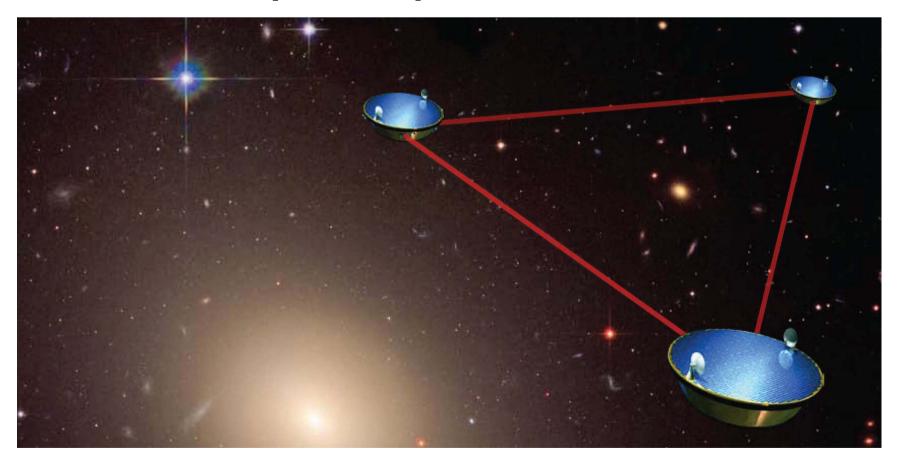
Two arms of ~ 4 km – At least 100 km requested for detection  $\rightarrow$  laser is bounced back and forth

**NOISE is the problem!** 



#### To go to lower frequency we need flying detectors!

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LISA – eLISA (>> 2020)
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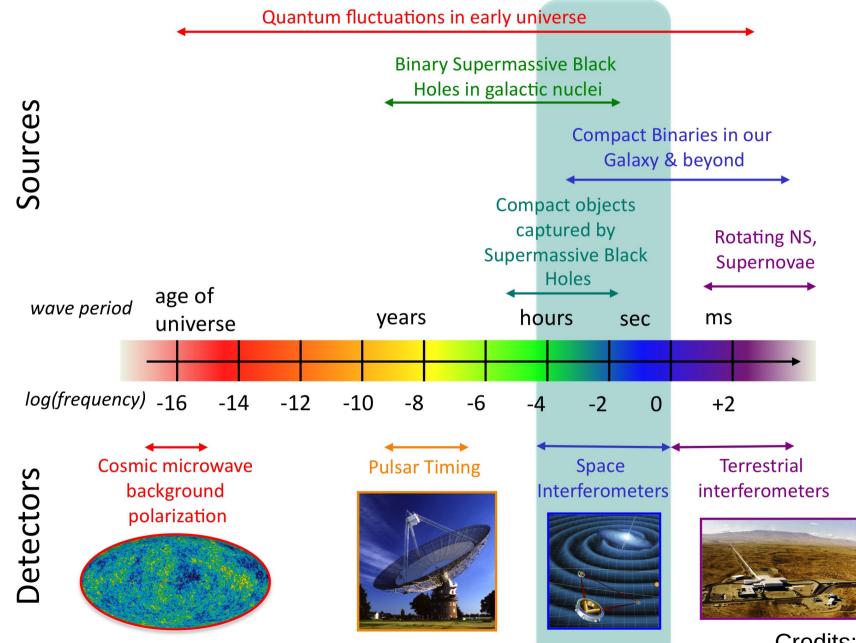
From http://lisa.nasa.gov/mission/index.html)



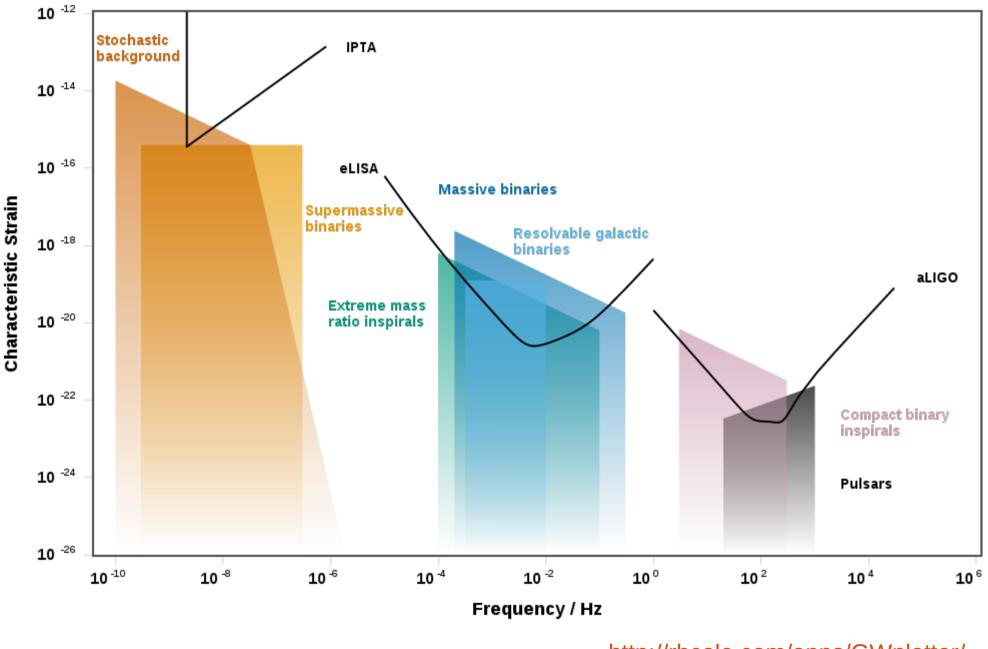
#### PLAY AT BUILDING YOUR OWN DETECTOR https://www.youtube.com/watch?v=IAvJrePR7F4

http://www.gwoptics.org/processing/space\_time\_quest/

# The Gravitational Wave Spectrum



Credits: NASA



http://rhcole.com/apps/GWplotter/ Christopher Berry **3. gravitational wave sources (for LIGO/Virgo)** 

Focus on double compact-object binaries because LIGO-VIRGO sources

black hole – black hole (BH-BH) binaries

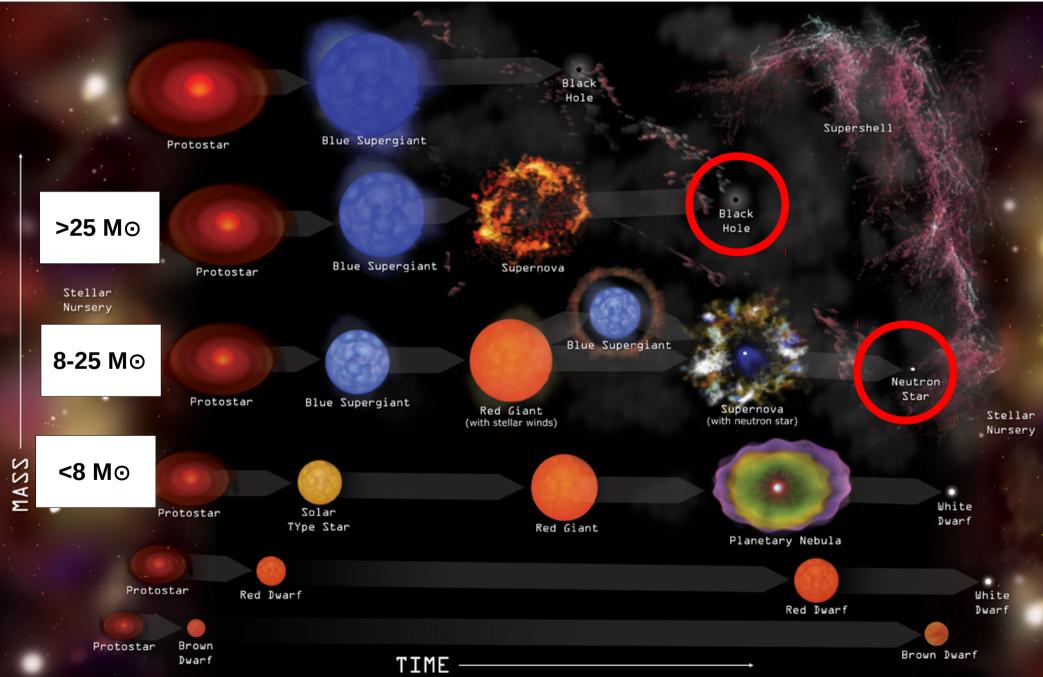
neutron star-neutron star (NS-NS) binaries

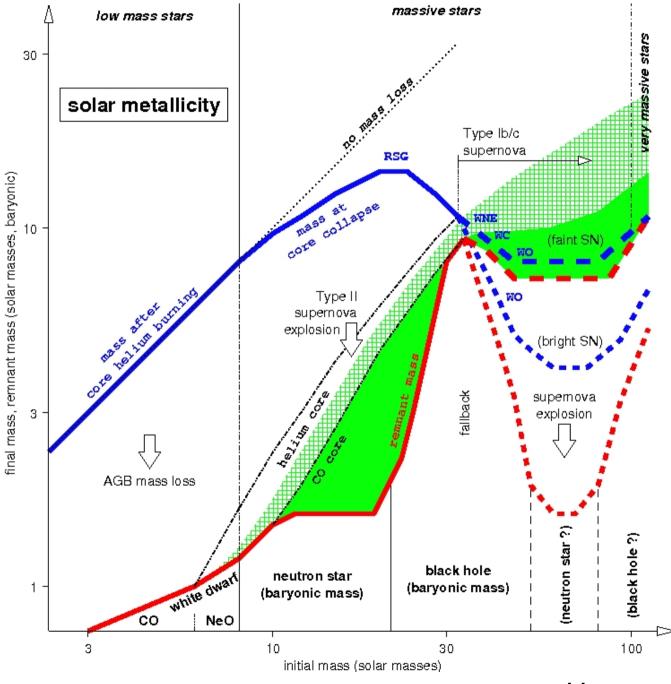
black hole – neutron star (BH-NS) binaries

HOW DO BHs and NSs form?

#### HOW DO BHs and NSs form?

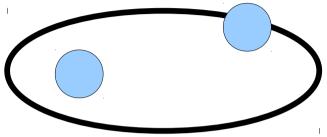
Credits: Chandra





Heger et al. (2003)

# For BH-BH, BH-NS and NS-NS binary I mean KEPLERIAN binary



#### **CARTOON OF NS-NS MERGER**

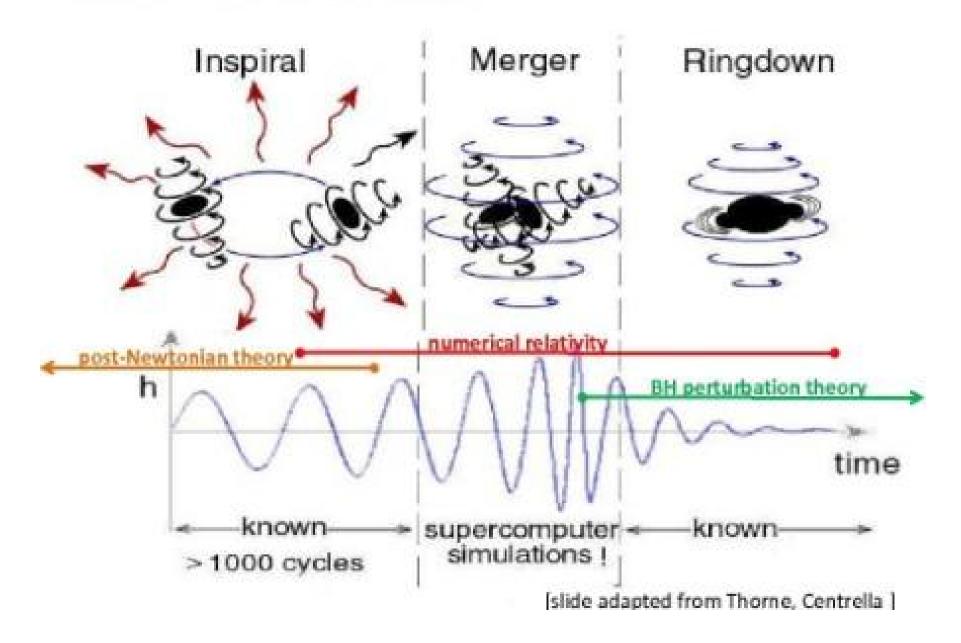


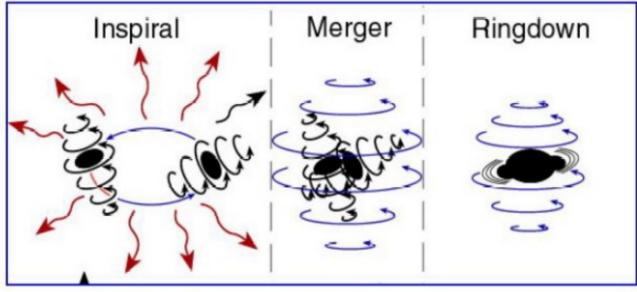
#### What differences if BH-BH merger?

https://www.youtube.com/watch?v=g8s81MzzJ5c

Credits: NASA

#### Cartoon of BH coalescence:





#### Some back of the envelope calculations:

- frequency of gravitational waves

$$\omega_{\rm GW} = 2 \, \omega_{\rm orb}$$

- last stable orbit  $\, r_{
  m LSO} = 6 \, G \, M/c^2$
- GW frequency at last stable orbit (=end of inspiral)

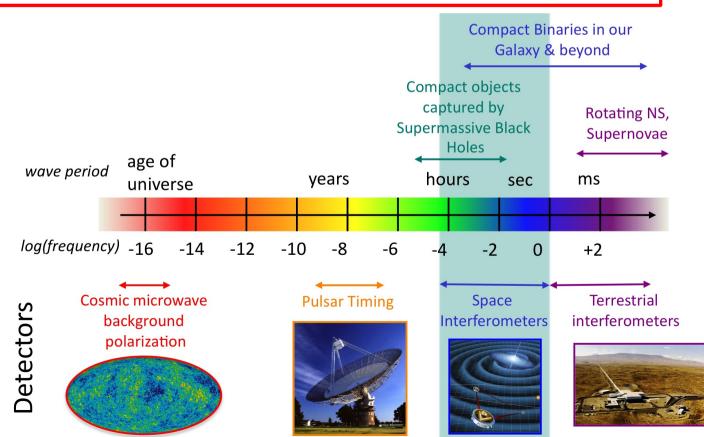
$$\omega_{\rm GW, \, LSO} = 2 \sqrt{\frac{G M}{r_{LSO}^3}} = \frac{2 c^3}{6^{3/2} \left(G M\right)}$$

#### **3. gravitational wave sources** *Some back of the envelope calculations:*

- GW frequency at last stable orbit (=end of inspiral)

$$\omega_{\rm GW, \, LSO} = 2 \sqrt{\frac{G M}{r_{LSO}^3}} = \frac{2 c^3}{6^{3/2} (G M)} \sim 2771 \, {\rm s}^{-1} (10 \, M_{\odot} / M)$$

$$\mathcal{V}_{\rm GW} \sim 440 \, {\rm Hz} \left( 10 \, M_{\odot} / M \right)$$



#### Some more back of the envelope calculations:

- amplitude

From 
$$h^{ij}(t,\vec{x}) \sim rac{2\,G}{r\,c^4} rac{d^2}{dt^2} \, I^{ij}(t-r/c)$$

In the specific case of a binary, in spherical coordinates, for the inspiral

$$h_{+} = \frac{1}{r} \left( \frac{4 G \mu \omega_{\text{orb}}^{2} a^{2}}{c^{4}} \right) \left( \frac{1 + \cos^{2} \theta}{2} \right) \cos \left( 2 \omega_{\text{orb}} t_{\text{ret}} + \phi \right)$$
$$h_{\times} = \frac{1}{r} \left( \frac{4 G \mu \omega_{\text{orb}}^{2} a^{2}}{c^{4}} \right) \cos \theta \sin \left( 2 \omega_{\text{orb}} t_{\text{ret}} + \phi \right)$$

$$|h| = \frac{1}{2}\sqrt{h_+^2 + h_\times^2} = \frac{1}{r} \left(\frac{2\,G\,\mu\,\omega_{\rm orb}^2\,a^2}{c^4}\right) \sqrt{\frac{(1+\cos^2\theta)^2}{4} + \cos^2\theta}$$

#### Some more back of the envelope calculations:

- strain (same as amplitude)

$$h \sim 10^{-21} \left(\frac{m_{\rm chirp}}{M_{\odot}}\right)^{5/3} \left(\frac{P}{\rm hour}\right)^{-2/3} \left(\frac{r}{1\,{\rm kpc}}\right)^{-1}$$

- chirp mass  $m_{\rm chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$
- coalescence timescale (Peters 1964)

$$t_{\rm GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4 (1 - e^2)^{7/2}}{m_1 m_2 (m_1 + m_2)}$$

(to derive this timescale: GWs lead to energy loss → change in semi-major axis)

# WHY 'normal' stars in binaries are not sources of gravitational waves?

WHY 'normal' stars in binaries are not sources of GWs?

$$\begin{split} & \underset{\mathbf{d}}{\overset{\mathbf{R}}{\text{m}_{2}}} \underbrace{\mathbf{m}_{1}}_{\mathbf{d}} & r_{t} \sim d \left(\frac{m_{2}}{3 m_{1}}\right)^{1/3} \\ & \underset{\mathbf{d}}{\overset{\mathbf{m}_{1}}{\text{If } \mathbf{R} > = \mathbf{r}_{t}}} & \text{the star is tidally disrupted} \\ & \omega_{\text{GW}} = 2 \, \omega_{\text{orb}} = 2 \, \left[\frac{G \left(m_{1} + m_{2}\right)}{d^{3}}\right]^{1/2} \sim 2 \, \left[\frac{G \left(m_{1} + m_{2}\right) m_{2}}{3 m_{1} r_{t}^{3}}\right]^{1/2} \end{split}$$

If  $r_t$  = R = 1 Rsun=6e10 cm,  $m_1$ =  $m_2$ =1 Msun, the maximum GW frequency that can be emitted by 2 sun-like stars (before tidal disruption) is  $\omega_{\rm GW} \sim 3 imes 10^{-4} {
m Hz}$ 

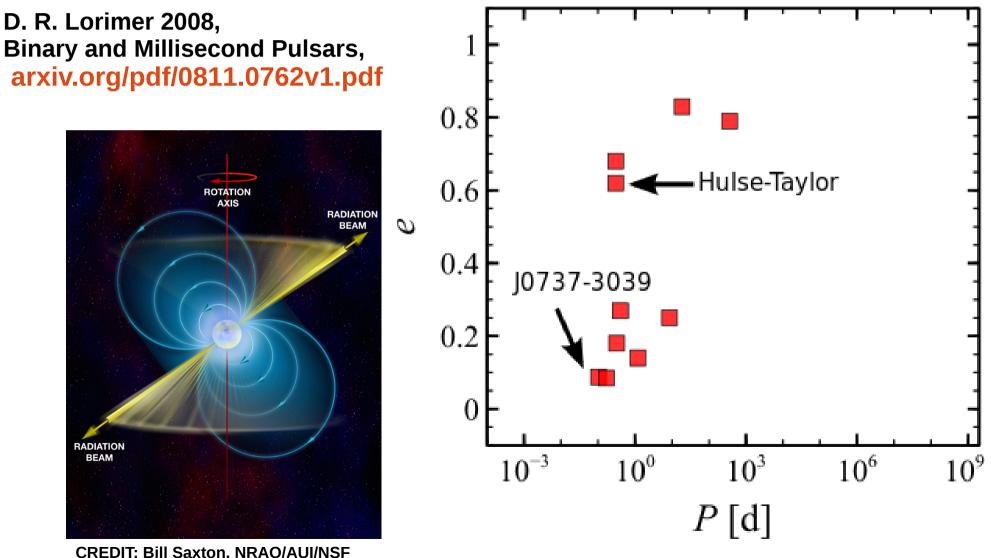
MUCH LOWER THAN LIGO-VIRGO range!!!

