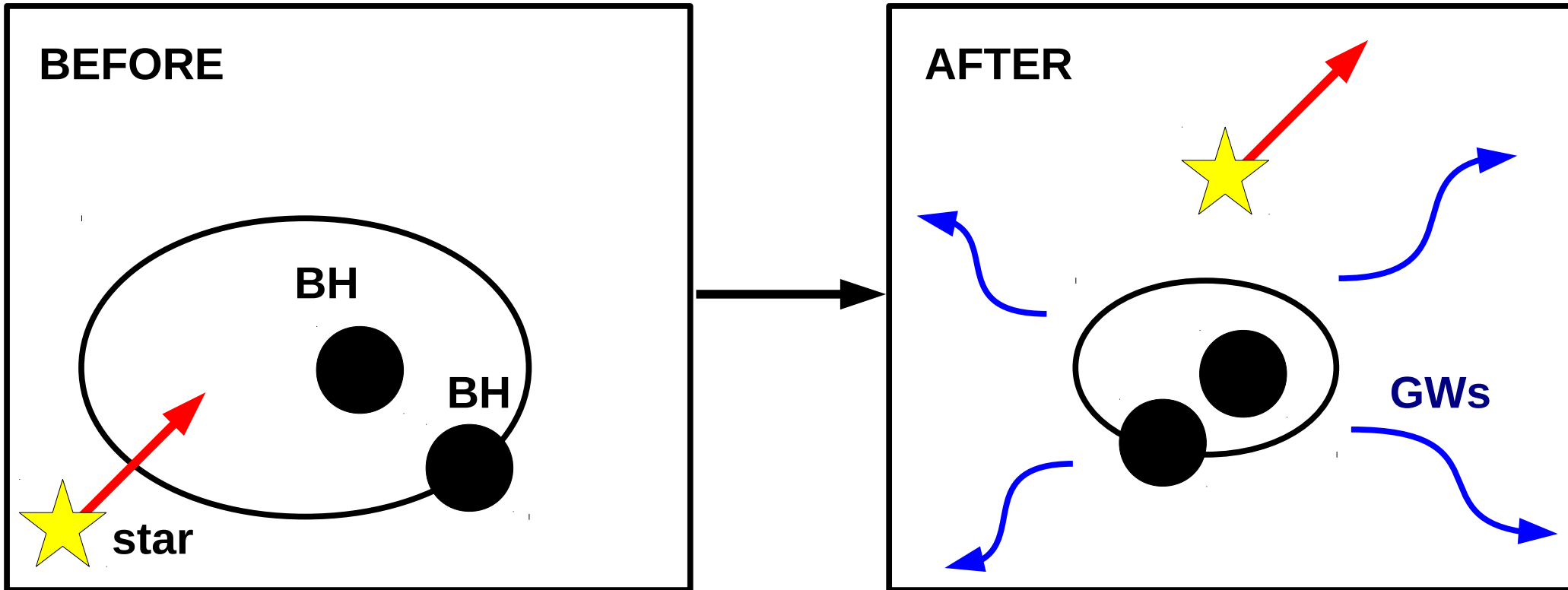


Dynamics of Stars and Black Holes in Dense Stellar Systems II

**Based on last lecture,
WHAT are the possible effects of
dynamics on BH binaries?**

1. HARDENING:



After 3-body encounters, the semi-major axis shrinks and the BH-BH (or BH-NS or NS-NS) binary becomes important as gravitational wave (GW) source

HARDENING TIMESCALE

$$t_h = \left| \frac{a}{\dot{a}} \right| = \frac{1}{2\pi G \xi} \frac{\sigma}{\rho} \frac{1}{a}$$

GRAVITATIONAL WAVE (GW) TIMESCALE

$$t_{GW} = \frac{5}{256} \frac{c^5 a^4 (1 - e^2)^{7/2}}{G^3 m_1 m_2 (m_1 + m_2)}$$

Combining 1) and 2) we can find the maximum semi-major axis for GWs to dominate evolution

$$a_{GW} = \left[\frac{256 G^2 m_1 m_2 (m_1 + m_2) \sigma}{5 2\pi \xi (1 - e^2)^{7/2} c^5 \rho} \right]^{1/5}$$

Gravitational wave (GW) progenitors

* blue

$$m_1 = 200 M_\odot \quad m_2 = 10 M_\odot$$

* green

$$m_1 = 50 M_\odot \quad m_2 = 10 M_\odot$$

* red

$$m_1 = 30 M_\odot \quad m_2 = 3 M_\odot$$

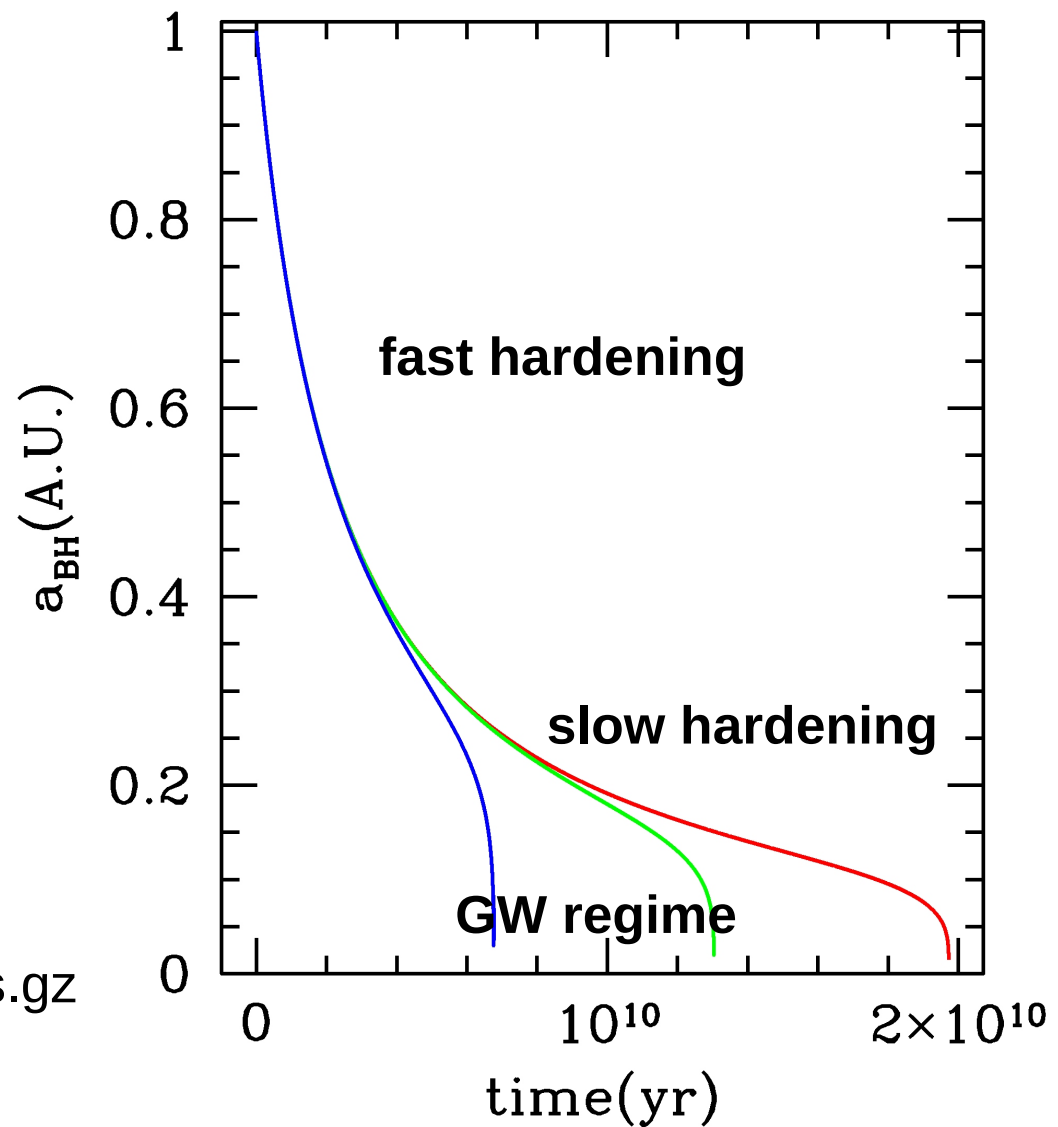
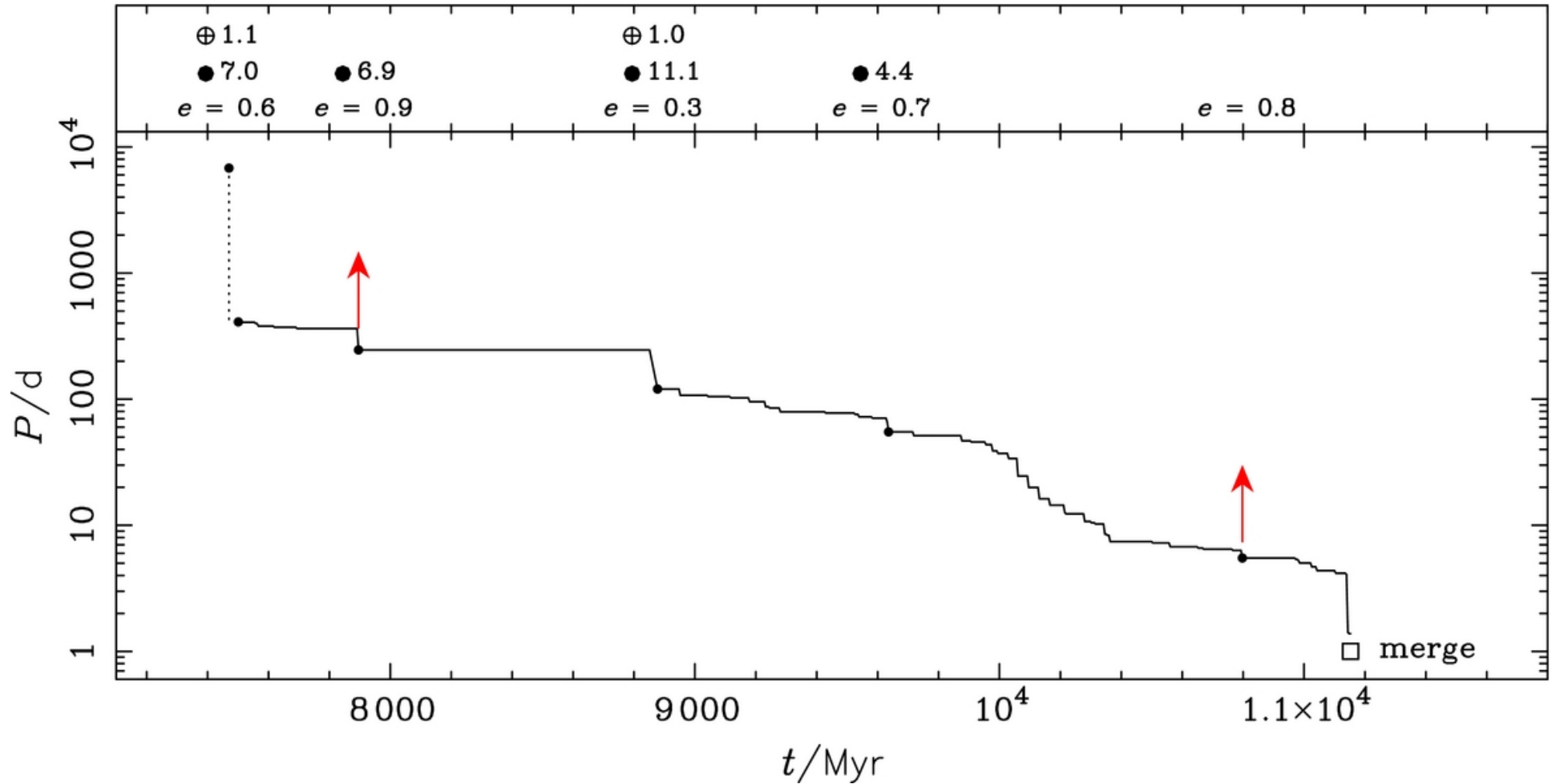