# 4. how to estimate the merger rate (and detection rate) FROM OBSERVATIONS

observable	NS-NS merger	BH-NS merger	BH-BH merger
1- number of observed NS-NS	YES	NO	NO
2- short gamma-ray burst rate	YES	MAYBE	NO
3- number of observed BH-WR	NO	MAYBE	YES

#### From observations

#### 1- number of observed NS NS binaries:



CREDIT: Bill Saxton, NRAO/AUI/NSF

From Ziosi, PhD thesis

#### **From observations**

#### **1- number of observed NS-NS binaries:**

- **1.** take properties of observed NS-NS (semi-major axis, mass, eccentr.)
- **2.** estimate GW merger timescale  $t_{GW}$  for each of them
- 3. sum 1/ t<sub>GW</sub> over all NS NS binaries in Milky Way (MW)
- normalize to MW star formation rate (SFR<sub>MW</sub>~0.25 Msun yr<sup>-1</sup>) and multiply by density of star formation rate in the local Universe (ρ<sub>SFR</sub>~0.015 Msun yr<sup>-1</sup> Mpc<sup>-3</sup>, Hopkins & Beacom 2006)
- 5. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS V~ 4/3  $\pi$  L<sup>3</sup> (with L=200 Mpc)

YOU GET THE DETECTION RATE:  

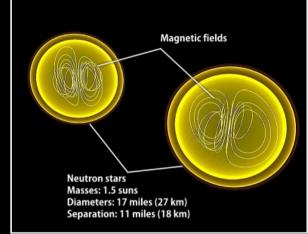
$$R = \sum_{i} \frac{1}{t_{\rm GW, i}} \frac{\rho_{\rm SFR}}{\rm SFR_{MW}} \frac{4}{3} \pi L^{3}$$

#### THERE ARE SEVERAL PROCEDURES SIMILAR TO THIS ONE!!!

#### **From observations**

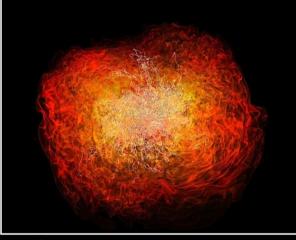
#### 2- short gamma ray burst rate

# Crashing neutron stars can make gamma-ray burst jets

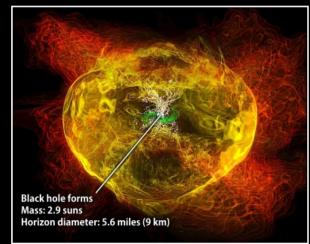








13.8 milliseconds









21.2 milliseconds

26.5 milliseconds

#### **From observations**

#### 2- short gamma ray burst rate gamma ray burst <2 s thought to be produced by NS-NS or NS-BH merger

**1. take observed short gamma-ray burst rate** *R*<sub>GRB</sub>

- 2. correct for beaming (we see only short  $\gamma$ -ray bursts pointing toward us)  $\sim (1 \cos\theta)^{-1}$
- 3. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS  $V \sim 4/3 \pi L^3$  (with L=200 Mpc)

YOU GET THE DETECTION RATE:

$$R = R_{\rm GRB} \left(1 - \cos\theta\right)^{-1} \frac{4}{3} \pi L^3$$

**EASY BUT ASSUMPTION that gamma-ray burst means merger** 

(e.g. Coward+2012; Siellez+ 2014)

#### **From observations**

#### **3- BH-WR binaries:**

WR stars are naked Helium stars that will end as BH or NS  $\rightarrow$  BH-WR are precursor of BH-BH (or possibly BH-NS)



The Galactic WR star WR124 and its nebula

#### **Cartoon of a BH-WR**



## From observations

# **3- BH-WR binaries:**

WR stars are naked Helium stars that will end as BH or NS

 $\rightarrow$  BH-WR are precursor of BH-BH (or possibly BH-NS)

Calculation similar to NS-NS binaries:

- **1.** take properties of observed BH-WR (semi-major axis, mass, eccentr.)
- **2.** estimate GW merger timescale  $t_{GW,i}$  for each of them
- 3. normalize to star formation rate of their host galaxy (SFR<sub>i</sub>)
- 3. sum 1/(SFR<sub>i</sub>  $t_{GW,i}$ ) over all BH-WR in local Universe
- 4. multiply by density of star formation rate in the local Universe (ρ<sub>SFR</sub>~0.015 Msun yr<sup>-1</sup> Mpc<sup>-3</sup>, Hopkins & Beacom 2006)
- 5. multiply by instrumental horizon of Adv LIGO/Virgo for BH-BH V~ 4/3  $\pi$  L<sup>3</sup> (with L=1 Gpc)

$$R = \sum_{i} \frac{1}{t_{\rm GW, \, i} \, \text{SFR}_{i}} \, \rho_{\rm SFR} \, \frac{4}{3} \, \pi L^{3}$$

# 4. how to estimate the merger rate (and detection rate) From theory

Method	NS-NS merger	BH-NS merger	BH-BH merger
1- population synthesis simulations	YES	YES	YES
2- dynamical simulations (Monte Carlo or N-body)	YES	YES	YES

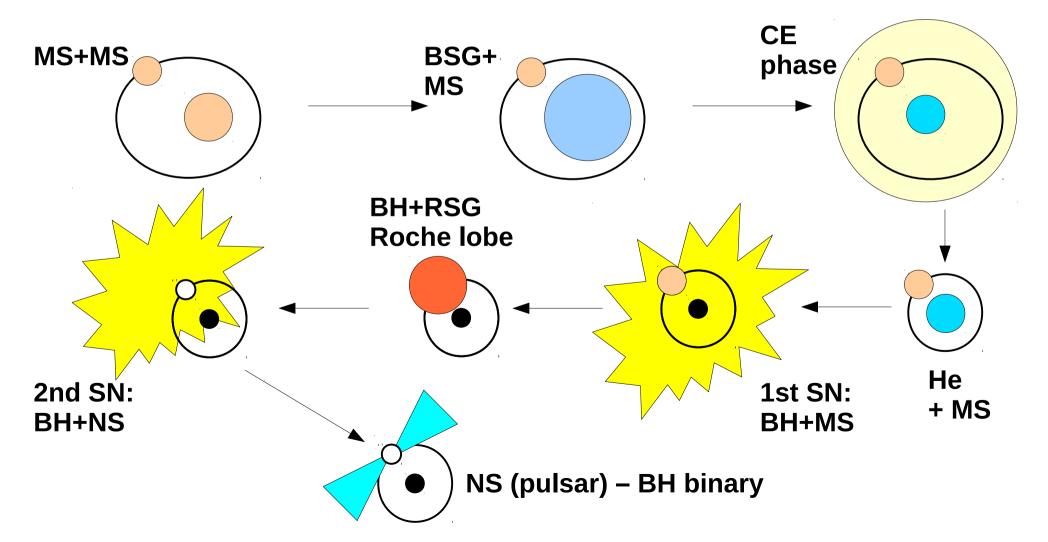
Theory seems better than observations, but.....

## 4. how to estimate the merger rate (and detection rate)

# From theory

# **1-** population synthesis simulations

Codes that evolve a binary of massive stars (stellar evolution + orbital evolution) until it forms a NS-NS or NS-BH or BH-BH



## 4. how to estimate the merger rate (and detection rate)

## From theory

# **1- population synthesis simulations**

Codes that evolve a binary of massive stars (stellar evolution + orbital evolution) until it forms a NS-NS or NS-BH or BH-BH

- 1. simulate a large grid of ISOLATED binaries (e.g. equal to total stellar mass of the Milky Way)
- 2. extract the number of mergers of NS-NS, NS-BH or BH-BH in the simulations in a Hubble time: this gives the merger rate of the Milky Way
- 3. either normalize to the SFR of the MW and multiply by the SFR density (see description about NS-NS observations) or normalize to the MW mass and multiply by the mass density of galaxies in the local Universe
- 4. multiply by instrumental horizon of Adv LIGO/Virgo for NS-NS, NS-BH, BH-BH

4. how to estimate the merger rate (and detection rate)

**From theory** 

2- dynamical simulations (Monte Carlo or N-body)

# WHY DYNAMICS??????

## WHY DYNAMICS??????

## **COLLISIONAL/COLLISIONLESS**

- **Collisional systems** are systems where interactions between stars are EFFICIENT with respect to the lifetime of the system

- **Collisionless systems** are systems where interactions are negligible

When is a stellar system collisional/collisionless?

#### **RELAXATION TIMESCALE**

Gravity is a LONG-RANGE force

 $\rightarrow$  Two-body encounters are important even if 2 bodies are distant

→ **two-body relaxation timescale**: timescale needed for a star to lose completely memory of its initial velocity ( $\Delta v/v \sim 1$ ) by the effect of two body encounters

**two-body relaxation timescale**: timescale needed for a star to lose completely memory of its initial velocity ( $\Delta v/v \sim 1$ ) by the effect of two body encounters

$$t_{\rm rlx} = n_{\rm cross} t_{\rm cross} = \frac{N}{8 \ln N} \frac{R}{v}$$

with more accurate calculations, based on diffusion coefficients (Spitzer & Hart 1971):

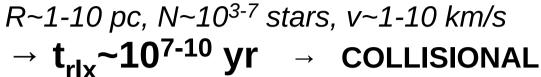
$$t_{\rm rlx} = 0.34 \, \frac{\sigma^3}{G^2 \, m \, \rho \ln \Lambda}$$

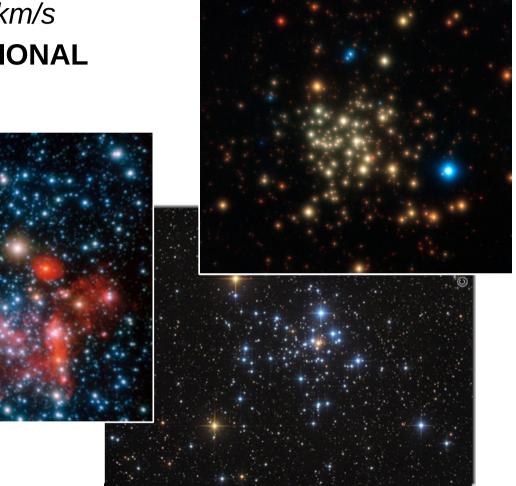
MOST USEFUL EXPRESSION:

$$t_{\rm rlx} = 10 \,{\rm Myr} \, \left(\frac{M_{\rm TOT}}{3500 \,{\rm M}_{\odot}}\right)^{1/2} \, \left(\frac{r_{\rm hm}}{1 \,{\rm pc}}\right)^{3/2}$$

0

# \* globular clusters, dense young star clusters, nuclear star clusters





#### \* galaxy field/discs

R~10 kpc, N~10<sup>10</sup> stars, v~100-500 km/s

 $\rightarrow$  t<sub>rlx</sub> >> Hubble time  $\rightarrow$  COLLISIONLESS

**5.** impact of environment on merger rate

## **BINARIES as ENERGY RESERVOIR**

Binaries have a energy reservoir (their internal energy) that can be exchanged with stars.

INTERNAL ENERGY: total energy of the binary – kinetic energy of the centreof-mass

 $E_{int} = \frac{1}{2} \,\mu \, v^2 - \frac{G \,m_1 \,m_2}{r}$ 

where  $m_1$  and  $m_2$  are the mass of the primary and secondary member of the binary,  $\mu$  is the reduced mass (:=  $m_1 m_2/(m_1 + m_2)$ ). *r* and *v* are the relative separation and velocity.

 $E_{int}$ <0 if the binary is bound

Note that  $E_{int}$  can be interpreted as the energy of the 'reduced particle': a fictitious particle of mass  $\mu$  orbiting in the potential –  $G m_1 m_2/r$ 

## **BINARIES as ENERGY RESERVOIR**

As far as the binary is bound, the orbit of the reduced particle is a Kepler ellipse with semi-major axis *a*. Thus, the energy integral of motion is

$$E_{int} = -\frac{G \, m_1 \, m_2}{2 \, a} = -E_b$$

where  $E_b$  is the **BINDING ENERGY** of the binary.

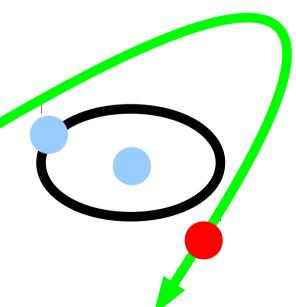
#### THE ENERGY RESERVOIR of BINARIES can be EXCHANGED with stars:

during a **3-BODY INTERACTION**, i.e. an interaction between a binary and a single star,

the single star can either

**EXTRACT INTERNAL ENERGY from the binary** 

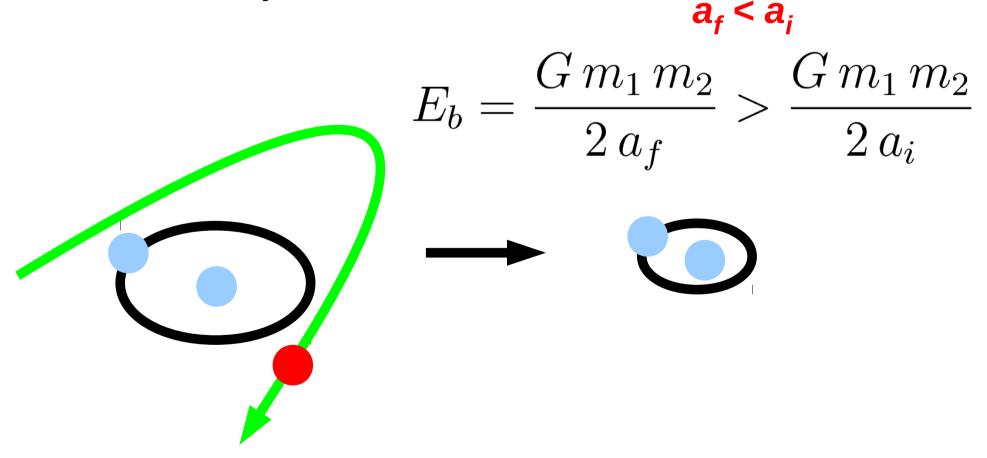
or lose a fraction of its kinetic energy, which is converted into internal energy of the binary.



## **BINARIES as ENERGY RESERVOIR**

If the star extracts  $E_{int}$  from the binary,

- 1) final kinetic energy of star > initial kinetic energy. STAR and BINARY acquire RECOIL VELOCITY
- 2)  $E_{int}$  becomes more negative, i.e.  $E_b$  higher: the binary becomes more bound.

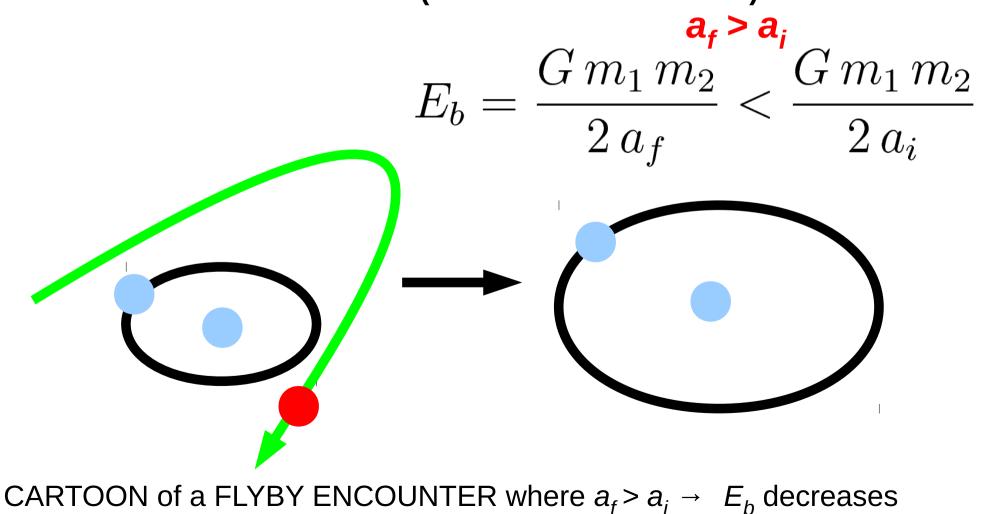


CARTOON of a FLYBY ENCOUNTER where  $a_f < a_i \rightarrow E_b$  increases

## **BINARIES as ENERGY RESERVOIR**

If the star transfer kinetic energy to the binary, 1) final kinetic energy of star < initial kinetic energy.

> 2) E<sub>int</sub> becomes less negative, i.e. E<sub>b</sub> smaller: the binary becomes less bound or is even IONIZED (:= becomes UNBOUND).



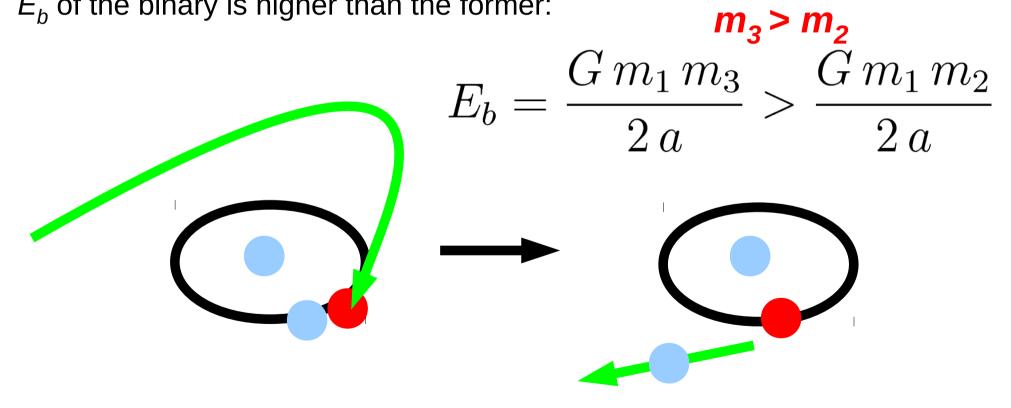
## **BINARIES as ENERGY RESERVOIR**

Alternative way for a binary to transfer internal energy to field stars:

### EXCHANGE

#### the single star replaces one of the former members of the binary.

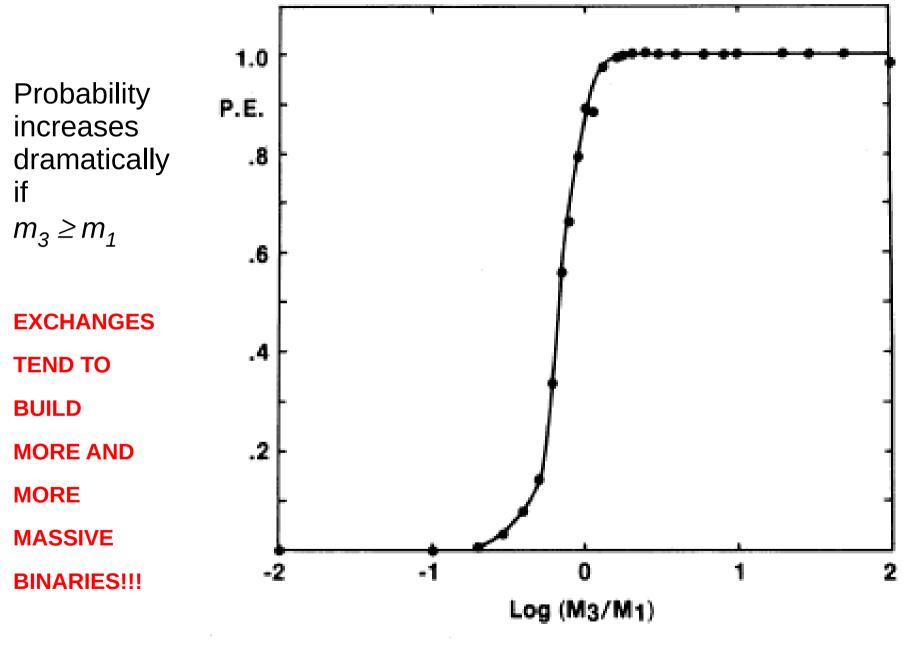
An exchange interaction is favoured when the mass of the single star  $m_3$  is HIGHER than the mass of one of the members of the binary so that the new  $E_h$  of the binary is higher than the former:



CARTOON of a EXCHANGE ENCOUNTER where  $m_3 > m_2 \rightarrow E_b$  increases

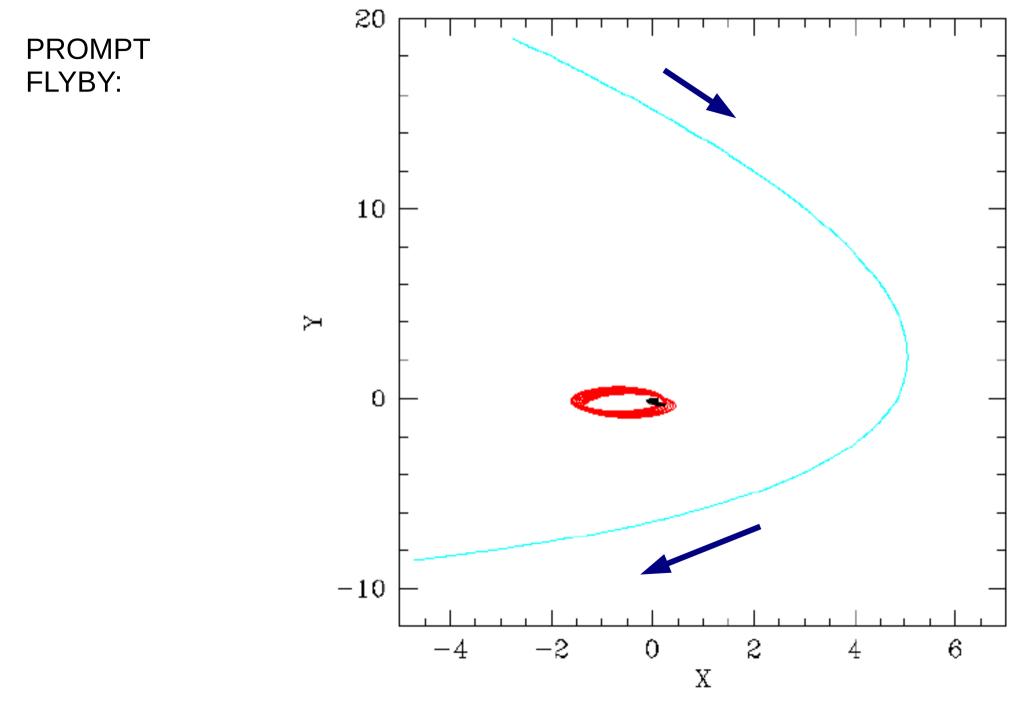
5. impact of environment on merger rate

## **EXCHANGE PROBABILITY**

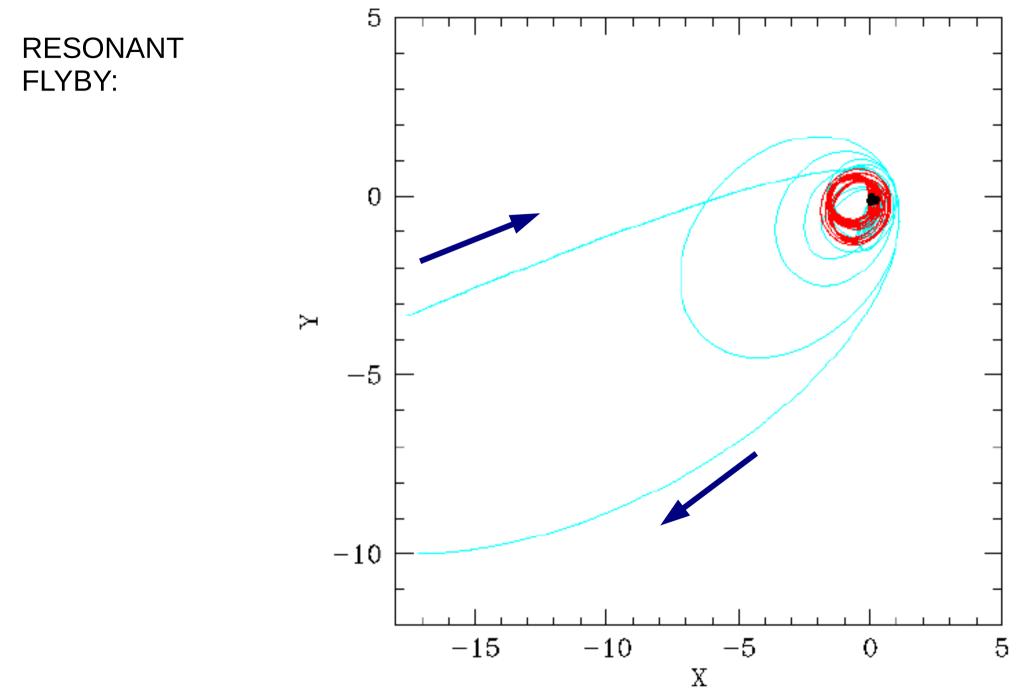


Hills & Fullerton 1980, AJ, 85, 1281

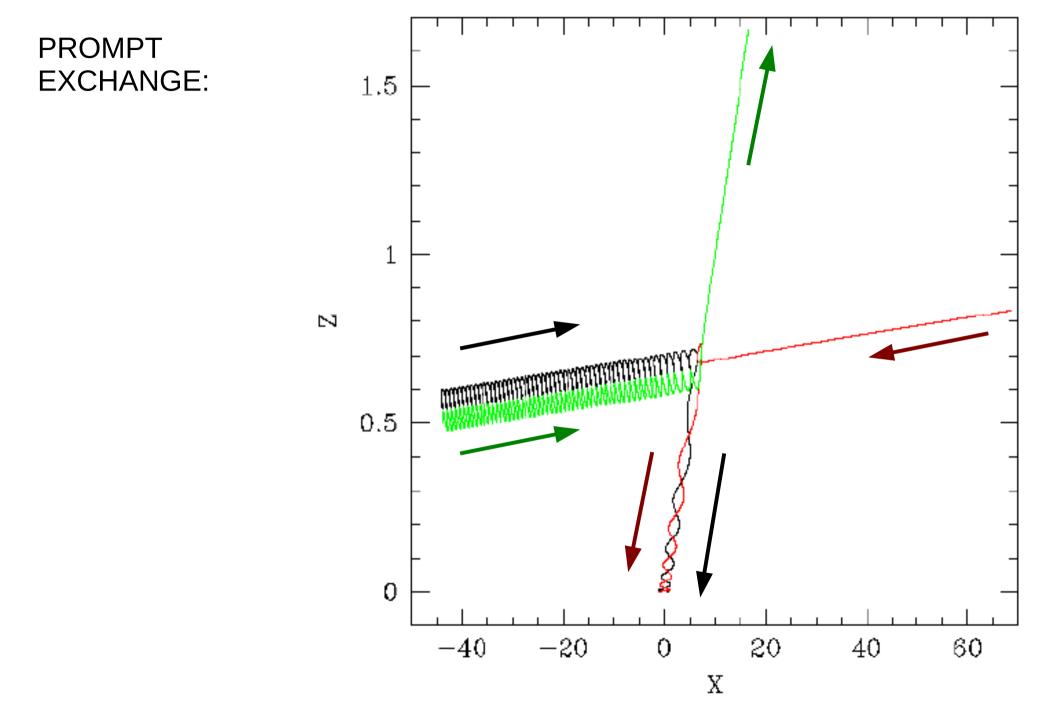
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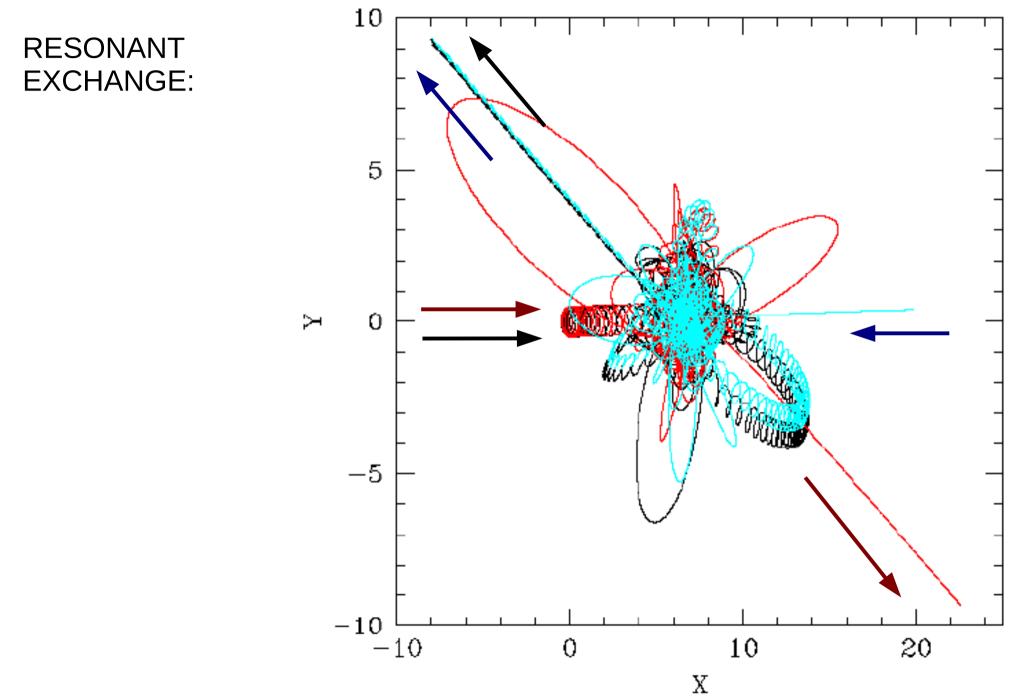
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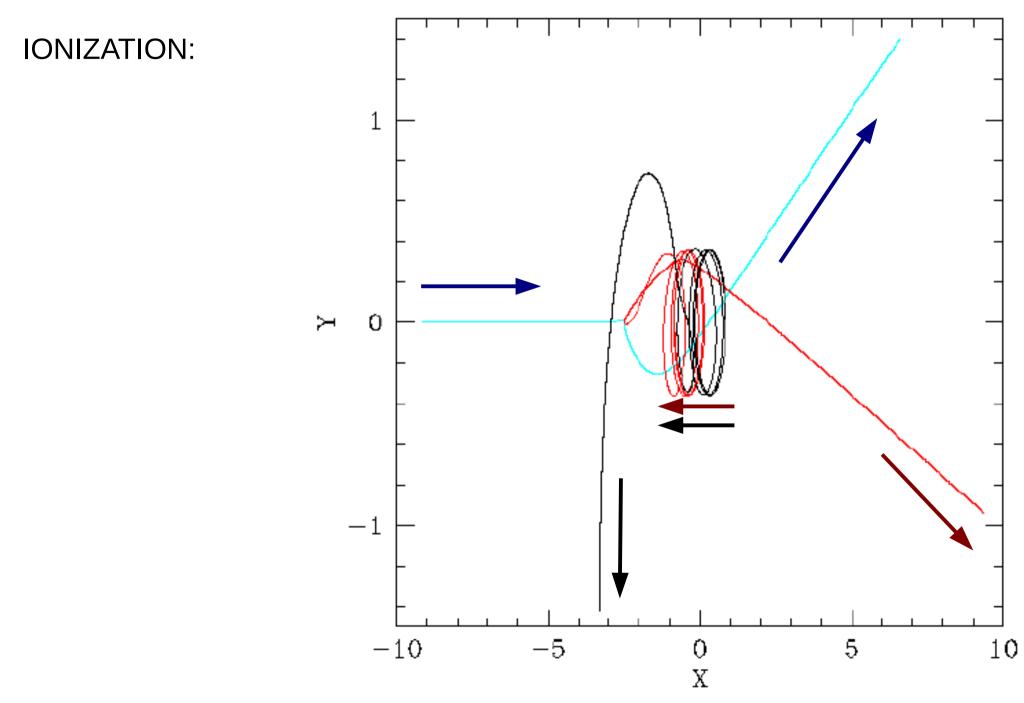
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5. impact of environment on merger rate



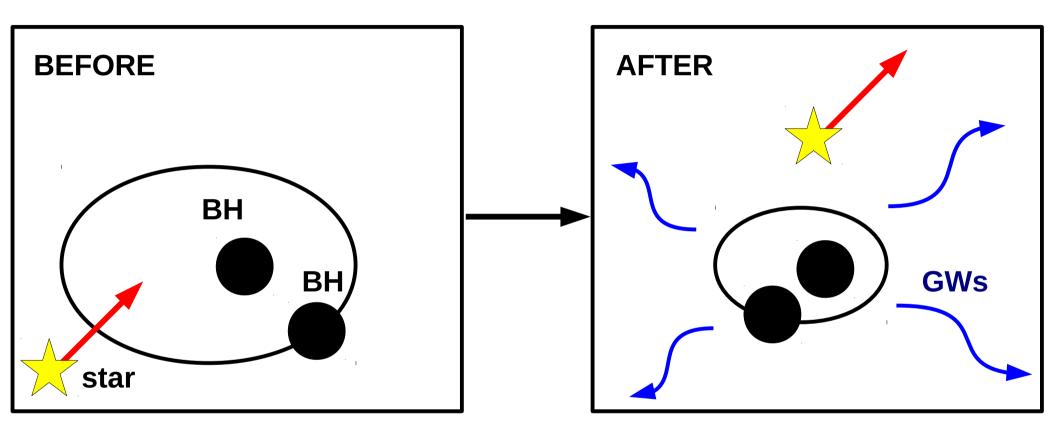
5. impact of environment on merger rate



## **IMPORTANT INFORMATION to understand importance of 3-body encounters for GRAVITATIONAL WAVES**

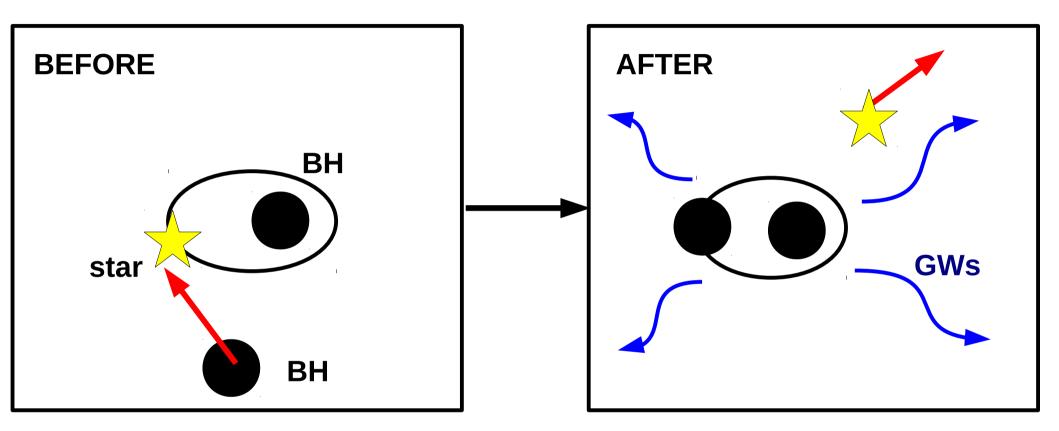
- 1 → If star extracts  $E_{int}$  from the binary, the binary SHRINKS: semi-major axis decreases
- 2→ EXCHANGES bring to formation of more and more massive binaries
- 3 → If star extracts  $E_{int}$  from the binary, the binary and the star RECOIL: may be ejected from the SC

Which is the effect of 3-body encounters on BH-BH binaries?