Figure and calculation
 from page 200 of M. Mapelli's thesis
<http://web.pd.astro.it/mapelli/images/tesi.ps.gz>

$$\frac{da}{dt} = \underbrace{-2 \pi \xi \frac{G \rho}{\sigma} a^2}_{\text{Binary shrinking by hardening}} - \underbrace{\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 (1 - e^2)^{7/2}} a^{-3}}_{\text{Binary shrinking by GWs (Peters 1964)}}$$

Number of encounters before GW regime:

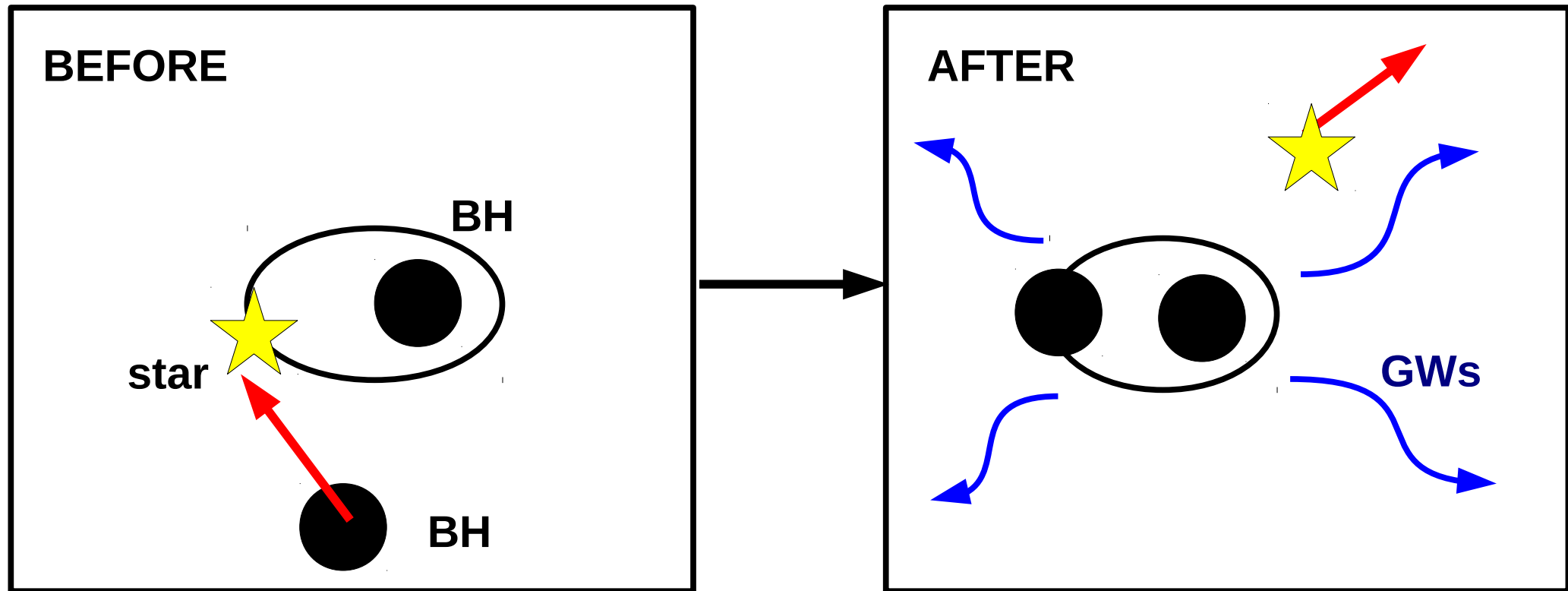
$$\begin{aligned}
 N_{int} &= \int_0^t R dt = \int_0^t \overset{\text{interaction rate}}{R} dt = \int_0^t \frac{2 \pi G m_T n a}{\sigma} dt \\
 &= \int_{a_0}^{a(t)} \frac{2 \pi G m_T n a}{\sigma} \frac{\sigma da}{-2 \pi G \xi \rho a^2} = \int_{a(t)}^{a_0} \frac{1}{\xi} \frac{m_T}{\langle m \rangle} \frac{da}{a} \\
 \frac{da}{dt} &= -2 \pi G \xi \frac{\rho}{\sigma} a^2 \\
 &= \frac{1}{\xi} \frac{m_T}{\langle m \rangle} \ln \left(\frac{a_0}{a(t)} \right)
 \end{aligned}$$



Hurley+ 2016, PASA, 33, 36

Hills 1992, AJ, 103, 1955; Kulkarni+ 1993, Nature, 364, 421; Sigurdsson & Hernquist 1993, Nature, 364, 423; Portegies Zwart & McMillan 2000, ApJ, 528, L17; Aarseth 2012, MNRAS, 422, 841; Breen & Heggie 2013, MNRAS, 432, 2779 ETC ETC...

2. EXCHANGE:



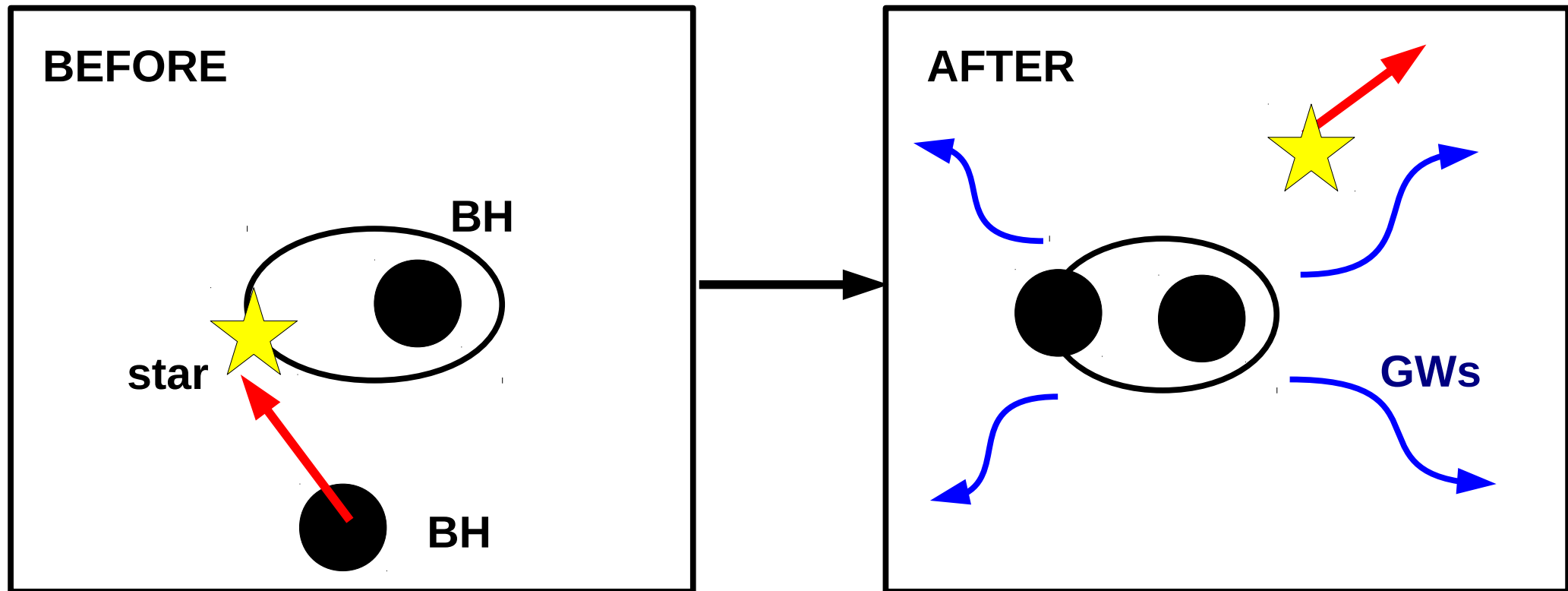
Exchanges are very important: bring BHs in binaries

BHs are FAVOURED BY EXCHANGES BECAUSE THEY ARE MASSIVE!