After 3-body encounters, the semi-major axis shrinks and the BH-BH (or BH-NS or NS-NS) binary becomes important as gravitational wave (GW) source

Which is the effect of 3-body encounters on BH-BH binaries?

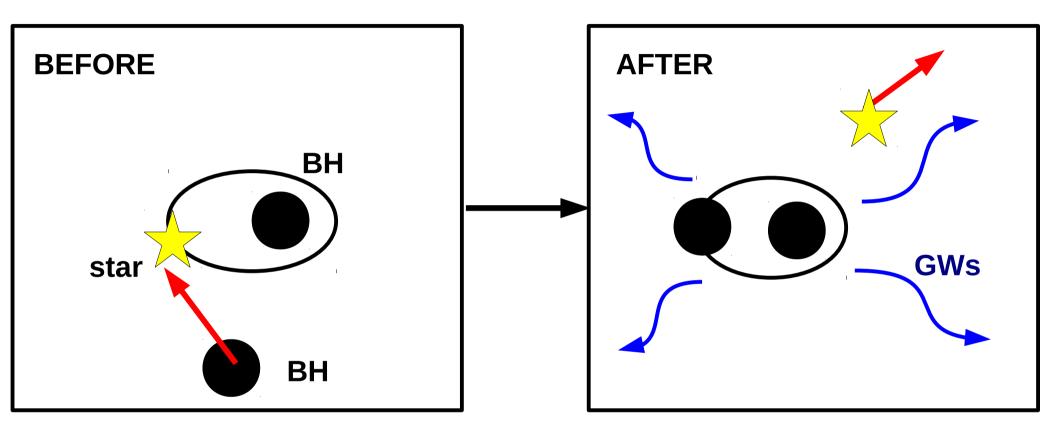


Exchanges are very important: bring BHs in binaries

BHs are FAVOURED BY EXCHANGES BECAUSE THEY ARE MASSIVE!

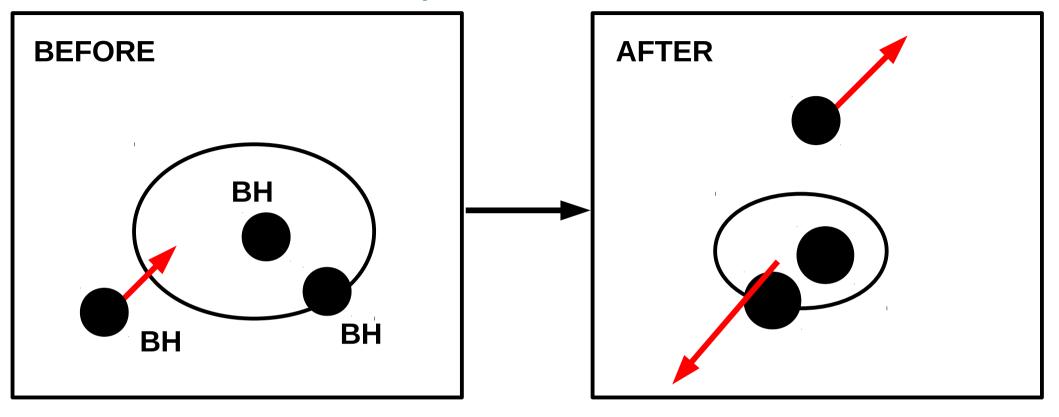
BH BORN FROM SINGLE STAR IN THE FIELD NEVER ACQUIRES A COMPANION BH BORN FROM SINGLE STAR IN A SC LIKELY ACQUIRES COMPANION FROM DYNAMICS

Which is the effect of 3-body encounters on BH-BH binaries?



EXCHANGES FAVOUR THE FORMATION of BH-BH BINARIES WITH THE MOST MASSIVE BHs !!

Which is the effect of 3-body encounters on BH-BH binaries?

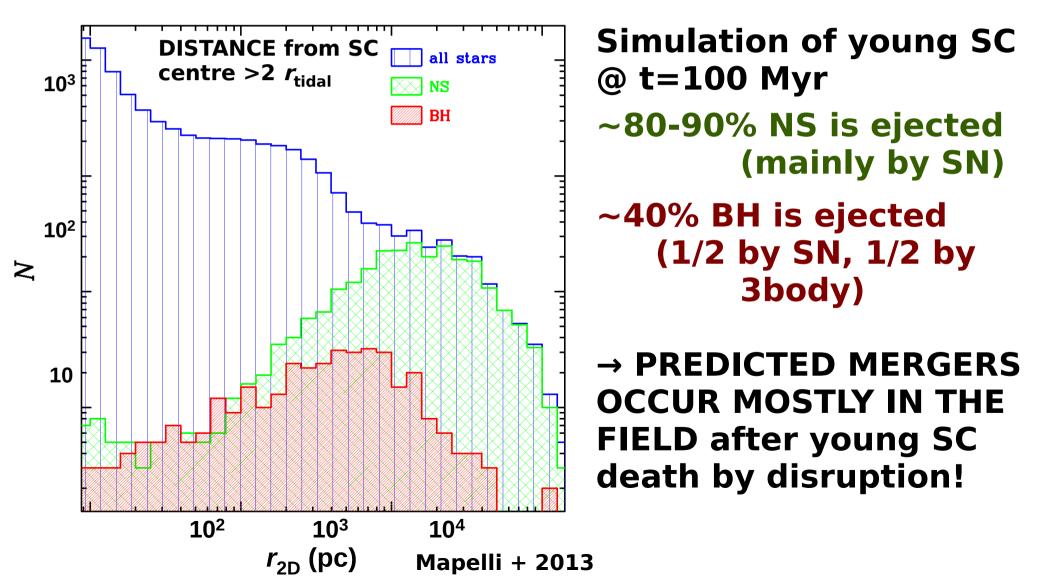


Internal energy is extracted from the binary

- Converted into KINETIC ENERGY of the INTRUDER AND of the CM of the BINARY
- BOTH RECOIL and can be ejected from SC

#### Star clusters lose large fraction of mass by

- 1. high-speed EJECTIONS (caused by SN kick and 3-body, enhanced by Spitzer instability)
  - 2. low-speed evaporation (less bound stars leave the star cluster)
  - 3. tidal fields



#### WE EXPECT THAT

#### **GLOBULAR CLUSTERS:**

- dynamics enhances formation of BH-BH binaries (with respect to NS-NS)
- some (?) BH-BH and NS-NS are ejected due to SN kicks and/or 3-body encounters

#### YOUNG STAR CLUSTERS AND OPEN CLUSTERS:

- dynamics enhances formation of BH-BH binaries (with respect to NS-NS)
- MOST (all?) BH-BH and NS-NS are ejected due to SN kicks, 3-body encounters and evaporation

FIELD:

- more NS-NS than BH-BH because no dynamics

#### DOES THIS AGREE with MODELS/SIMULATIONS OF MERGER RATE?

## HOW DO WE MODEL/SIMULATE THIS?

#### **STAR CLUSTERS:**

**1. MONTE CARLO simulations** (dynamics only or coupled with stellar and binary evolution)

### **PROS:**

- \* very fast treatment of dynamics (N logN)
- \* large number of objects

#### CONS:

 \* assume equilibrium and spherical symmetry

→ GLOBULAR CLUSTERS

#### 2. DIRECT N-BODY simulations (dynamics only or coupled with stellar and binary evolution)

#### PROS:

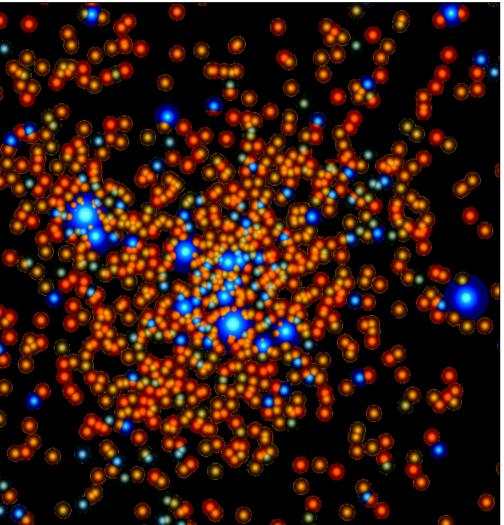
- \* very accurate treatment of dynamics
- \* each particle is a single star with physical mass, radius
- \* no necessary assume equilibrium and symmetry

#### CONS:

- \* slow (N^2, but GRAPHICS PROCESSING UNITS)
- → YOUNG STAR CLUSTERS

#### HOW DO WE MODEL/SIMULATE THIS?

#### **STAR CLUSTERS:**



2. DIRECT N-BODY simulations (dynamics only or coupled with stellar and binary evolution)

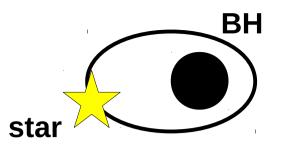
#### **PROS**:

- \* very accurate treatment of dynamics
- \* each particle is a single star with physical mass, radius
- \* no necessary assume equilibrium and symmetry

#### CONS:

- \* slow (N^2, but GRAPHICS PROCESSING UNITS)
- → YOUNG STAR CLUSTERS

### HOW DO WE MODEL/SIMULATE THIS?



#### FIELD BINARIES:

Population synthesis models, i.e. stellar and binary evolution of isolated binaries

**PROS:** 

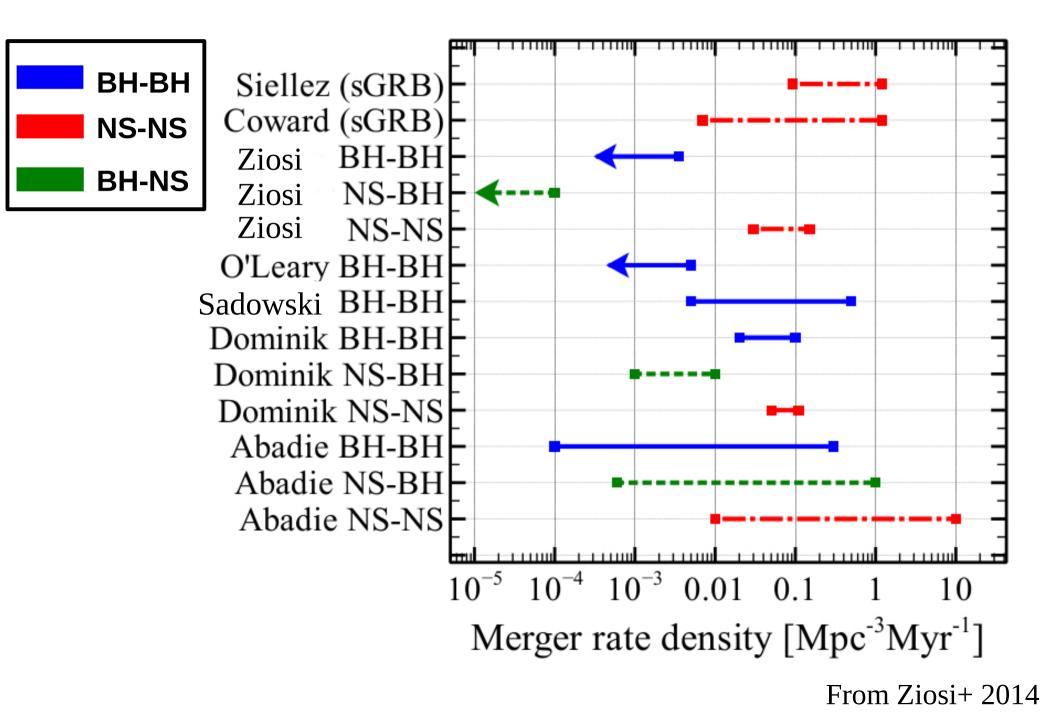
fast → large statistical sample

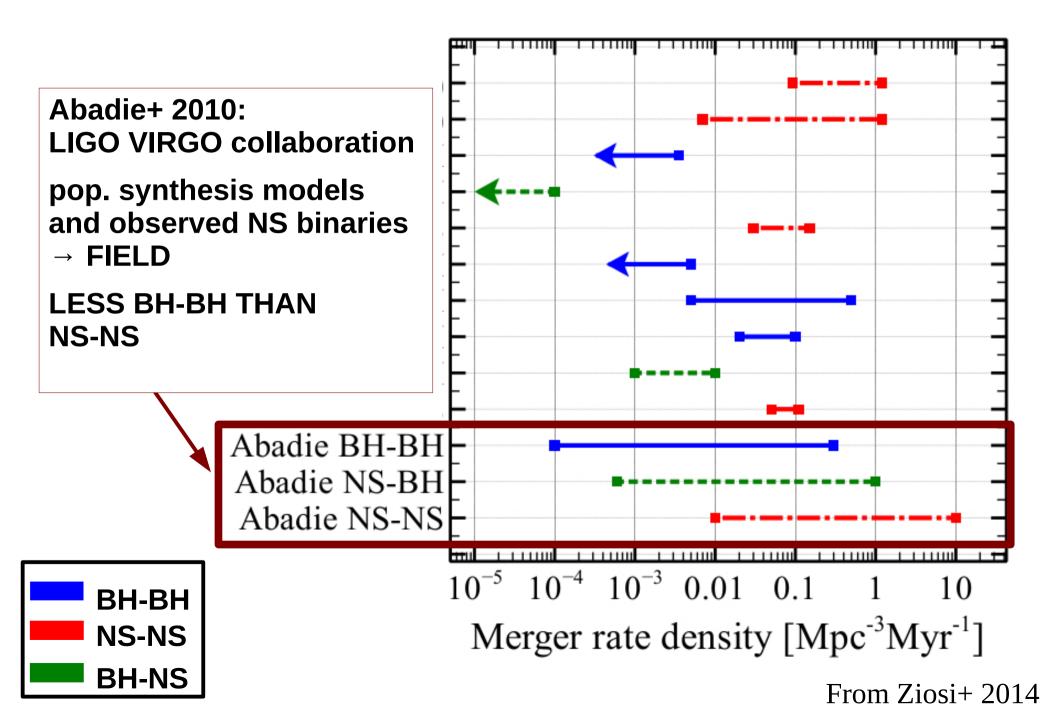
CONS:

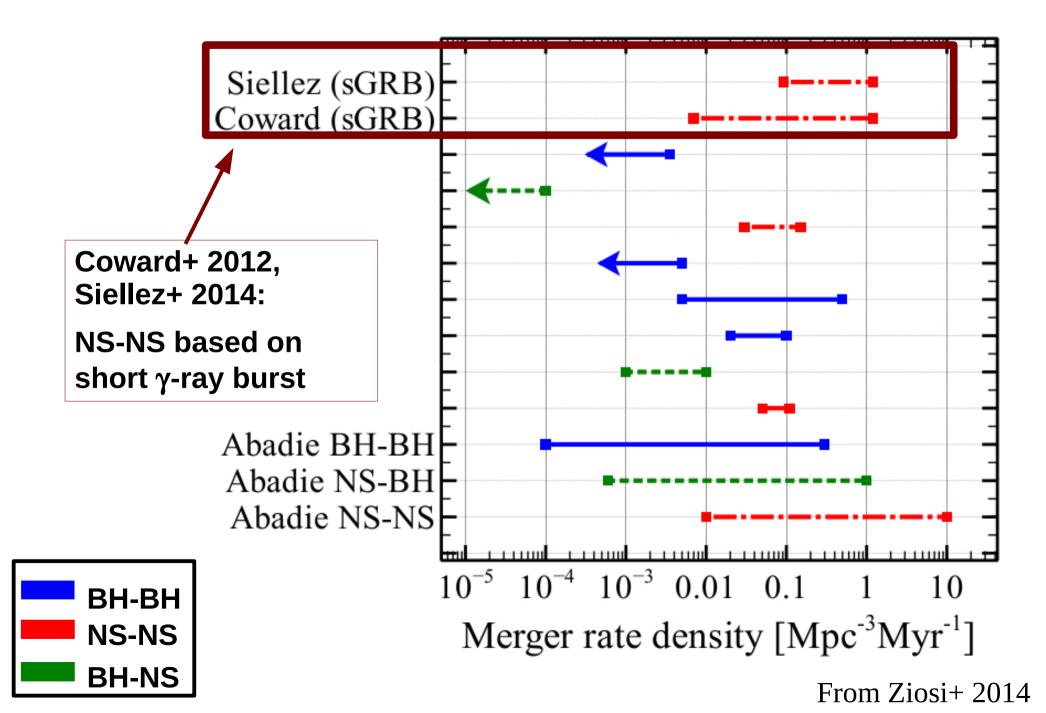
NO dynamics, while we know that stars form in clusters

SOMETIMES COMPLEMENTED BY (or TUNED on the basis of) THE EMPIRICAL APPROACH (see Andrea Possenti's talk):

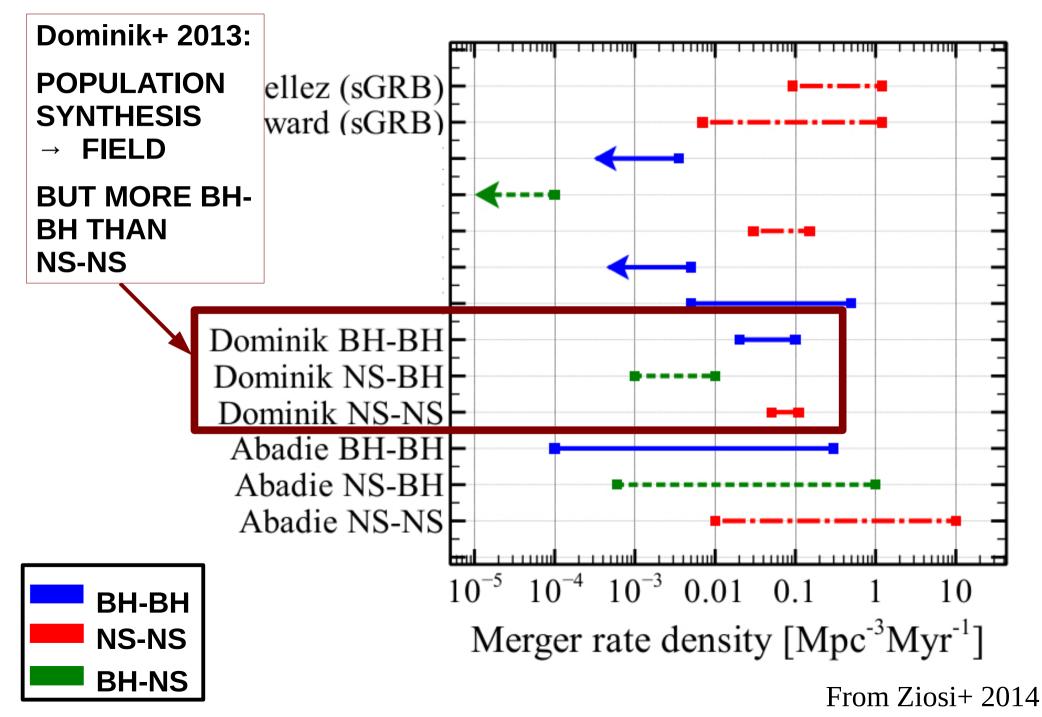
> extrapolation of NS-NS merger rate from observed NS-NS binaries and/or from short gamma-ray burst rate



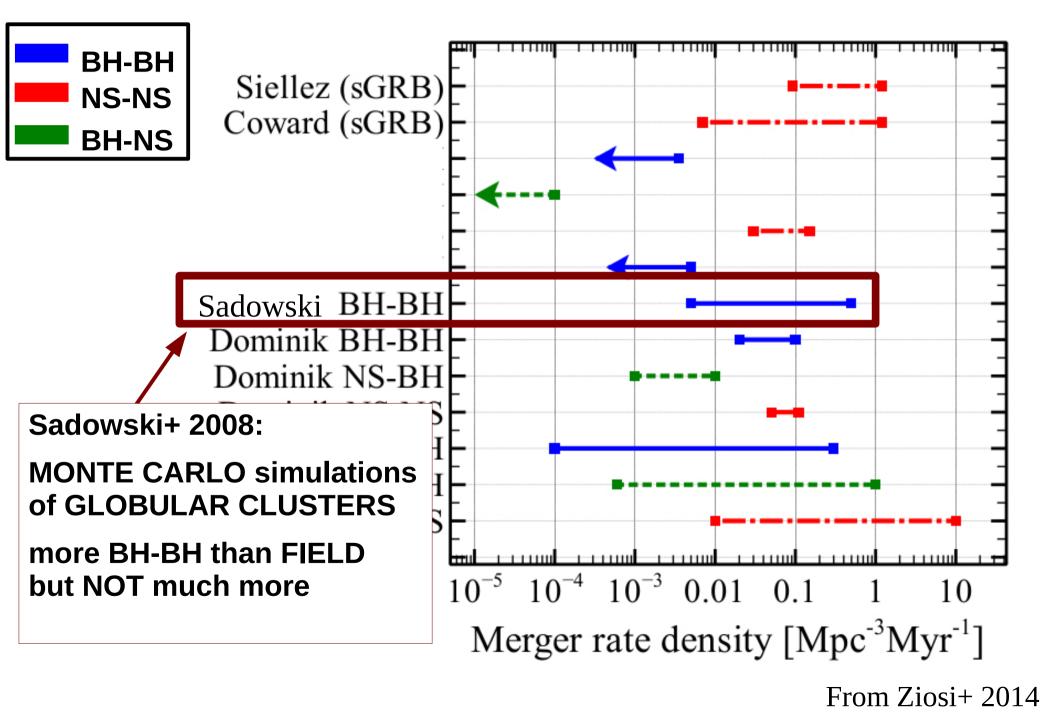




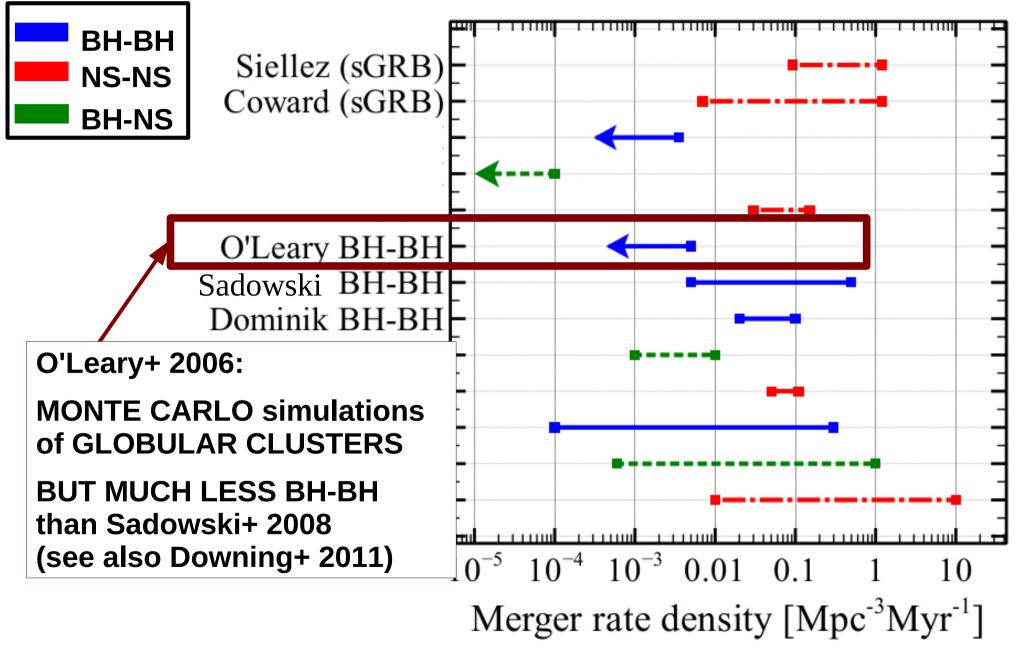
5. impact of environment on merger rate



5. impact of environment on merger rate

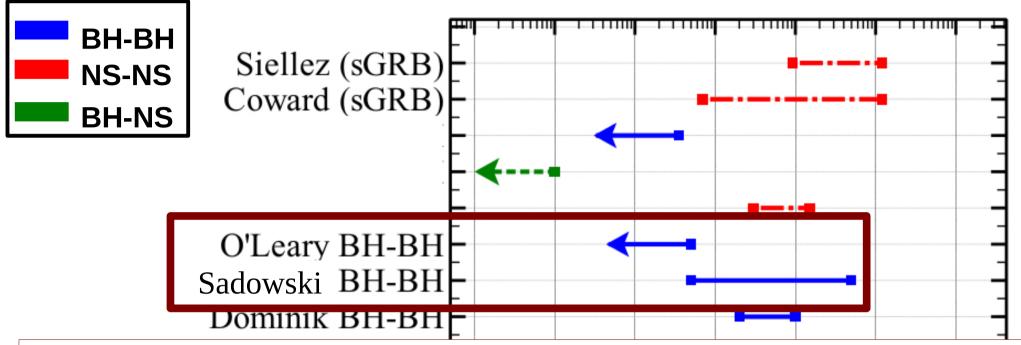


5. impact of environment on merger rate



From Ziosi+ 2014

5. impact of environment on merger rate

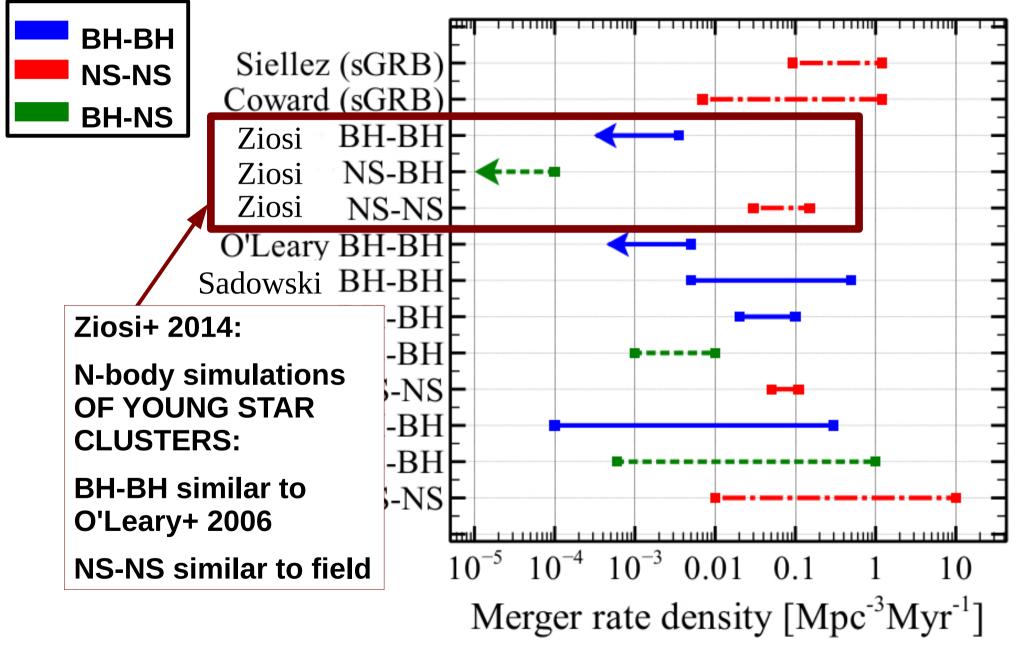


**MONTE CARLO SIMULATIONS of globular clusters:** 

O'Leary+ 2006 assume SPITZER's INSTABILITY LEADS TO EJECTION OF MOST BHs before BH-BH binaries are important for GW (see also Downing+ 2011)

Sadowski+ 2008 assume Spitzer's instability never occurs, BHs remain in equilibrium with other stars

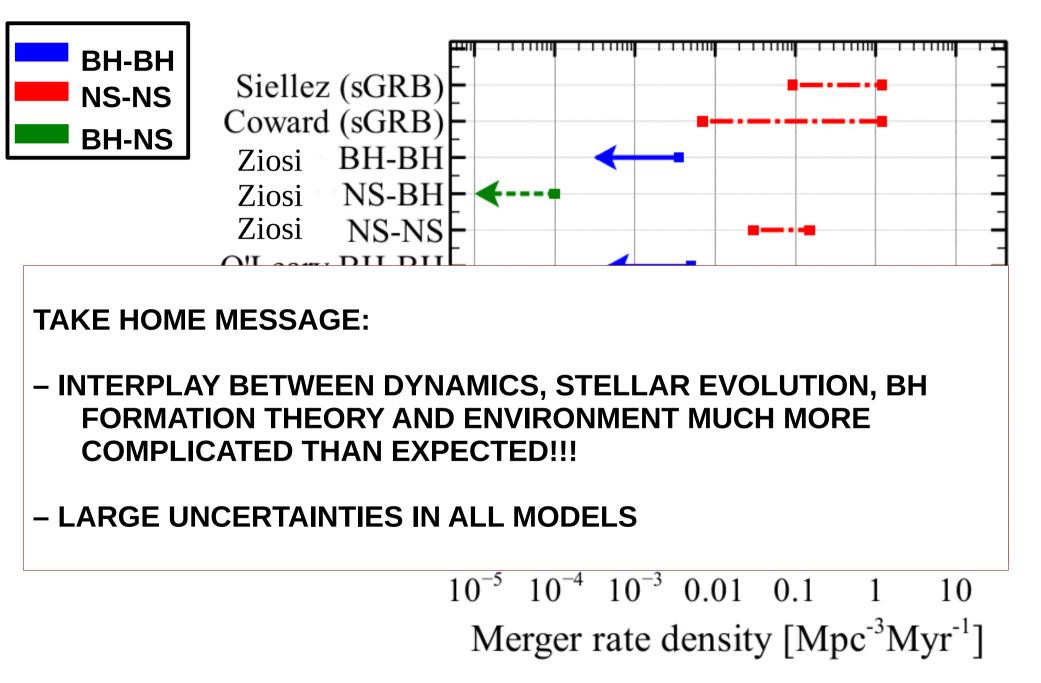
5. impact of environment on merger rate



From Ziosi+ 2014

5. impact of environment on merger rate

**Recent estimates of the MERGER RATE of BH-BH, BH-NS and NS-NS:** 



From Ziosi+ 2014

# WHAT TO DO NEXT

**Possible themes to investigate for Jena:** 

- detectors (build one ;))
- population synthesis or N-body simulation to investigate effect of environment
- follow stellar evolution in a binary to see if it forms a 'good' BH-BH, NS-NS or NS-BH binary
- census of gamma ray bursts, or black hole WR binaries → indicators of GW sources
- calculate merger and detection rate in alternative way to the one I suggested
- your ideas (but discuss them with me before Jena if you need help..)

# **USEFUL REFERENCES**

- Michele Maggiore 2007, Gravitational Waves Volume 1. Theory and Experiments, Oxford University Press

- Christopher Berry's website, http://cplberry.com/2015/01/10/1408-0740/
- Space Time Quest, the interferometer game, http://www.gwoptics.org/processing/space\_time\_quest/
- Abadie+ 2010 (LIGO/Virgo paper on sources), http://arxiv.org/abs/1003.2480
- Coward+ 2012 (GW rate from γ-ray bursts), http://arxiv.org/abs/1202.2179
- Ziosi+ 2014 (dynamics of BH-BH, NS-NS), http://arxiv.org/abs/1404.7147
- Lorimer 2008 (pulsar binary census and pulsar physics), http://relativity.livingreviews.org/Articles/Irr-2008-8/

- N-body codes to play with : starlab (http://www.sns.ias.edu/~starlab/), HiGPUs (http://astrowww.phys.uniroma1.it/dolcetta/HPCcodes/HiGPUs.html), Nbody6 (http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm) ASK ME IF YOU WANT TO PLAY WITH N-BODY!

- stellar and binary evolution codes: MESA (http://mesa.sourceforge.net/) ASK ME IF YOU WANT TO PLAY WITH MESA!

## **3. gravitational wave sources**

WHY 'normal' stars in binaries are not sources of gravitational waves?

# WHY 'normal' stars in binaries are not sources of gravitational waves?

# **5.** impact of environment on merger rate

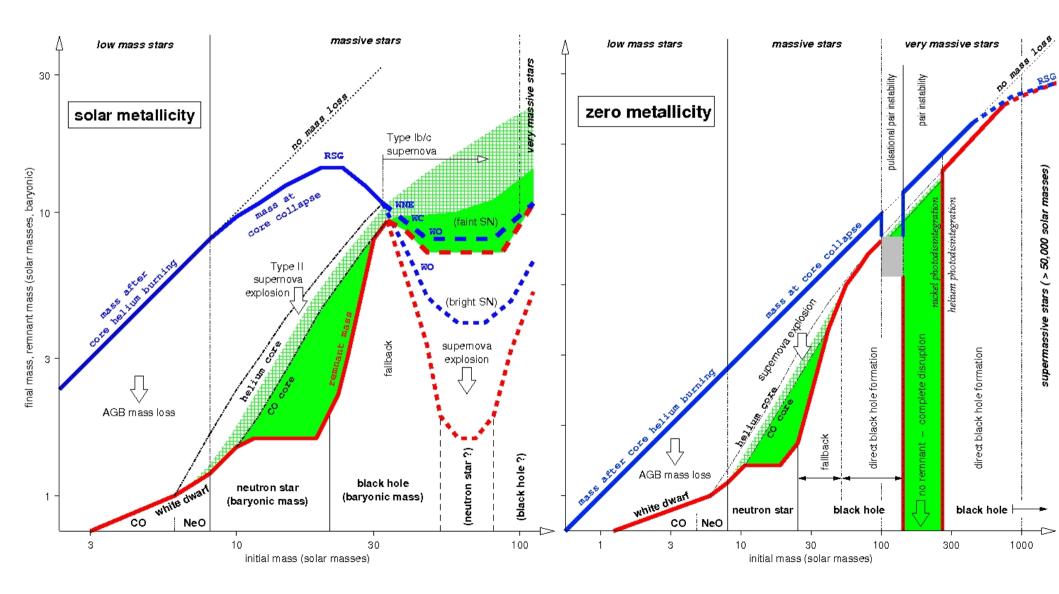
## 5. impact of environment on merger rate

WHY DYNAMICS??????

## 5. impact of environment on merger rate

WHY DYNAMICS??????

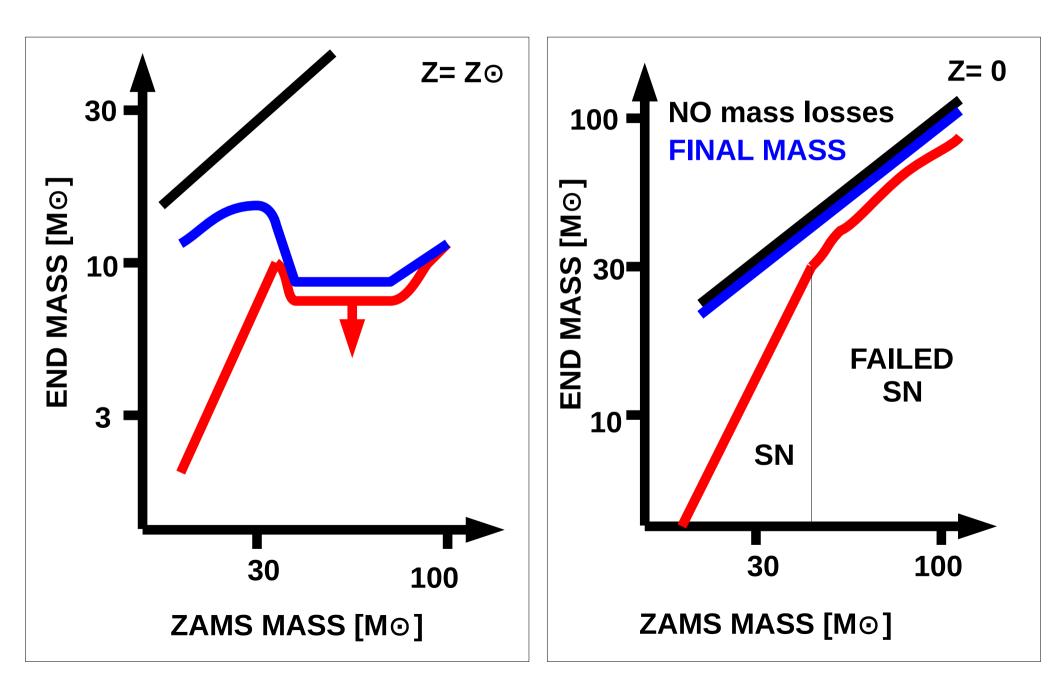
# **3. gravitational wave sources** HOW DO BHs and NSs form?



Heger et al.

## **3. gravitational wave sources**

HOW DO BHs and NSs form?