BH born from single star in the field never acquires a companion

BH born from single star in a sc likely acquires companion from dynamics

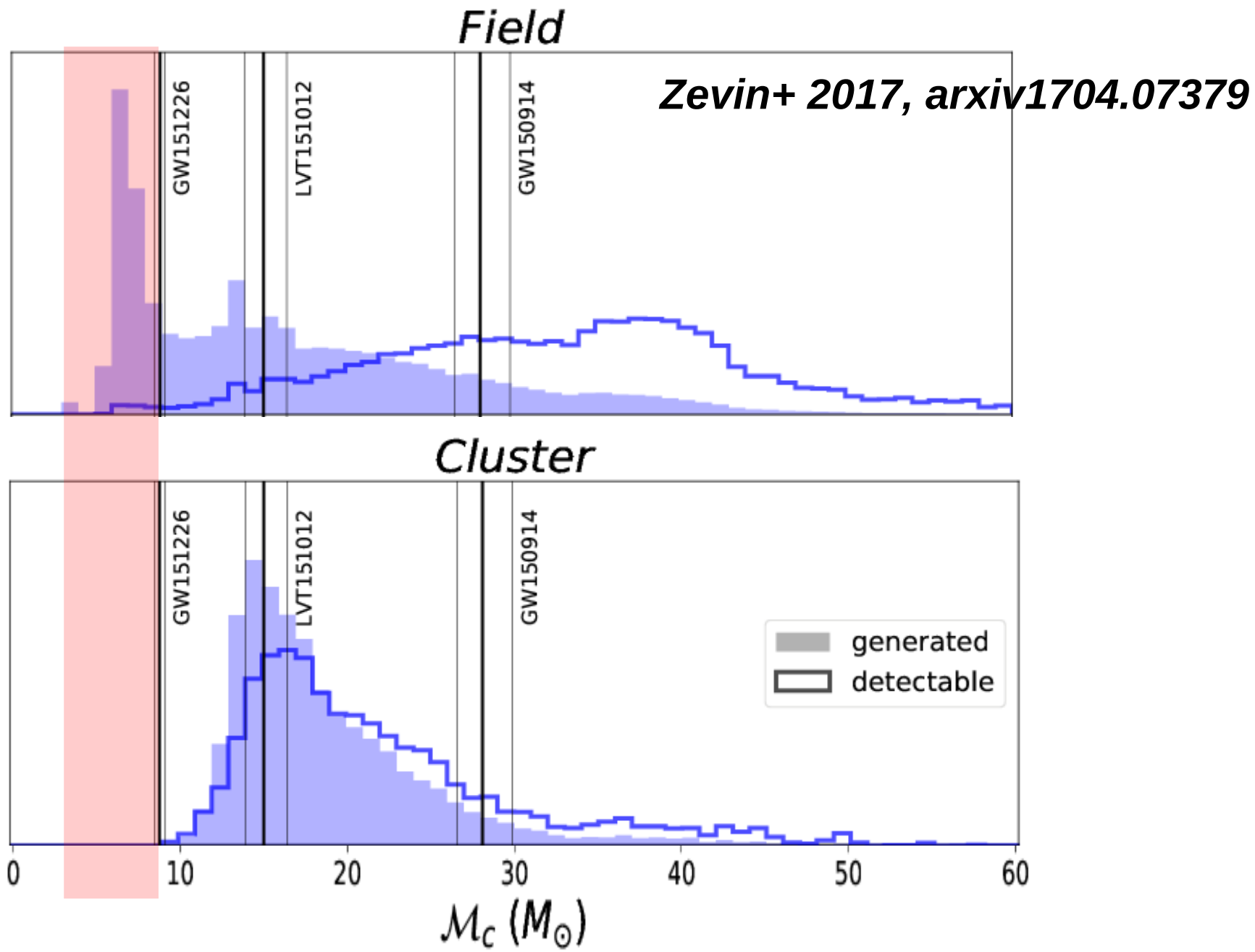
2. EXCHANGE:



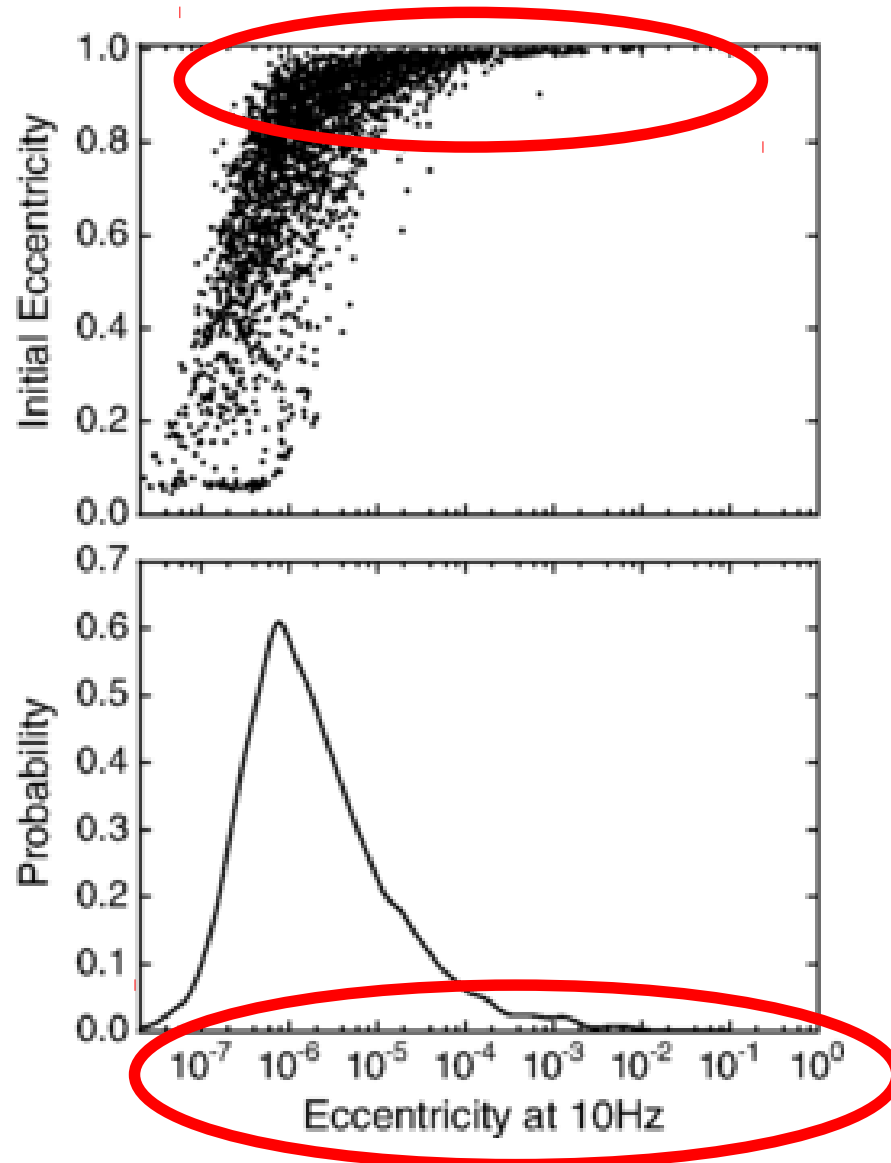
*>90% BH-BH binaries in young star clusters form by exchange
(Ziosi+ 2014)*

EXCHANGES FAVOUR THE FORMATION of BH-BH BINARIES WITH

- * **THE MOST MASSIVE BHs**
- * **HIGH ECCENTRICITY**
- * **MISALIGNED BH SPINS**



Ziosi, MM+ 2014, MNRAS, 441, 3703; Rodriguez+ 2015, Phys. Review Letter, 115, 1101; Rodriguez+ 2016, PhRvD, 93, 4029; Askar+ 2017, MNRAS, 464, L36; Banerjee 2017, MNRAS, 467, 524 and many others



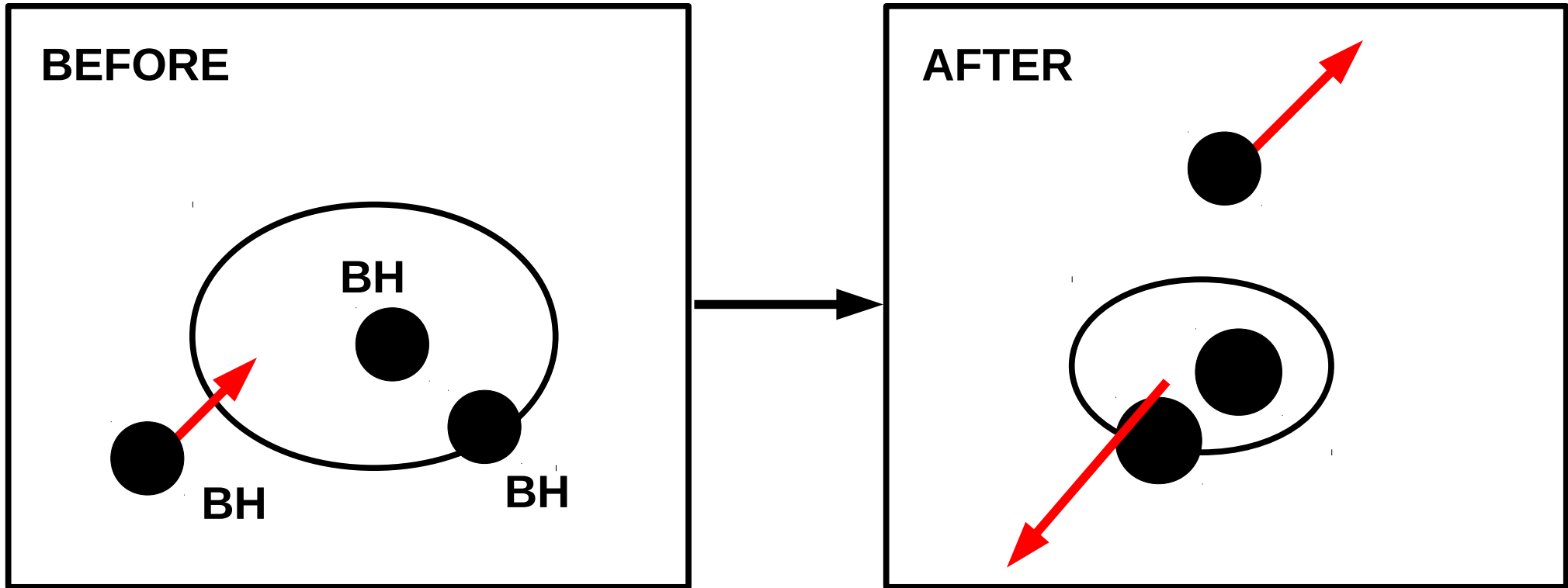
Rodriguez+ 2016, PhRvD, 93, 4029

- high eccentricity at formation

- small eccentricity when reaching
LIGO-Virgo range

Ziosi, MM+ 2014, MNRAS, 441, 3703; Rodriguez+ 2015, Phys. Review Letter, 115, 1101; Hurley+ 2016, PASA, 33, 36; Askar+ 2017, MNRAS, 464, L36; Banerjee 2017, MNRAS, 467, 524 and many others