# **COLLISIONAL/COLLISIONLESS?**

- Collisional systems are systems where interactions between particles are EFFICIENT with respect to the lifetime of the system
- **Collisionless systems** are systems where interactions are negligible

#### When is a system collisional/collisionless?

#### **RELAXATION TIMESCALE**

Gravity is a LONG-RANGE force

 $\rightarrow$  Two-body encounters are important even if 2 bodies are distant

→ **two-body relaxation timescale**: timescale needed for a star to lose completely memory of its initial velocity ( $\Delta v/v \sim 1$ ) by the effect of two body encounters

## **COLLISIONAL/COLLISIONLESS?**

**two-body relaxation timescale**: timescale needed for a star to lose completely memory of its initial velocity ( $\Delta v/v \sim 1$ ) by the effect of two body encounters

$$t_{\rm rlx} = n_{\rm cross} t_{\rm cross} = \frac{N}{8 \ln N} \frac{R}{v}$$

with more accurate calculations, based on diffusion coefficients (Spitzer & Hart 1971):

$$t_{\rm rlx} = 0.34 \, \frac{\sigma^3}{G^2 \, m \, \rho \ln \Lambda}$$

MOST USEFUL EXPRESSION:

$$t_{\rm rlx} = 10 \,{\rm Myr} \, \left(\frac{M_{\rm TOT}}{3500 \,{\rm M}_\odot}\right)^{1/2} \, \left(\frac{r_{\rm hm}}{1 \,{\rm pc}}\right)^{3/2}$$

0

# Which is the typical t<sub>rlx</sub> of stellar systems?

**\* globular clusters, dense young star clusters, nuclear star clusters** (far from SMBH influence radius)
 R~1-10 pc, N~10^3-10^6 stars, v~1-10 km/s

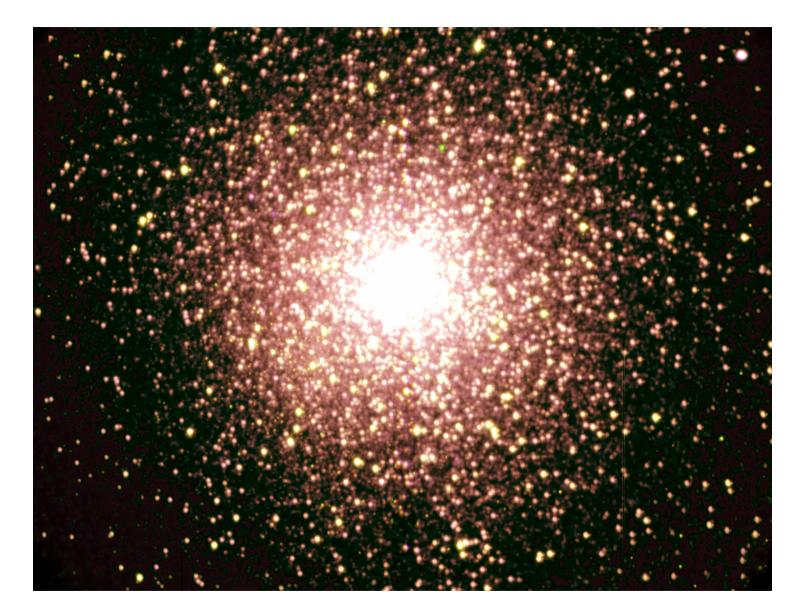
 $\rightarrow$  COLLISIONAL

galaxy field/discs
 R~10 kpc, N~10^10 stars, v~100-500 km/s

# t<sub>rlx</sub> >> Hubble time

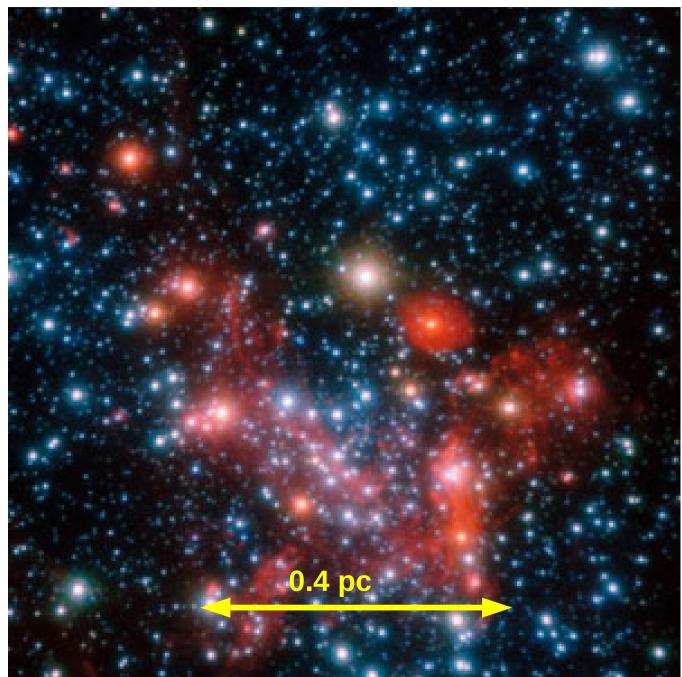
→ COLLISIONLESS

#### **EXAMPLES of COLLISIONAL stellar systems**



*t*<sub>rlx</sub>~1 Gyr Globular clusters (47Tuc)

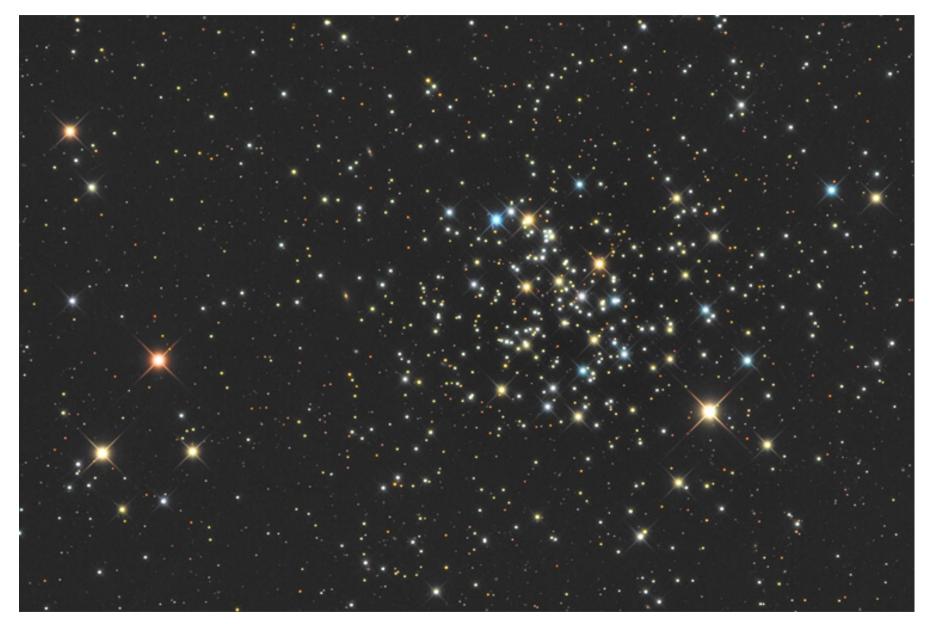
### **EXAMPLES of COLLISIONAL stellar systems**



# *t*<sub>rlx</sub>∼ 0.1 Gyr

Nuclear star clusters (MW) NaCo @ VLT Genzel+2003

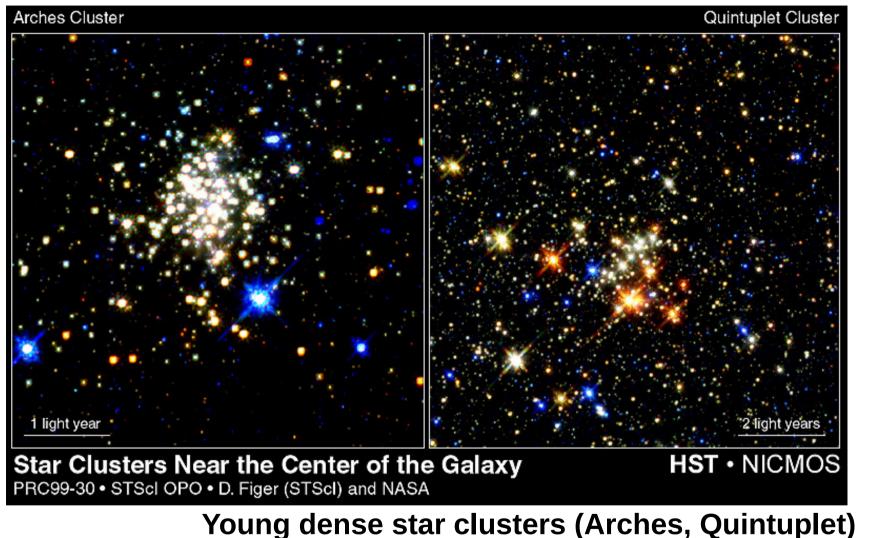
#### **EXAMPLES of COLLISIONAL stellar systems**



*t*<sub>rlx</sub>∼ 100 Myr

**Open clusters (M67)** Courtesy Bob Franke

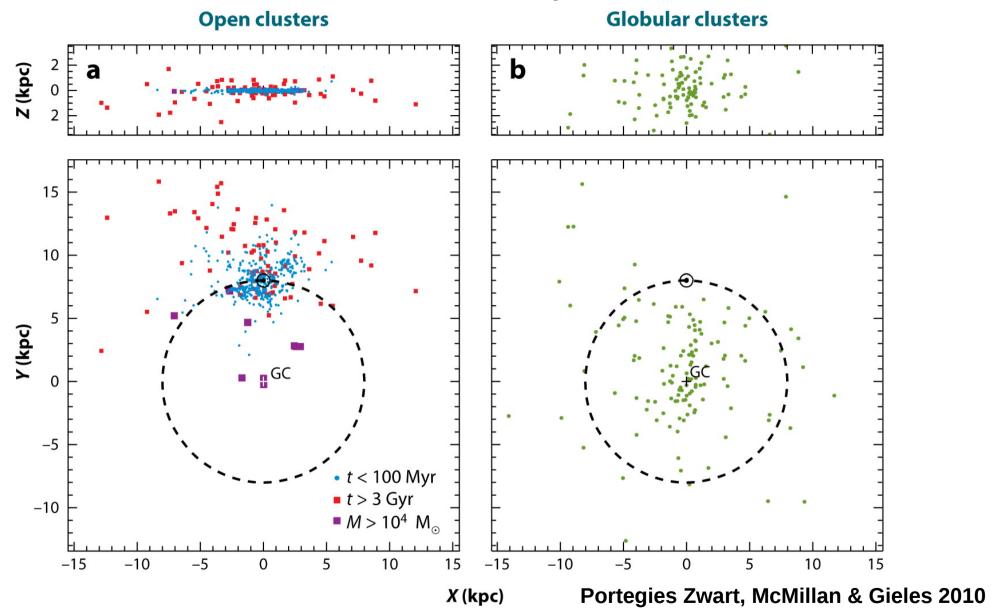
### **EXAMPLES of COLLISIONAL stellar systems**



*t*<sub>rlx</sub>~ 10-100 Myr

#### VERY IMPORTANT BECAUSE ARE THE BIRTHPLACE OF STARS IN LOCAL UNIVERSE!!

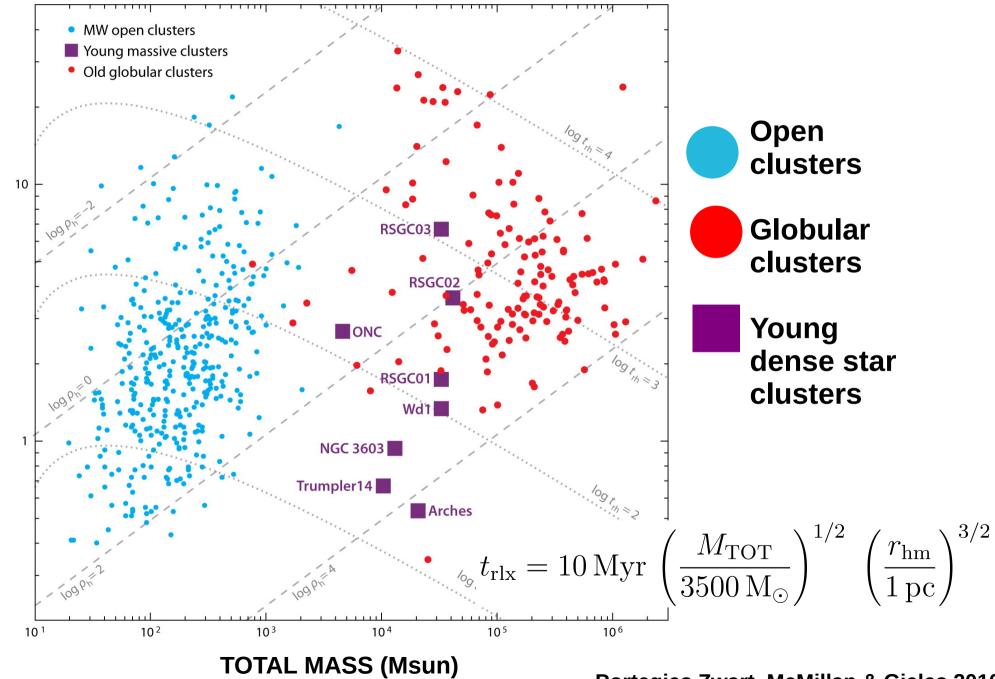
#### **DISTRIBUTION of COLLISIONAL stellar systems in the MILKY WAY**



GLOBULAR CLUSTERS ARE A HALO POPULATION YOUNG and OPEN CLUSTERS ARE A DISC POPULATION

HALF-MASS RADIUS (pc)

#### MAIN PROPERTIES of COLLISIONAL stellar systems in the MILKY WAY



Portegies Zwart, McMillan & Gieles 2010

MAIN PROPERTIES of COLLISIONAL stellar systems in the MILKY WAY

cluster	age	$m_{ m to}$	M	$r_{ m vir}$	ρ <sub>c</sub>	Z	location	$t_{ m dyn}$	$t_{ m rh}$
	[Gyr]	$[M_{\odot}]$	$[M_{\odot}]$	[pc]	$[M_{\odot}/pc^{3}]$	$[\mathbf{Z}_{\odot}]$		[Myr]	[Myr]
OC	≲ 0.3	≈4	$\lesssim 10^3$	1	$\lesssim 10^3$	~1	disk	~1	$\lesssim 100$
GC	$\gtrsim 10$	~ 0.8	$\gtrsim 10^5$	10	$\gtrsim 10^3$	<1	halo	$\gtrsim 1$	$\gtrsim 1000$
YMC	≲ 0.1	$\gtrsim 5$	$\gtrsim 10^4$	1	$\gtrsim 10^3$	≳1	galaxy	$\stackrel{<}{\sim} 1$	$\lesssim 100$

Portegies Zwart, McMillan & Gieles 2010

# ONE OF THE MAIN PROPERTIES OF COLLISIONAL SYSTEMS IS THAT

## THREE-BODY ENCOUNTERS (= CLOSE GRAVITATIONAL ENCOUNTERS BETWEEN A BINARY AND A SINGLE STAR)

**ARE FREQUENT IN COLLISIONAL SYSTEMS** 

# **SPITZER INSTABILITY:** – ENHANCES EJECTIONS –

#### MASS SEGREGATION Consequence of equipartition theorem (for two-body encounters):

Particles tend to have the same average kinetic energy  $m_i \left< v_i^2 \right> = m_j \left< v_j^2 \right>$ 

If stars are equal mass  $\rightarrow$  equipartition implies that have the same average VELOCITY

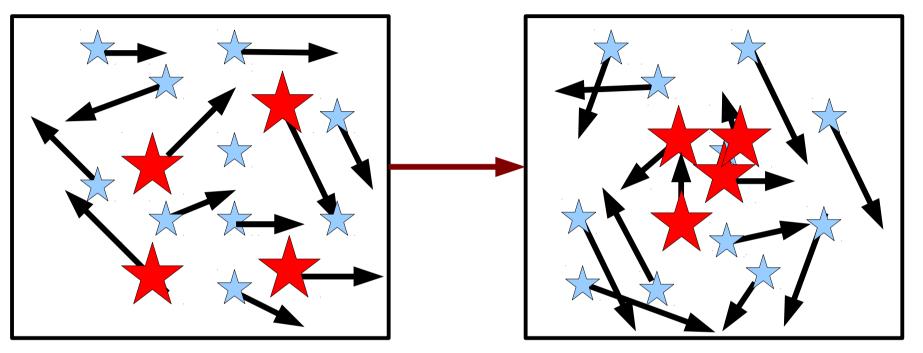
If stars have different masses, this has a relevant consequence:

if 
$$m_i > m_j \Rightarrow \langle v_i^2 \rangle < \langle v_j^2 \rangle$$

During two-body encounters, massive stars transfer kinetic energy to light stars. Massive stars slow down, light stars move to higher velocities.

# **SPITZER INSTABILITY:**

#### MASS SEGREGATION



During two-body encounters, massive stars transfer kinetic energy to light stars. Massive stars slow down, light stars move to higher velocities.

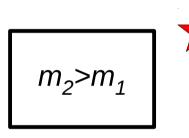
This means that **heavier stars drift to the centre of the cluster**, producing **MASS SEGREGATION** (i.e. local mass function different from IMF)

Equipartition in multi-mass systems is reached via dynamical friction

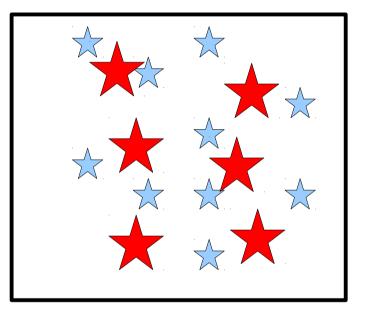
# **SPITZER INSTABILITY** (or mass stratification instability):

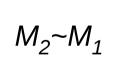
# It is not always possible to reach equipartition in a multi-mass system.

Let us suppose that there are two populations with two different masses:



- HEAVY POPULATION m<sub>2</sub> (total mass M<sub>2</sub>)
- LIGHT POPULATION  $m_1$  (total mass  $M_1$ )





If the total mass of the heavy population is similar to the total mass of the light population, equipartition is not possible:

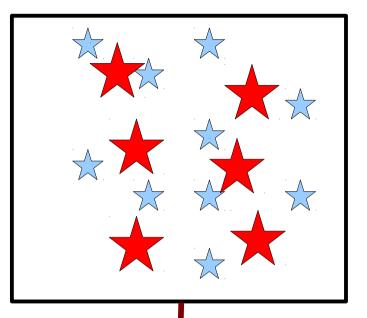
$$M_2 \langle v_2^2 \rangle >> M_1 \langle v_1^2 \rangle$$

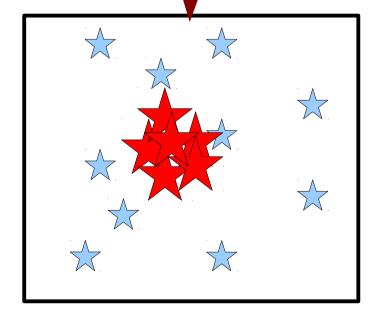
THE LIGHT POPULATION CANNOT ABSORB ALL THE KINETIC ENERGY THAT MUST BE TRANSFERRED FROM THE HEAVY POPULATION TO REACH EQUIPARTITION

# **SPITZER INSTABILITY** (or mass stratification instability):

The heavy population forms a CLUSTER WITHIN THE CLUSTER (sub-cluster at the centre of the cluster), DYNAMICALLY DECOUPLED from the rest of the cluster.

The massive stars in the sub-cluster keep transferring kinetic energy to the lighter stars but cannot reach equipartition: the core of massive stars continues to **CONTRACT TILL INFINITE DENSITY**!



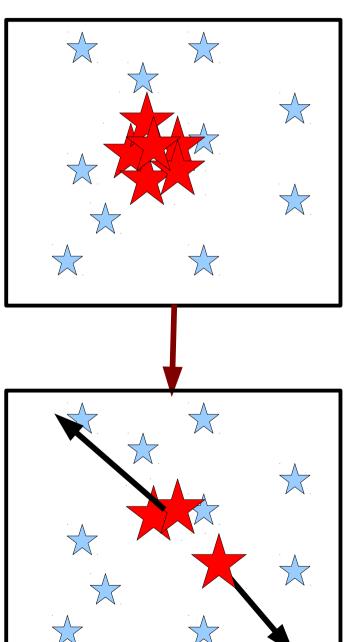


# **SPITZER INSTABILITY** (or mass stratification instability):

#### The contraction stops

 when most of the massive stars eject each-other from the SC by 3-body encounters

SPITZER INSTABILITY ENHANCES THE EJECTION OF MASSIVE OBJECTS (E.G. BLACK HOLES) FROM SCs !!!!



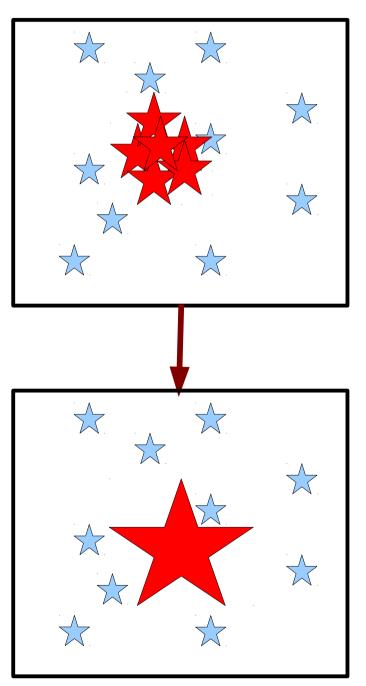
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SPITZER INSTABILITY ENHANCES THE EJECTION OF MASSIVE OBJECTS (E.G. BLACK HOLES) FROM SCs !!!!

 or when most of the massive stars collapse into a single object



#### IMBHs are BHs with mass 10^2-10^5 Msun

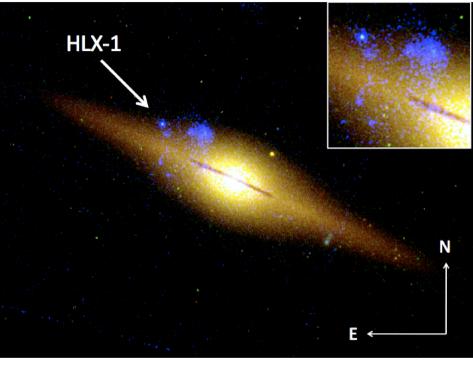
#### $\rightarrow$ some limited observational signature

1\* Hyper-luminous X-ray source HLX-1 close to ESO 243-49



peak  $L_X \sim 10^{42}$  ergs, X-ray VARIABILITY, redshift consistent with ESO 243-49 (not a background object)  $\rightarrow$  BH mass $\sim 10^4$  M $\odot$ 

Farrell+ 2009, 2012, 2014; Soria+ 2010, 2012; Mapelli+ 2012, 2013



2\* centre of G1 globular cluster (dwarf nucleus?) in Andromeda

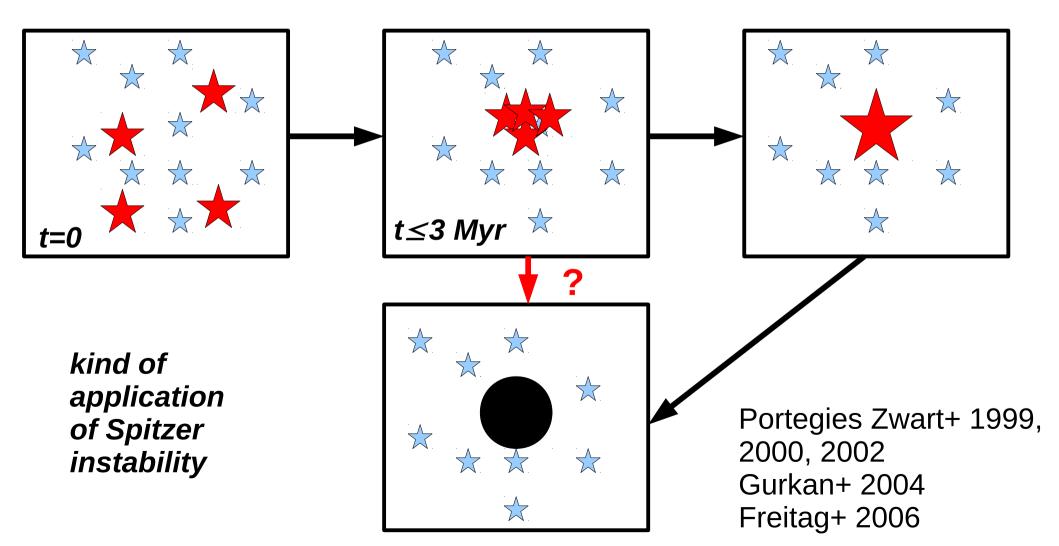
- → cannot form through stellar evolution (at least in current Universe)
- $\rightarrow$  theoretical models predict formation through dynamics

#### 4. mechanisms for formation of IMBHs

## **1-** runaway collapse of stars at centre of star cluster

IDEA: mass segregation brings very massive stars to the centre *If timescale for mass segregation < timescale for stellar evolution* 

+ if encounter rate sufficiently high Massive stars collide, merge and form a super-massive star, which collapses to a BH

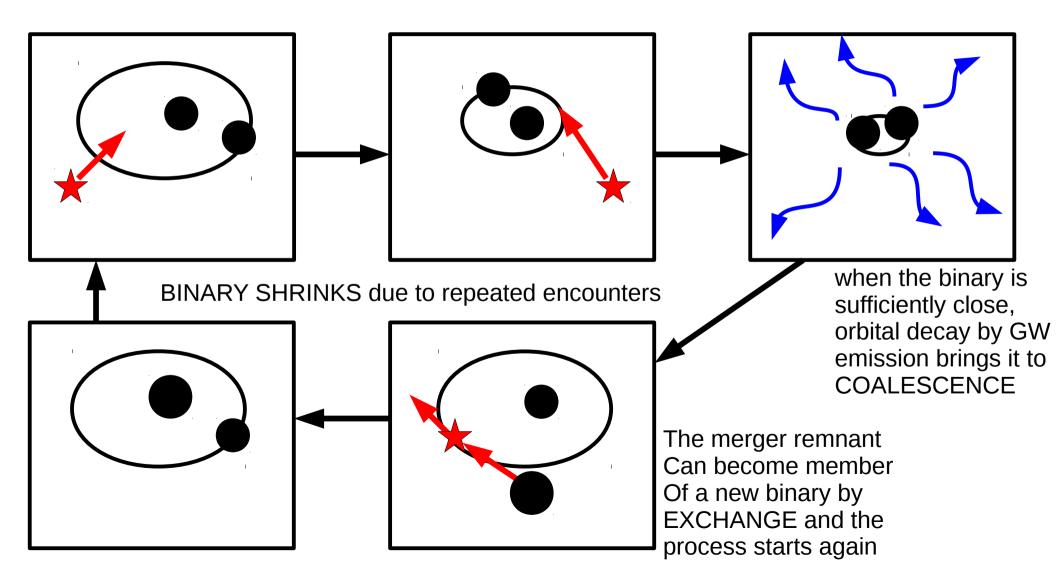


4. mechanisms for formation of IMBHs

#### 2- repeated mergers

Formalism by Miller & Hamilton (2002)

In a old cluster stellar BHs can grow in mass because of repeated mergers with the companion triggered by 3-body encounters



# **CONCLUSIONS:**

COLLISIONAL SYSTEMS are very important for the evolution of BH and NS systems:

- THREE-BODY ENCOUNTERS enhance formation of BH binaries
- BUT THREE-BODY ENCOUNTERS can also eject compact objects
- SPITZER INSTABILITY PLAYS A ROLE in enhancing ejections
- IMBHs might form dynamically
- INTERPLAY BETWEEN DYNAMICS, STELLAR EVOLUTION, BH FORMATION THEORY AND ENVIRONMENT MUCH MORE TRICKY THAN EXPECTED!!!
- LARGE UNCERTAINTIES IN ALL MODELS of BH and NS binaries

Most general expression of recoil velocity for the reduced particle (Sigurdsson & Phinney 1993)

$$v_{fin} = \sqrt{\frac{m_3 (m_1 + m_2)}{m_e (m_a + m_b)}} v_{\infty}^2 + \frac{2 m_T}{m_e (m_a + m_b)} \Delta E_b$$

 $m_a$ ,  $m_b$  and  $m_e$  are the final mass of the primary binary member, the final mass of the secondary binary member and the final mass of the single star, respectively FROM ENERGY CONSERVATION:

$$\frac{1}{2} \frac{m_3 \left(m_1 + m_2\right)}{m_T} v_{\infty}^2 + \Delta E_b = \frac{1}{2} \frac{m_e \left(m_a + m_b\right)}{m_T} v_{fin}^2$$

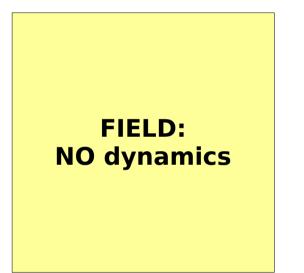
#### What happens to the binary, then?

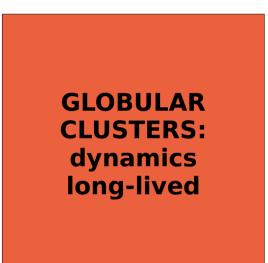
The recoil of the binary (if the binary is more massive than the single star -i.e. the motion of the single star coincides almost with that of the reduced particles) follows from conservation of linear momentum

$$v_{rec} = \frac{m_e}{m_T} v_{fin}$$

Star clusters lose large fraction of mass by

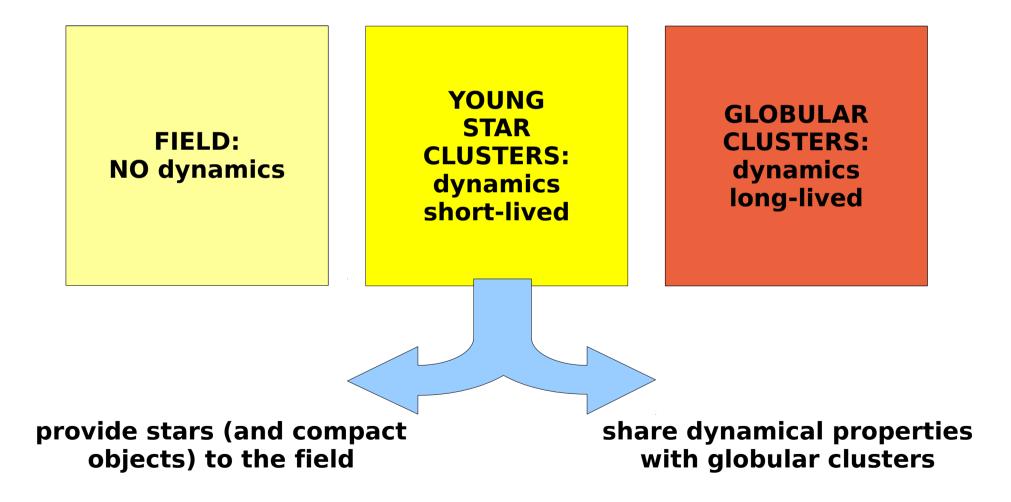
- 1. high-speed EJECTIONS (caused by SN kick and 3-body)
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- 3. tidal fields





Star clusters lose large fraction of mass by

- 1. high-speed EJECTIONS (caused by SN kick and 3-body)
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## SELECTED REFERENCES

- \* Spitzer L., Dynamical evolution of globular clusters, 1987, Princeton University Press
- \* Binney & Tremaine, Galactic Dynamics, First edition, 1987, Princeton University Press
- \* Portegies Zwart & McMillan, 2002, ApJ, 576, 899
- \* Miller & Hamilton, 2002, MNRAS, 330, 232
- \* Kulkarni, Hut & McMillan 1993, Nature 364, 421
- \* Sigurdsson & Hernquist 1993, Nature 364, 42
- \* Mapelli et al. 2013, MNRAS, 429, 2298
- \* Mapelli et a. 2011, MNRAS, 416, 1756

# **SPITZER INSTABILITY** (or mass stratification instability):

It is not always possible to reach equipartition in a multi mass system.

Let us suppose that there are two populations with two different masses:  $m_1$  (total mass  $M_1$ ) and  $m_2$  (total mass  $M_2$ ), with  $m_1 < m_2$ .

We explore 2 limit cases where equipartition is impossible.

1)  $M_2 >> M_1 \Rightarrow$  potential is dominated by massive stars  $\Rightarrow < v^2 >$  of the massive stars is  $\sim \frac{1}{4} < v_{esc}^2 >$  $\Rightarrow if m_2/m_1 > 4$ , the  $< v^2 >$  of light stars is higher than  $< v_{esc}^2 >$ 

#### $\Rightarrow$ ALL LIGHT STARS EVAPORATE FROM THE CLUSTER!!!

Not very important in practice because IMF is not sufficiently top-heavy

# **SPITZER INSTABILITY** (or mass stratification instability):

It is not always possible to reach equipartition in a multi mass system.

Let us suppose that there are two populations with two different masses:  $m_1$  (total mass  $M_1$ ) and  $m_2$  (total mass  $M_2$ ), with  $m_1 < m_2$ .

2)  $M_2 \sim M_1$  (the case of the so called Spitzer's instability)

If the total mass of the heavy population is similar to the total mass of the light population, equipartition is not possible: the heavy population forms a cluster within the cluster, i.e. a sub-cluster at the centre of the cluster, dynamically decoupled from the rest of the cluster. The sub-cluster of the heavy population tends to contract.

# Can we understand whether a binary will lose or acquire $E_b$ ?

YES, but ONLY in a STATISTICAL SENSE

We define HARD BINARIES: binaries  $\frac{G m_1 m_2}{2 a} > \frac{1}{2} \langle m \rangle \sigma^2$  with binding energy higher than the average kinetic energy of a star in the cluster

$$\frac{G \, m_1 \, m_2}{2 \, a} < \frac{1}{2} \left\langle m \right\rangle \sigma^2$$

**SOFT BINARIES:** binaries with binding energy lower than the average kinetic energy of a star in the cluster

# **HEGGIE'S LAW (1975)**:

Hard binaries tend to become harder (i.e. increase  $E_b$ ) Soft binaries tend to become softer (i.e. decrease  $E_b$ ) as effect of three-body encounters

# **Recoil velocities**

Most general expression of recoil velocity for the reduced particle (Sigurdsson & Phinney 1993)

$$v_{fin} = \sqrt{\frac{m_3 (m_1 + m_2)}{m_e (m_a + m_b)}} v_{\infty}^2 + \frac{2 m_T}{m_e (m_a + m_b)} \Delta E_b$$

 $m_a$ ,  $m_b$  and  $m_e$  are the final mass of the primary binary member, the final mass of the secondary binary member and the final mass of the single star, respectively (these may be different from the initial ones in the case of an exchange). This equation comes from (+) at slide 20:

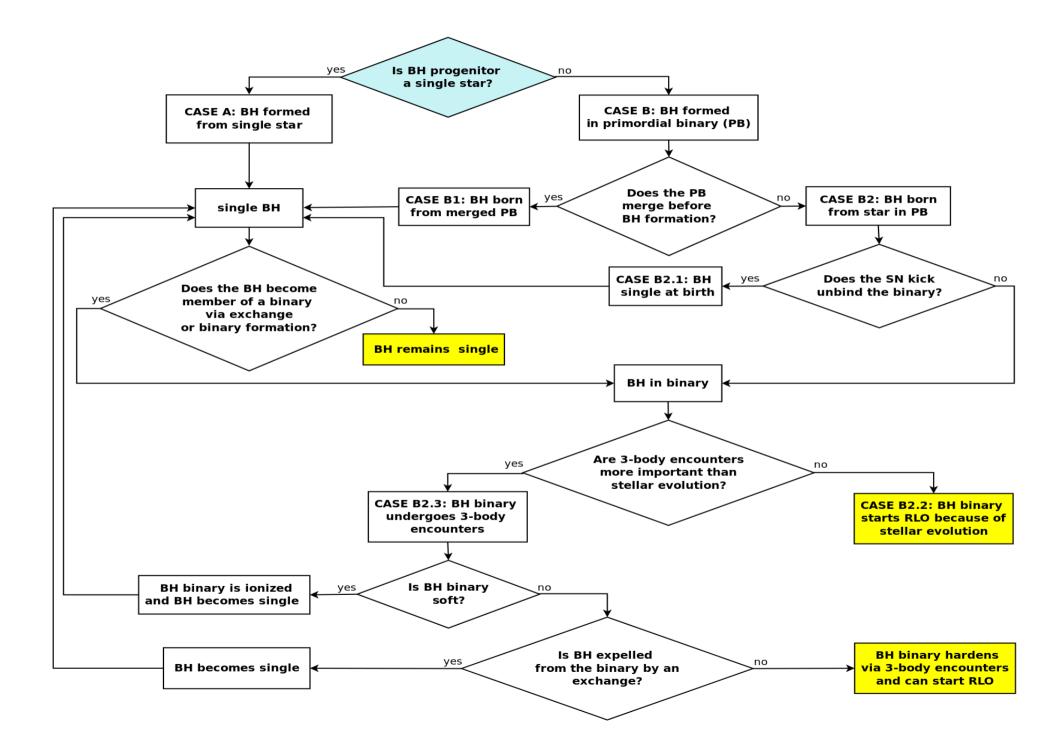
$$\frac{1}{2} \frac{m_3 \left(m_1 + m_2\right)}{m_T} v_\infty^2 + \Delta E_b = \frac{1}{2} \frac{m_e \left(m_a + m_b\right)}{m_T} v_{fin}^2$$