3. EJECTIONS:



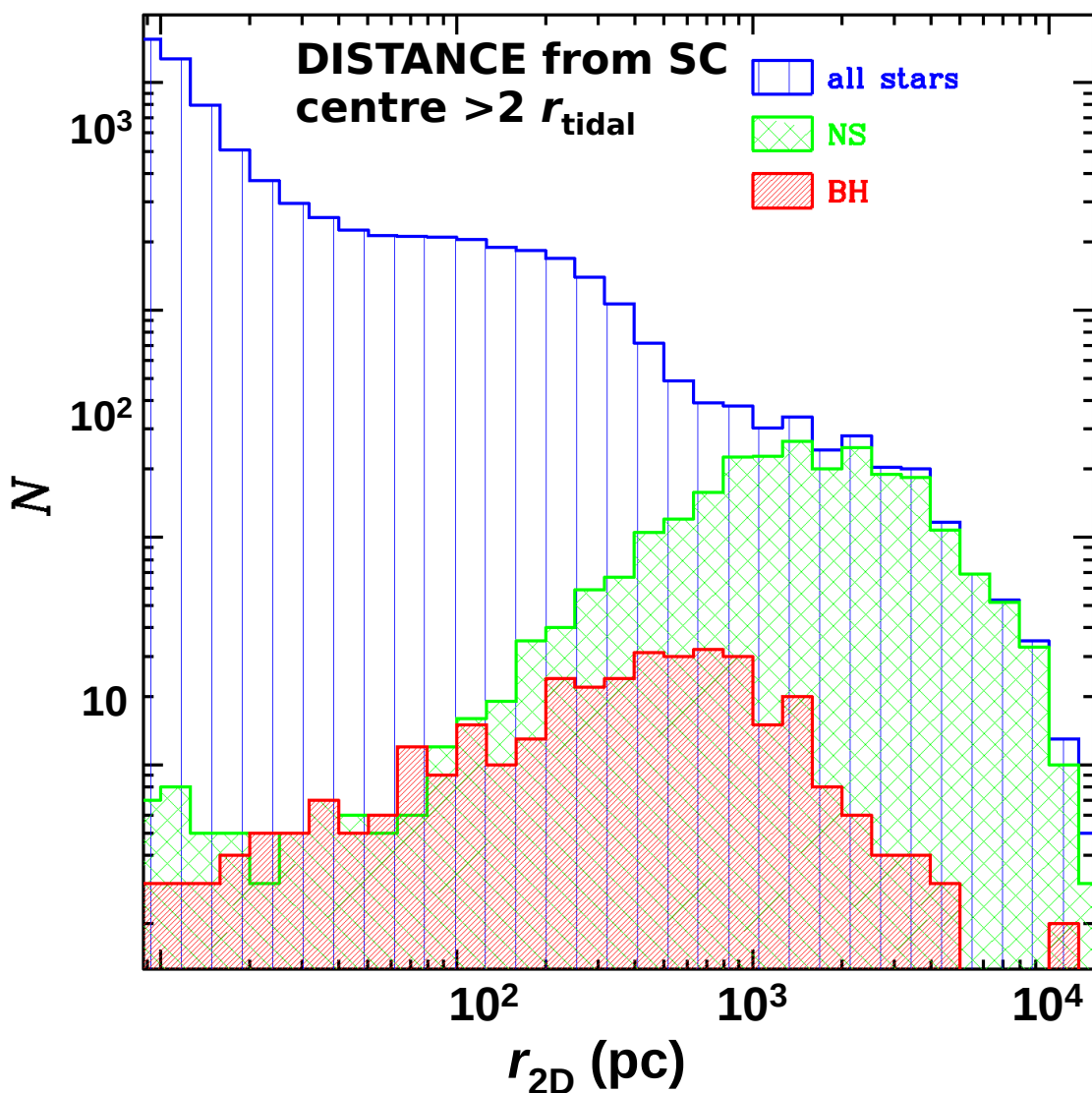
Internal energy is extracted from the binary

➔ converted into KINETIC ENERGY of the INTRUDER AND of the centre-of-mass of the BINARY

➔ BOTH RECOIL and can be ejected from star cluster

BHs and NSs are ejected from host star clusters by
DYNAMICS and **NATAL (SN) KICKS**

Simulations of young star clusters @ $t=100$ Myr



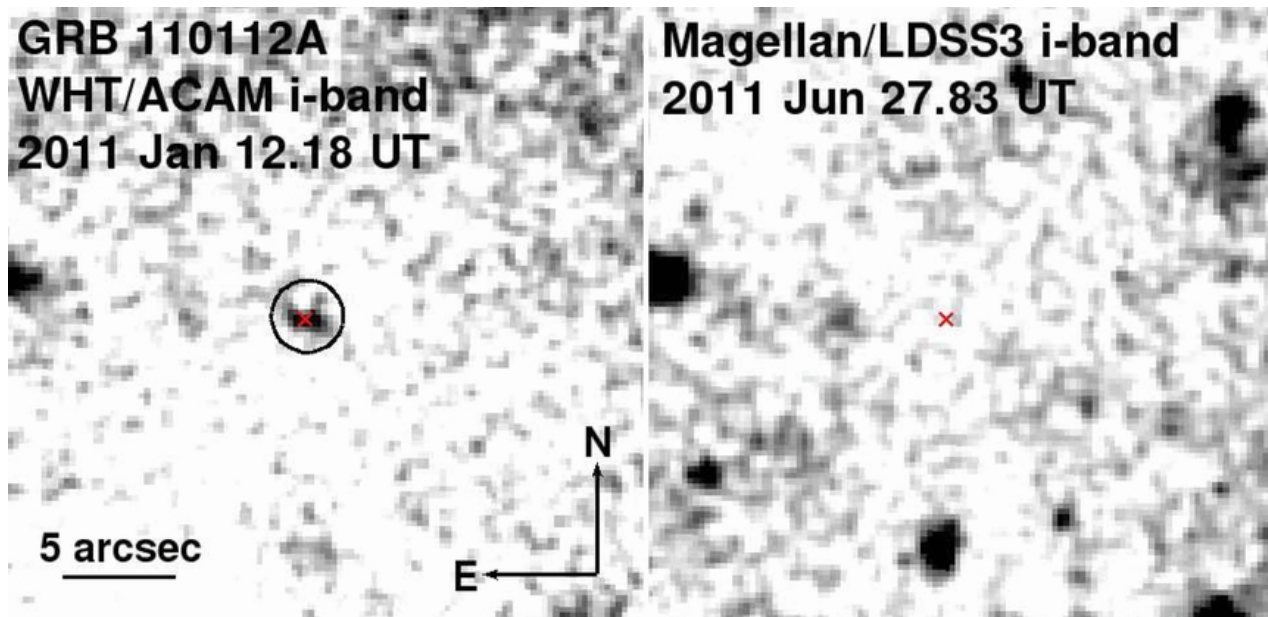
~80-90% NS is ejected
(mainly by SN)

~40% BH is ejected
(1/2 by SN, 1/2 by
3body)

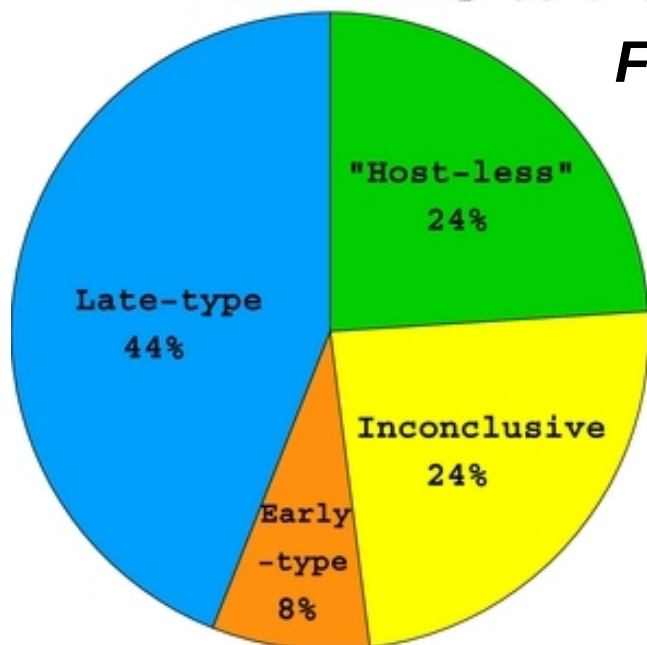
**PREDICTED MERGERS
OCCUR MOSTLY IN THE
FIELD**

Downing+ 2011, MNRAS, 416, 133
MM + 2013, MNRAS, 429, 2298

Are host-less short GRBs associated with dynamical ejections?



Fong+ 2013, ApJ, 769, 56



ISSUE: dynamical kicks 0 – 200 km/s

not enough to unbind system from host galaxy

4. KOZAI-LIDOV RESONANCE:

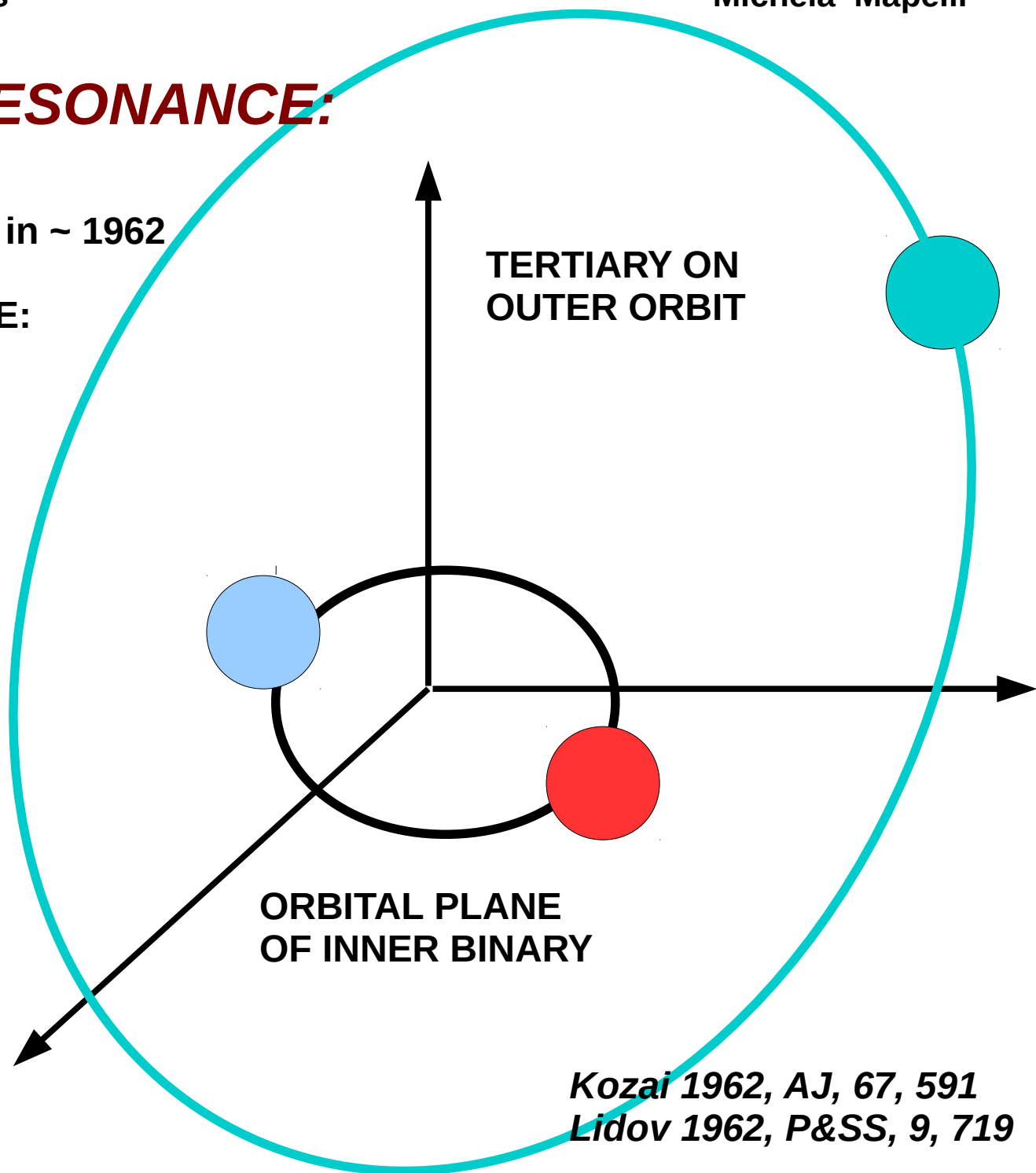
Discovered by Kozai and Lidov in ~ 1962

I need a HIERARCHICAL TRIPLE:

- Inner tight binary
- Outer body whose motion about the binary can be approximated with outer binary (CM + 3rd body)
- if inclination between 2 orbital planes is not 0

→ KOZAI RESONANCE

ECCENTRICITY AND INCLINATION OSCILLATE



Kozai 1962, AJ, 67, 591
Lidov 1962, P&SS, 9, 719

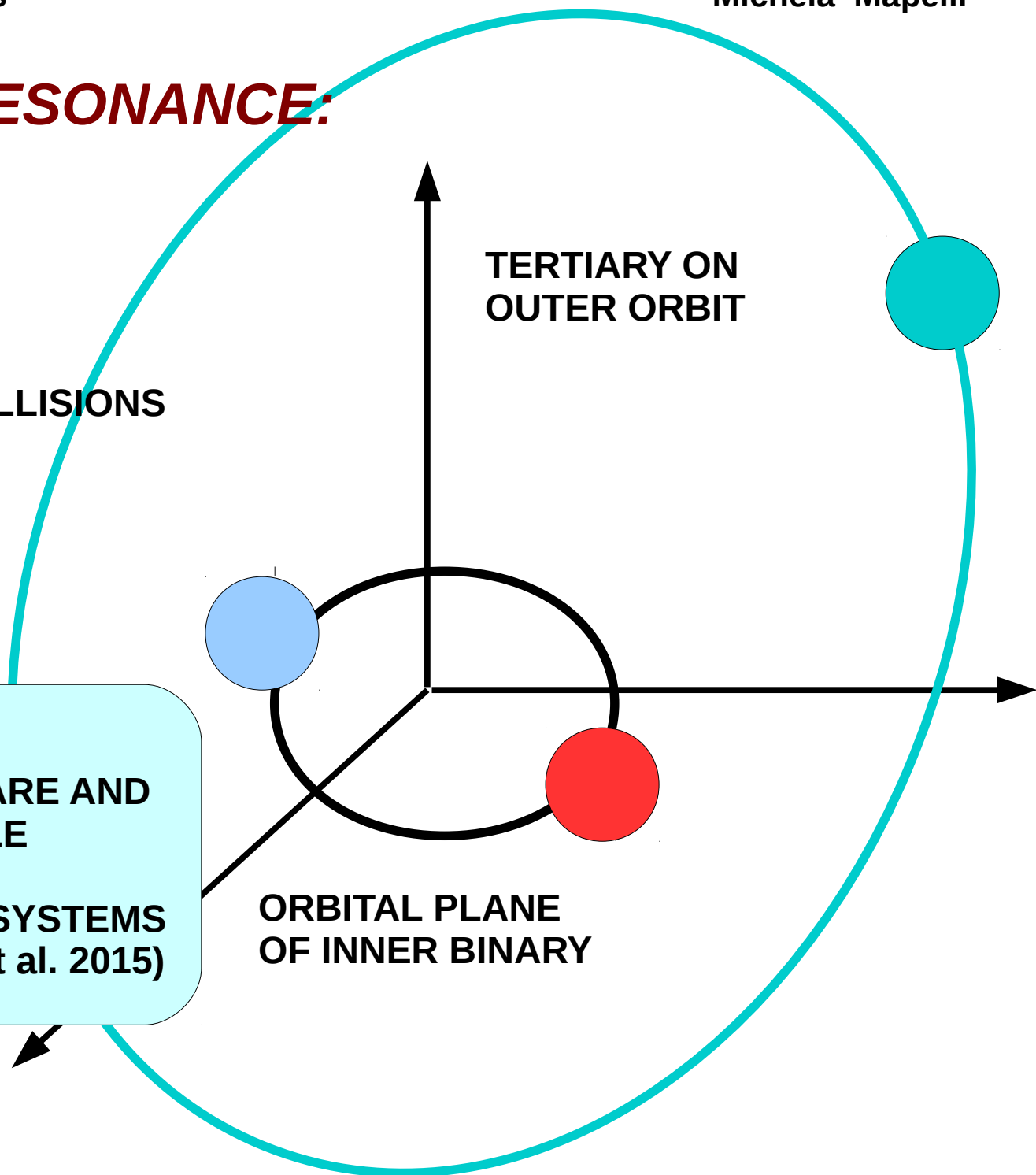
4. KOZAI-LIDOV RESONANCE:

ECCENTRICITY AND
INCLINATION OSCILLATE

TRIGGERING MERGERS / COLLISIONS
between binary members

→ IMPORTANT FOR
GRAVITATIONAL WAVES

YOU MIGHT SAY:
TRIPLES SHOULD BE VERY RARE AND
KL RESONANCE IS NEGLIGIBLE
INSTEAD
~ 10 % STARS ARE IN TRIPLE SYSTEMS
(Raghavan et al. 2010; Riddle et al. 2015)



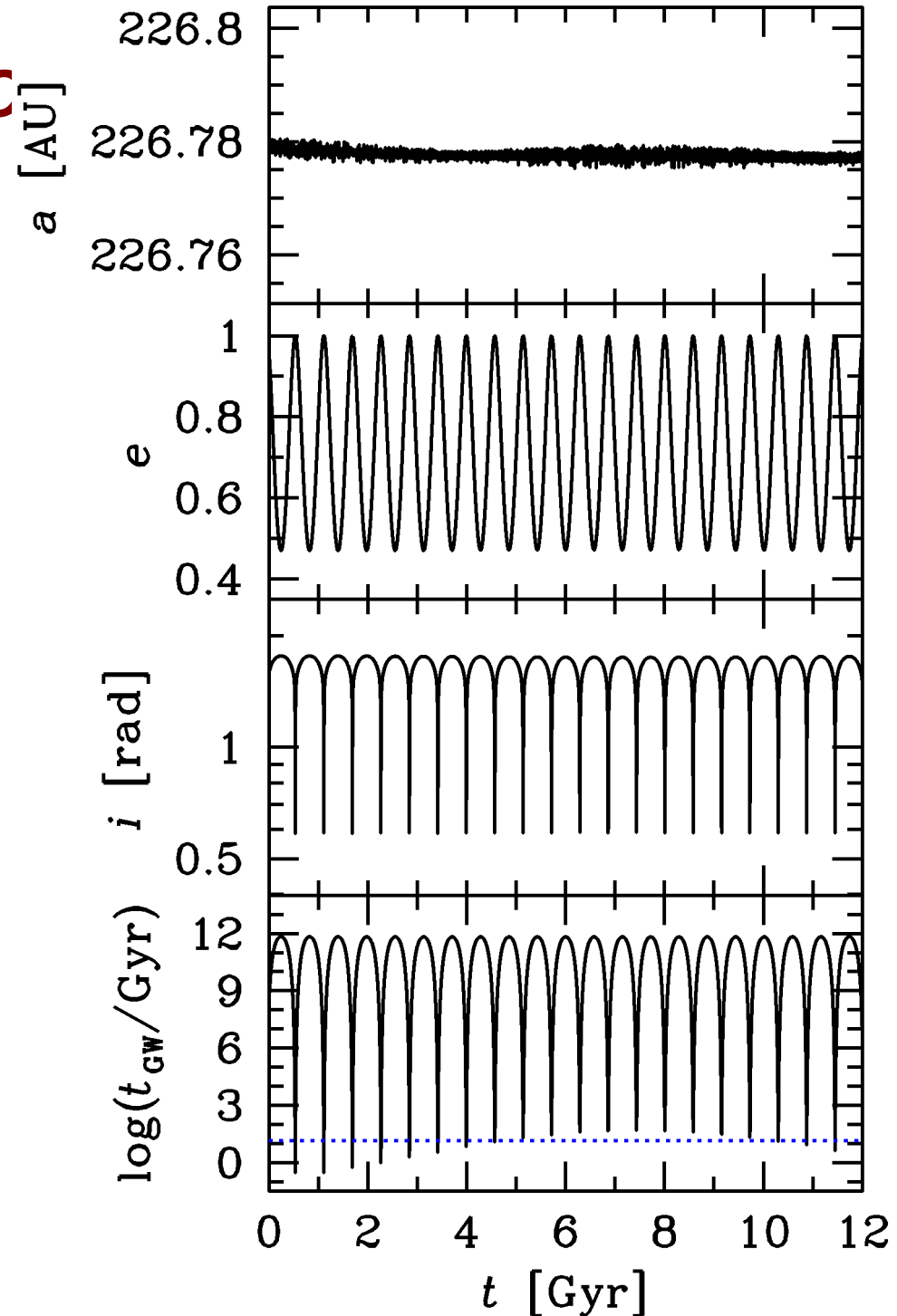
4. KOZAI-LIDOV RESONANCE

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4. KOZAI-LIDOV RESONANCE:

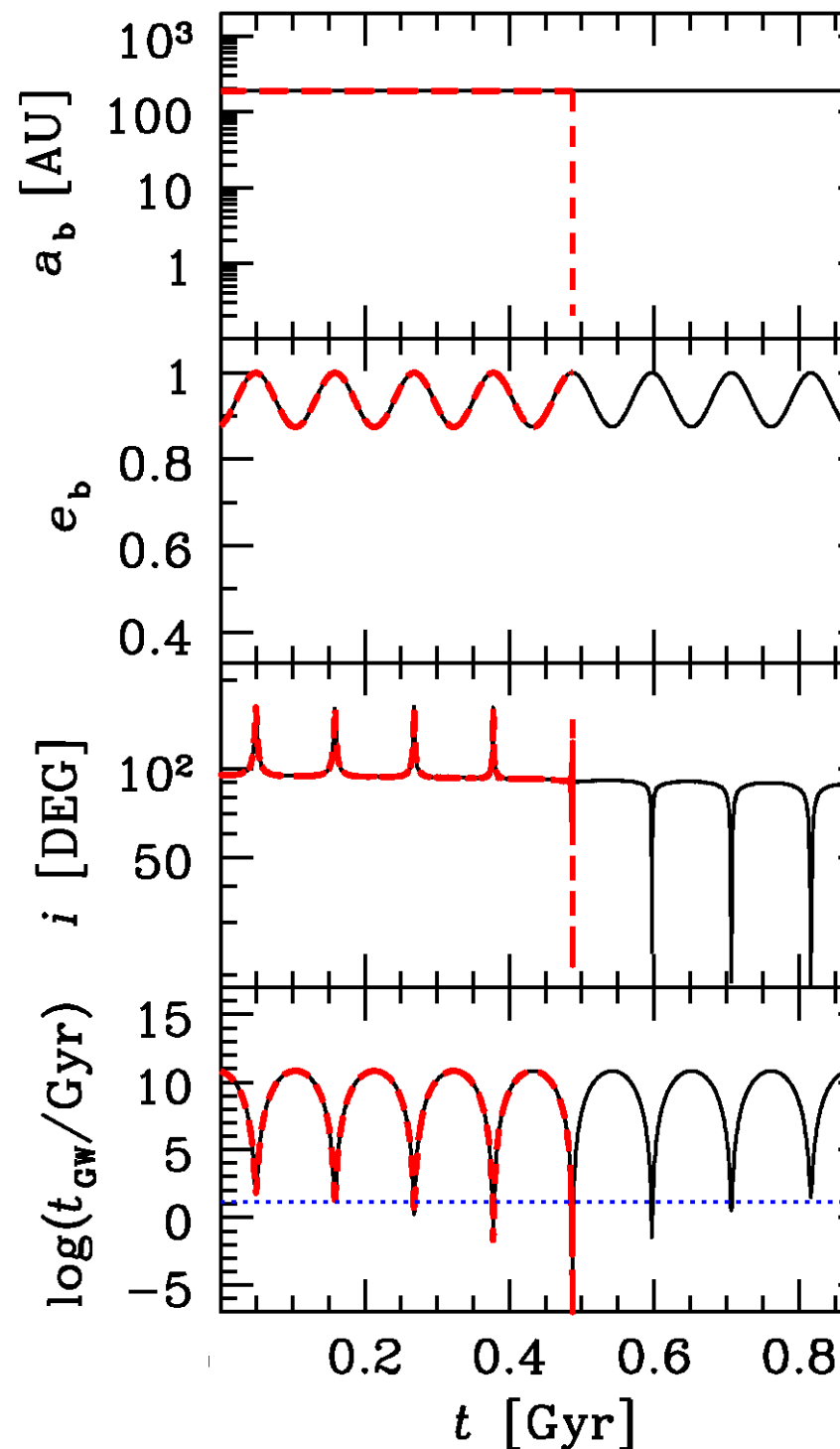
- No post-Newtonian (PN)
- - - - - With 2.5 PN term

PN: treatment of Einstein's non-linear equations as lowest-order deviations from Newton's equation

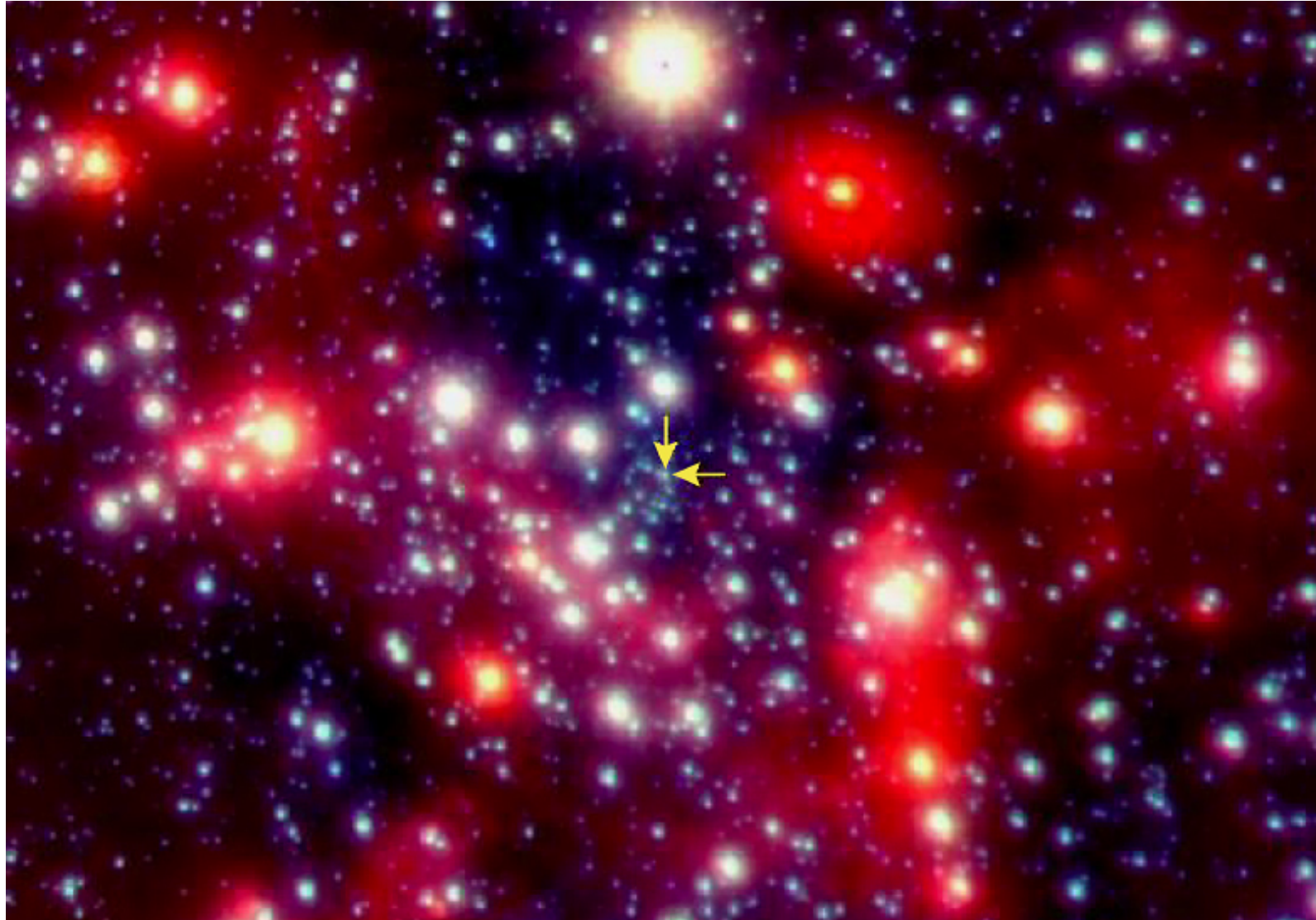
~ 50% more MERGERS
of BH-BH binaries
in young dense star clusters
If Kozai accounted for

Kimpson, Spera, MM, Ziosi 2016, MNRAS, 463, 2443

Antognini+ 2014, MNRAS, 439, 1079;
Antonini+ 2016, ApJ, 816, 65;
Antognini+ 2016, MNRAS, 456, 4219;
Kimpson+ 2016, MNRAS, 463, 2443;
Antonini+ 2017arXiv170306614A



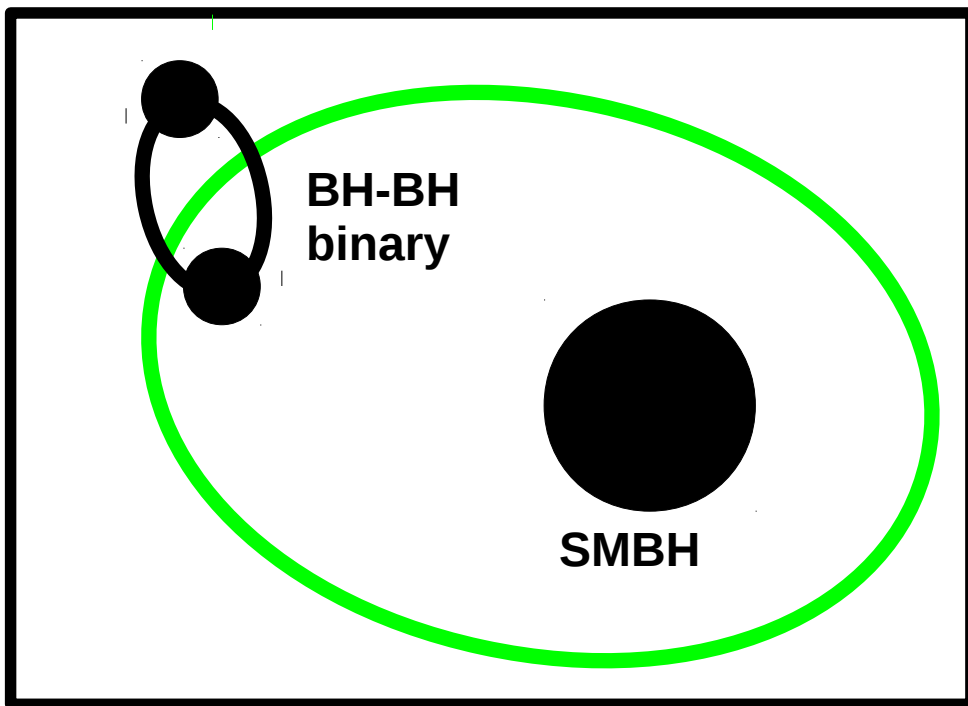
KOZAI-LIDOV particularly efficient in NUCLEAR STAR CLUSTERS:



KOZAI-LIDOV particularly efficient in NUCLEAR STAR CLUSTERS:

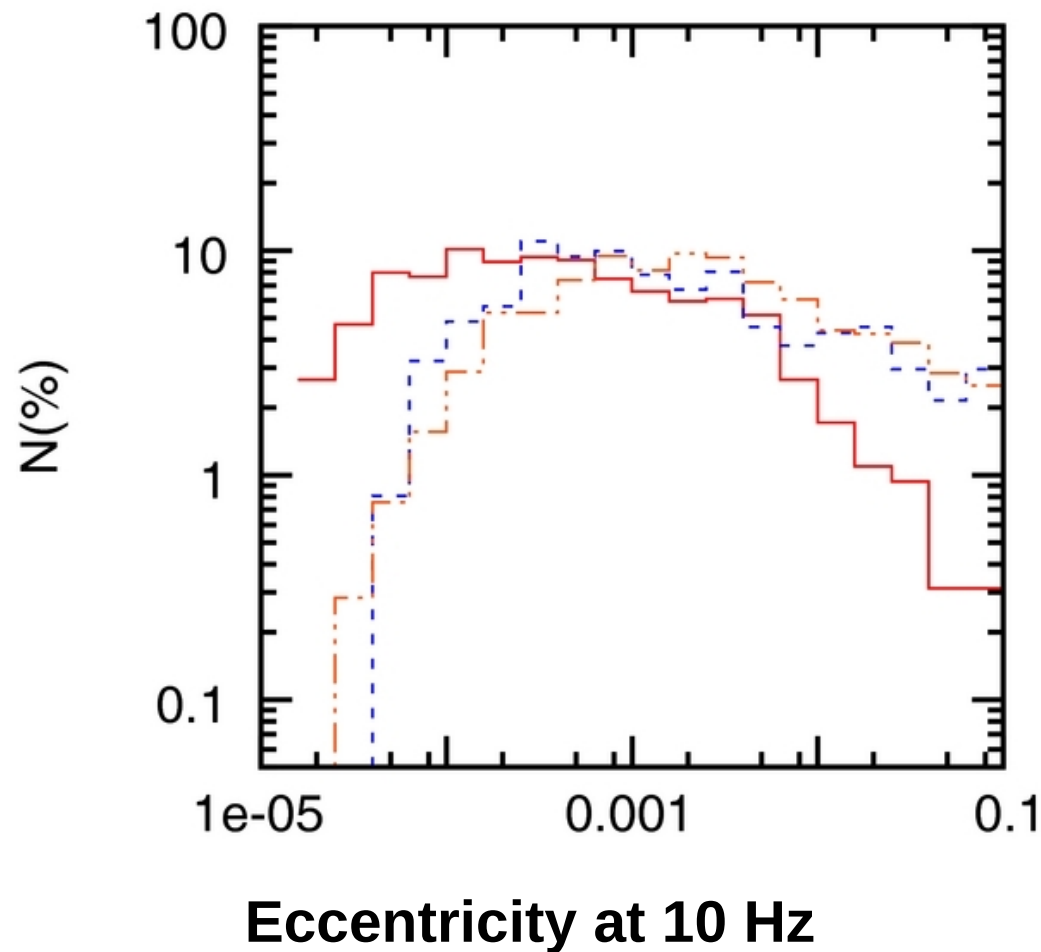
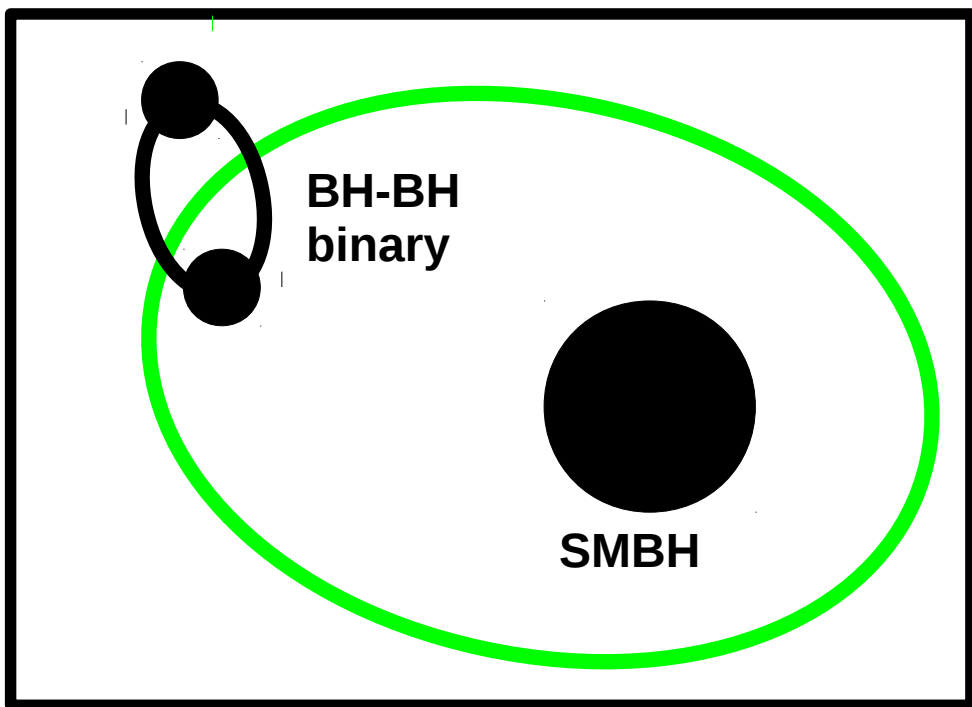
*** high escape velocity
(BHs are retained)**

*** triple might be with SMBH**



KOZAI-LIDOV particularly efficient in NUCLEAR STAR CLUSTERS:

- * high escape velocity
(BHs are retained)
- * triple might be with SMBH

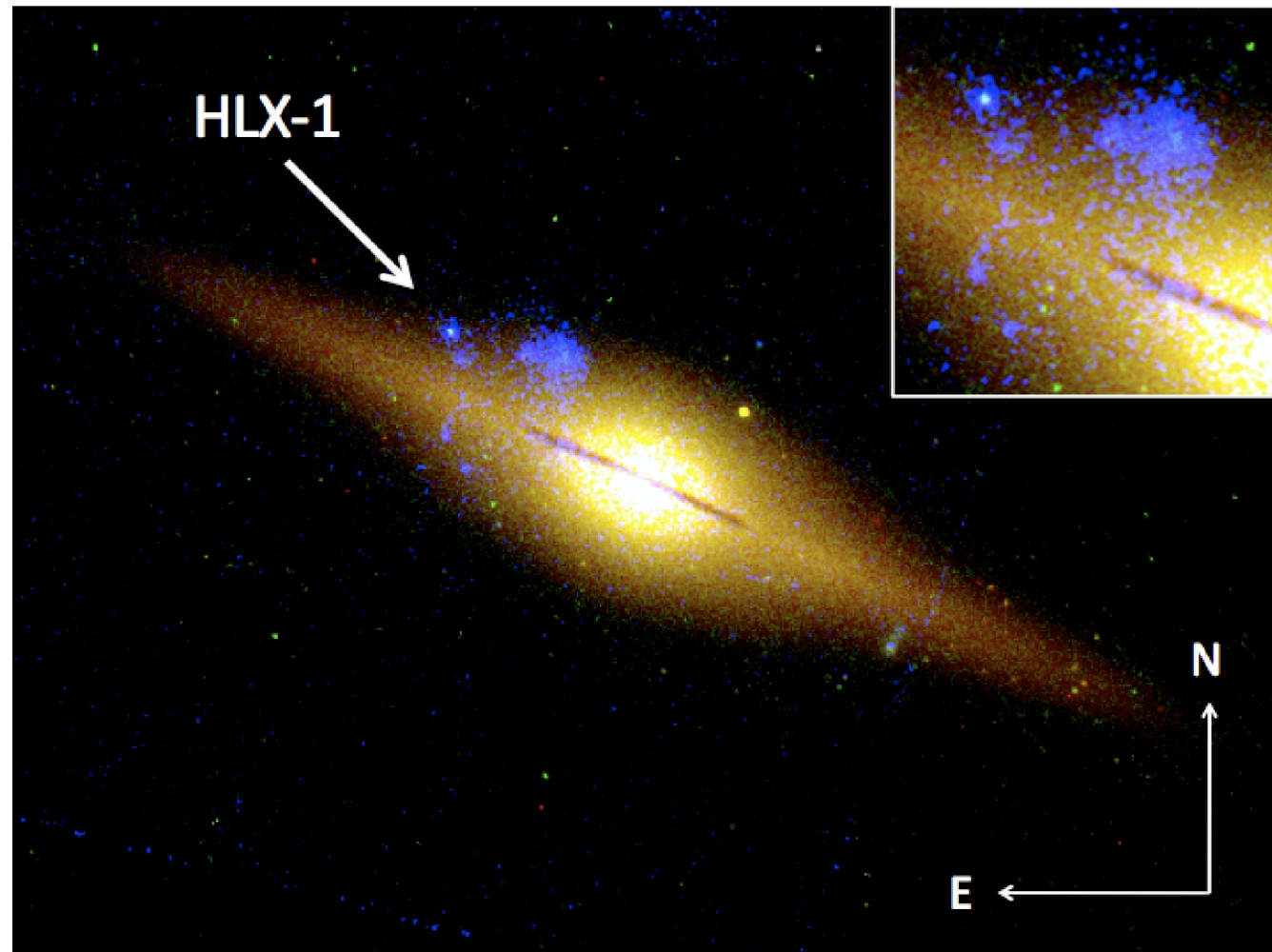


5. Intermediate-mass black holes (IMBHs): BHs with mass $10^2 - 5 M_{\odot}$

OBSERVATIONAL EVIDENCES: none, just hints

1 Hyperluminous X-ray source HLX-1 close to ESO 243-49

peak $L_x \sim 10^{42}$ ergs,
X-ray VARIABILITY,
redshift consistent
with ESO 243-49
(not a background object)
→ BH mass $\sim 10^4 M_{\odot}$