#### What happens to the binary, then?

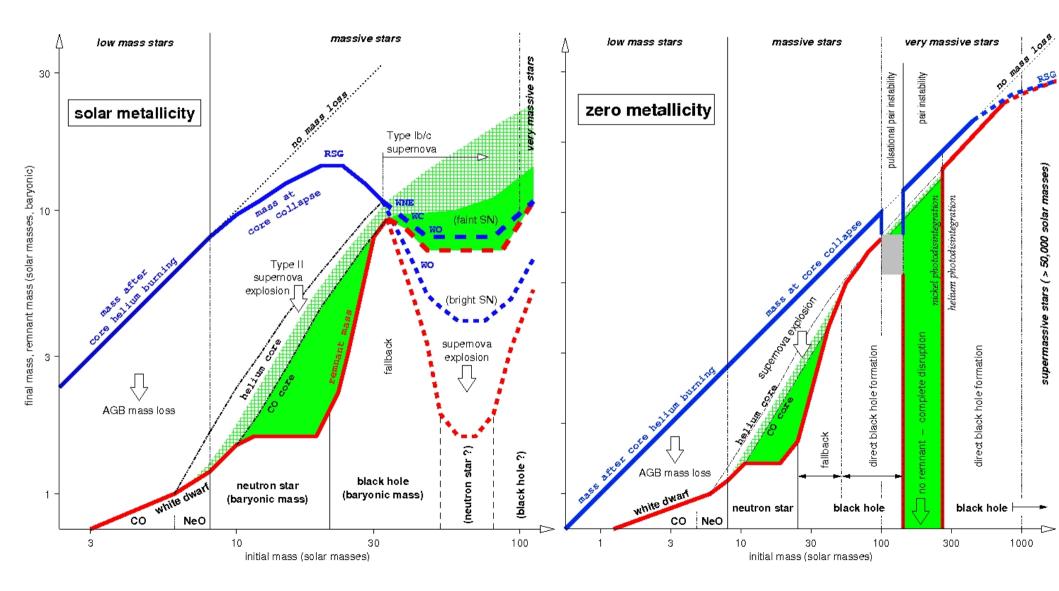
The recoil of the binary (if the binary is more massive than the single star -i.e. the motion of the single star coincides almost with that of the reduced particles) follows from conservation of linear momentum

$$v_{rec} = \frac{m_e}{m_T} \, v_{fin}$$

#### 2. 3-body encounters enhance ULX formation

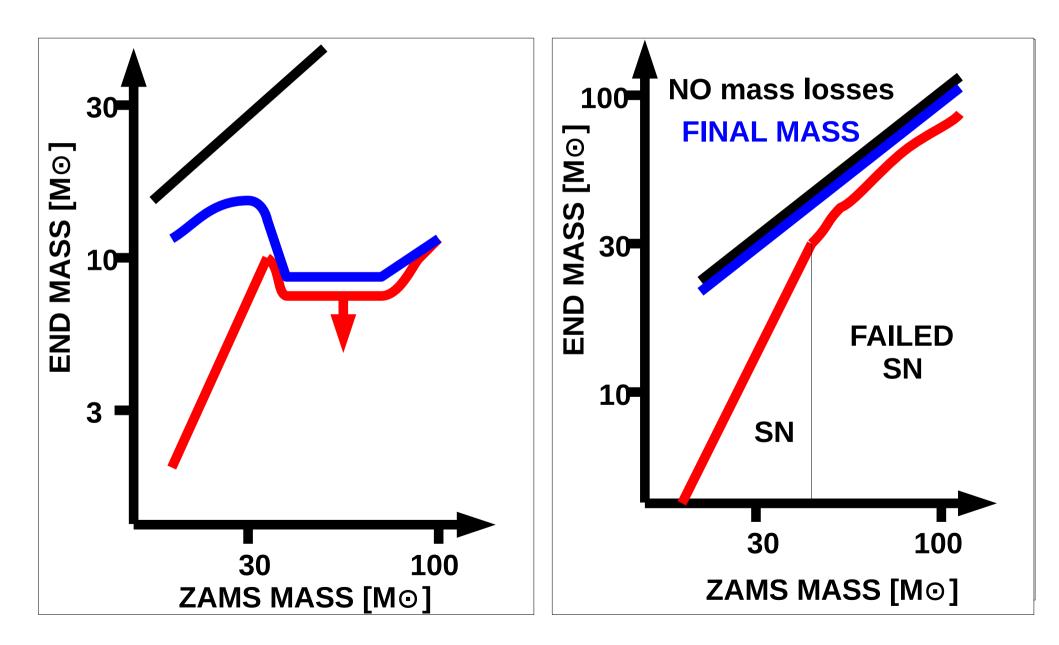


Formalism by Heger et al. (2003)

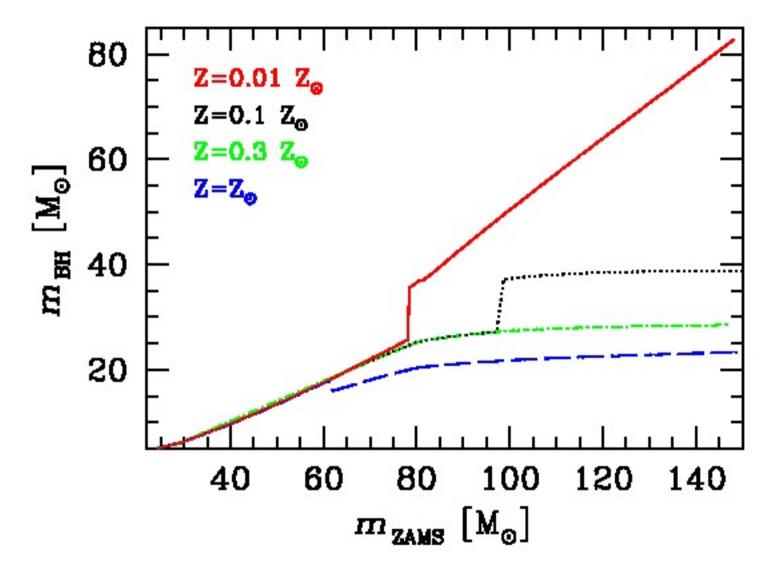


2. 3-body encounters enhance ULX formation

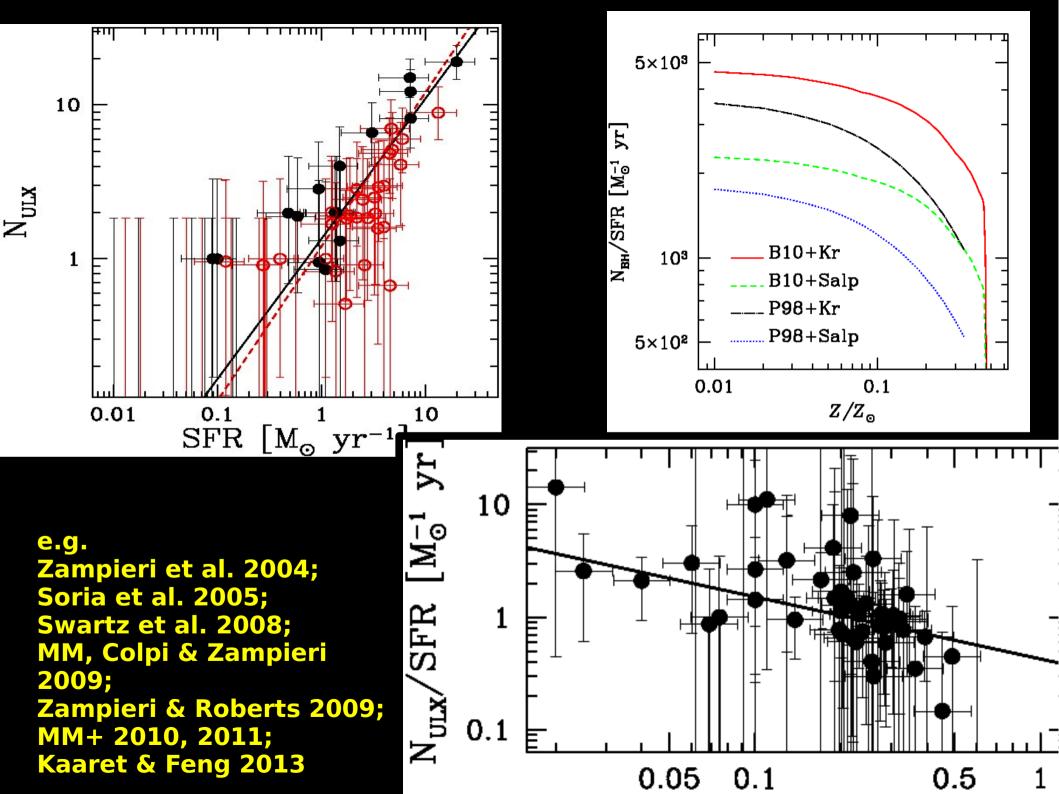
Formalism by Heger et al. (2003)



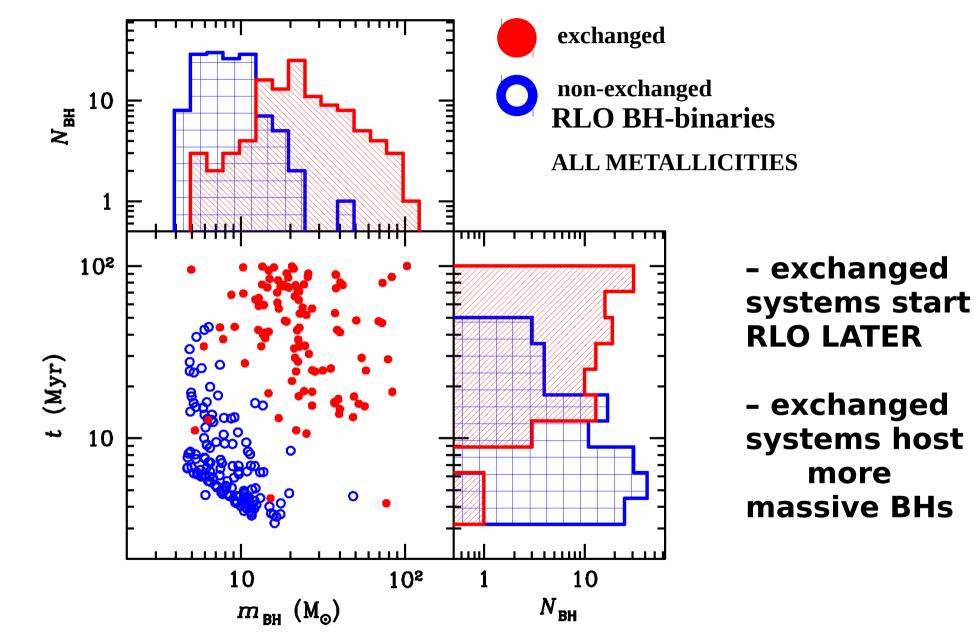
### NOT ONLY AT ZERO METALLICITY



MM+09; Zampieri & Roberts 2009; Belczynski+2010; Fryer+2012; MM+2013



### FROM N-Body simulations with STARLAB (MM+ 2011, 2013, 2014) Bachelor, Master and PhD thesis available on this topic (ASK ME!)



1. introduction about dynamics of SCs and 3-body encounters

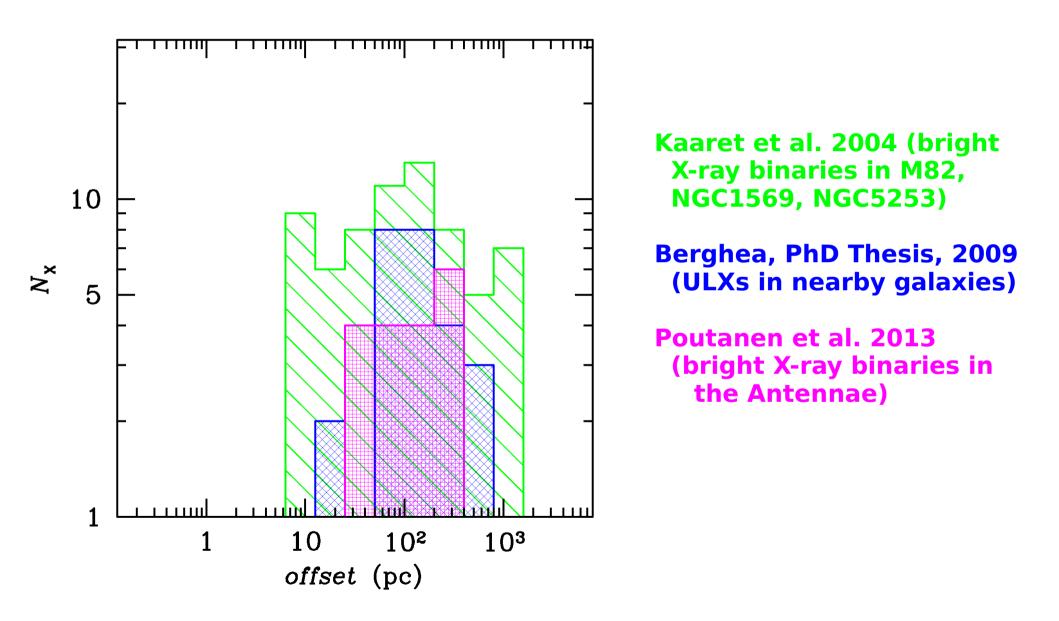
MAIN PROPERTIES of COLLISIONAL stellar systems in the MILKY WAY

# ONE OF THE MAIN PROPERTIES OF COLLISIONAL SYSTEMS IS THAT

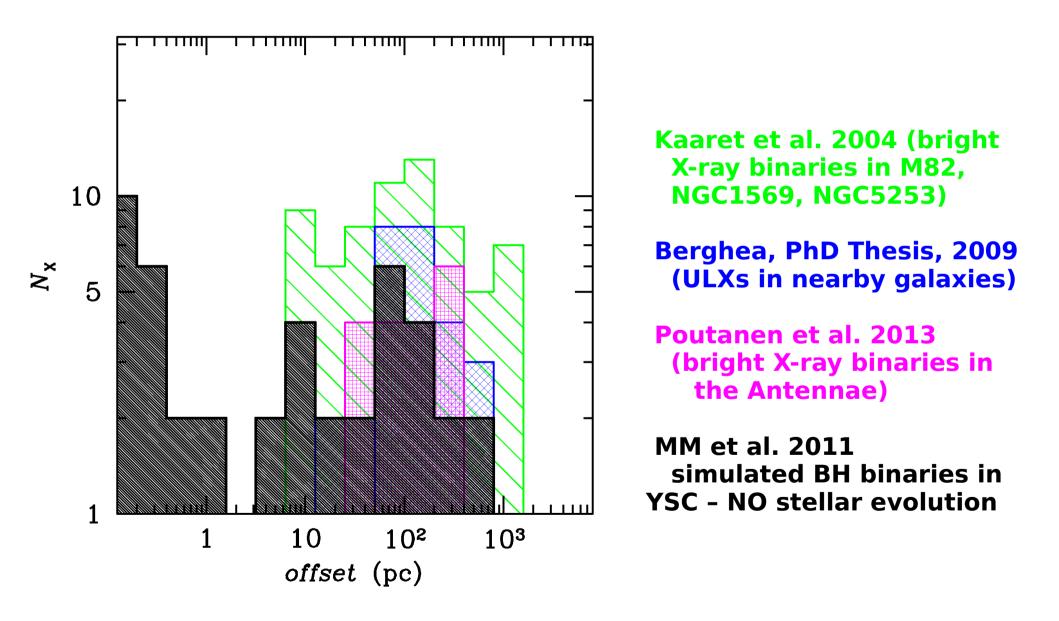
## THREE-BODY ENCOUNTERS (= CLOSE GRAVITATIONAL ENCOUNTERS BETWEEN A BINARY AND A SINGLE STAR)

**ARE FREQUENT IN COLLISIONAL SYSTEMS** 

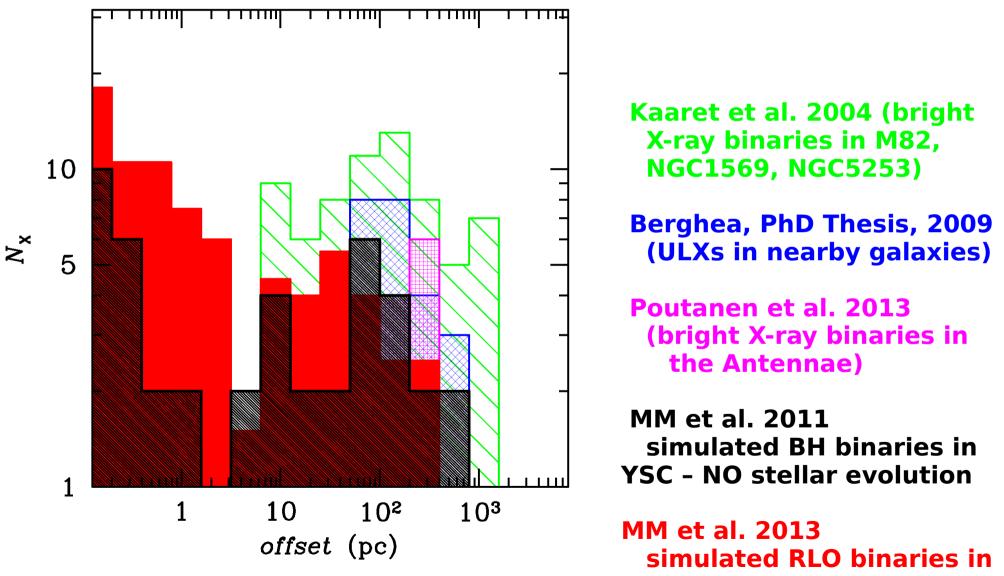
**OBSERVED OFFSET of X-ray binaries with respect to the closest YSC:** 



OBSERVED OFFSET of X-ray binaries with respect to the closest YSC + SIMULATIONS: BH binaries ejected by 3 BODY ENCOUNTERS



OBSERVED OFFSET of X-ray binaries with respect to the closest YSC + SIMULATIONS: BH binaries ejected by 3 BODY ENCOUNTERS



YSC - with stellar evolution

**Possible explanations for the discrepancy (to be checked):** 