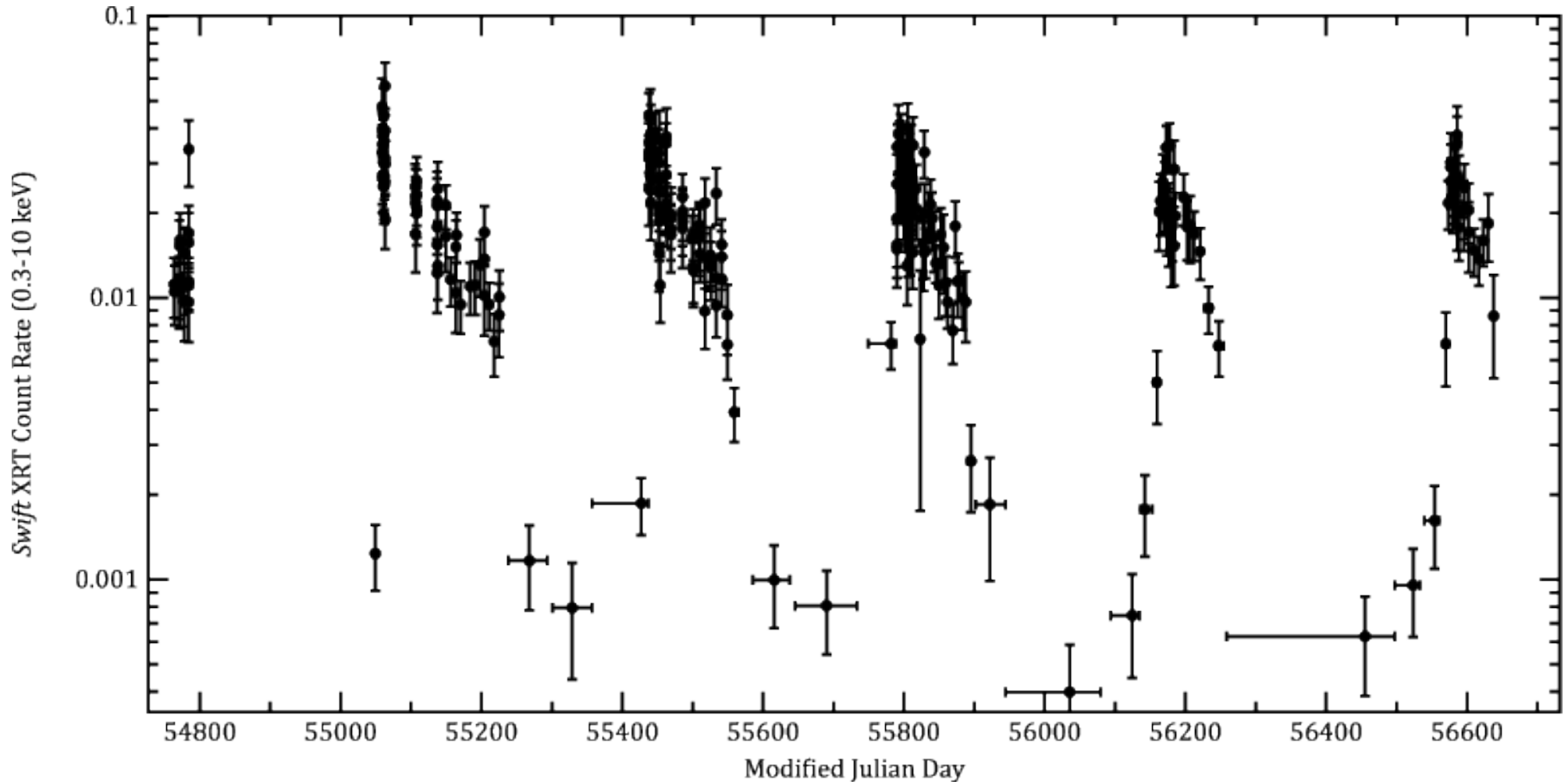


Farrell+ 2009, 2012;
Soria+ 2010, 2012;
MMi+ 2012, 2013

5. Intermediate-mass black holes (IMBHs): BHs with mass $10^2 - 5 M_{\odot}$

OBSERVATIONAL EVIDENCES: none, just hints

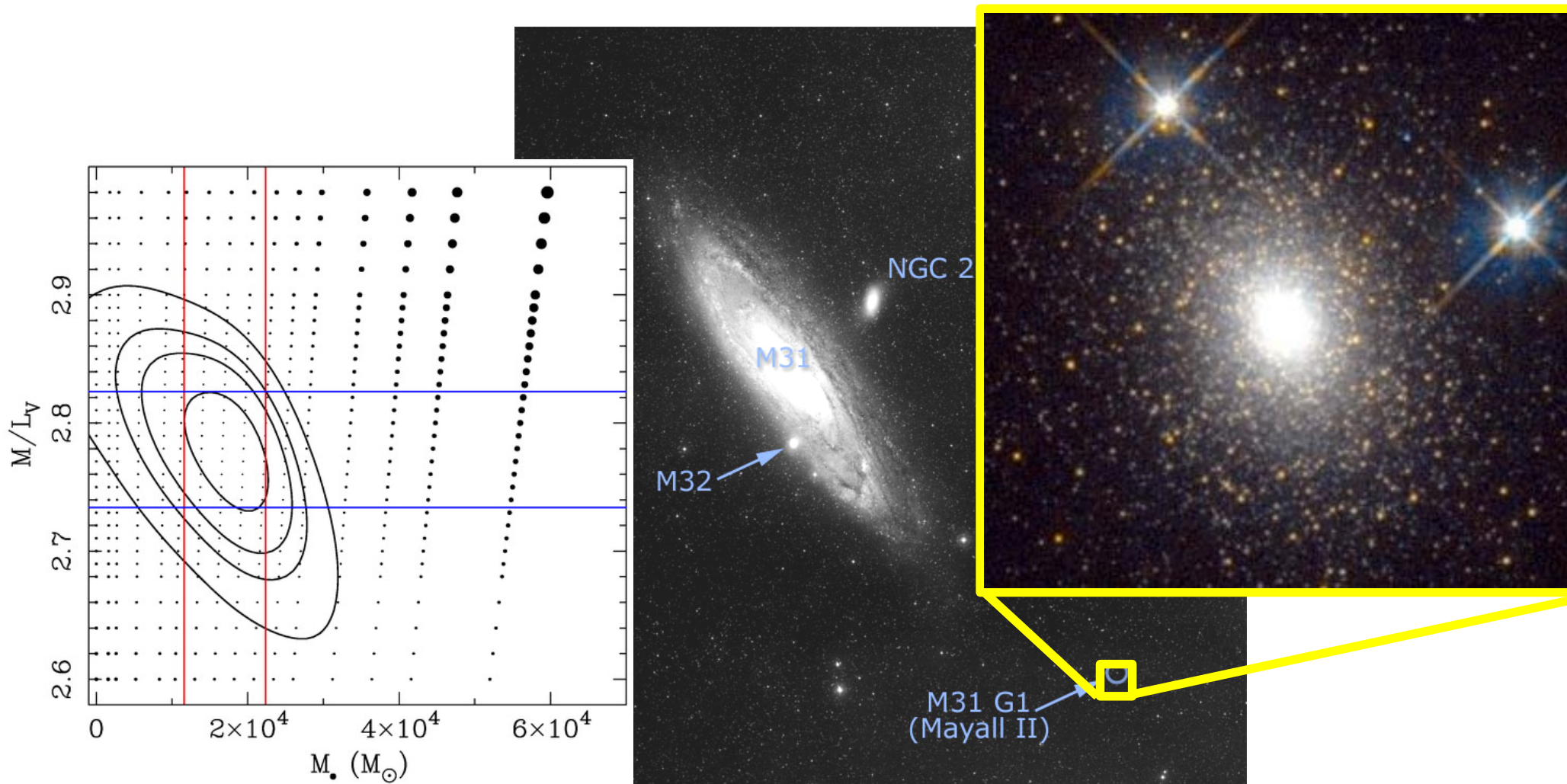
1 Hyperluminous X-ray source HLX-1 close to ESO 243-49



5. Intermediate-mass black holes (IMBHs): BHs with mass $10^2 - 5 M_{\odot}$

#2 centre of G1 globular cluster (dwarf nucleus?) in Andromeda

Central velocity distribution + central M/L ratio suggest BH mass $\sim 10^4 M_{\odot}$



5. Intermediate-mass black holes (IMBHs): BHs with mass $10^2 - 5 M_{\odot}$

How do IMBHs form?

Requires dynamics?

1- runaway collisions of stars

yes

2- repeated mergers of BHs

yes

3- remnants of very massive
($>260 M_{\text{sun}}$) extremely
metal-poor stars (stellar BHs)

No

*(unless very massive star
was dynamically formed)*

4- low mass end of super-massive
BHs (not part of this course)

maybe

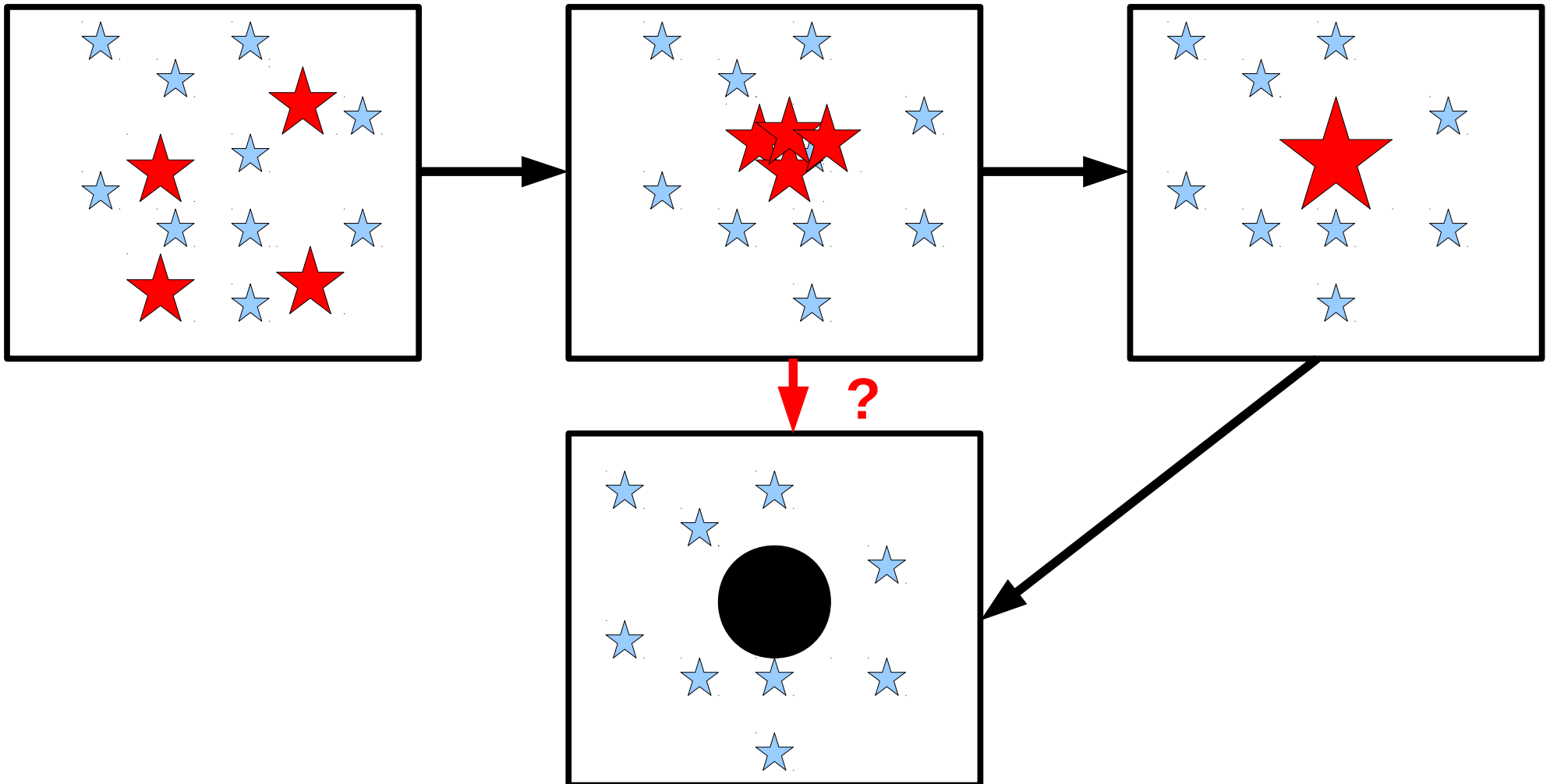
5.1 IMBHs from Runaway collisions

IDEA: mass segregation brings very massive stars to the centre

If **timescale for mass segregation** < **timescale for SN explosion**

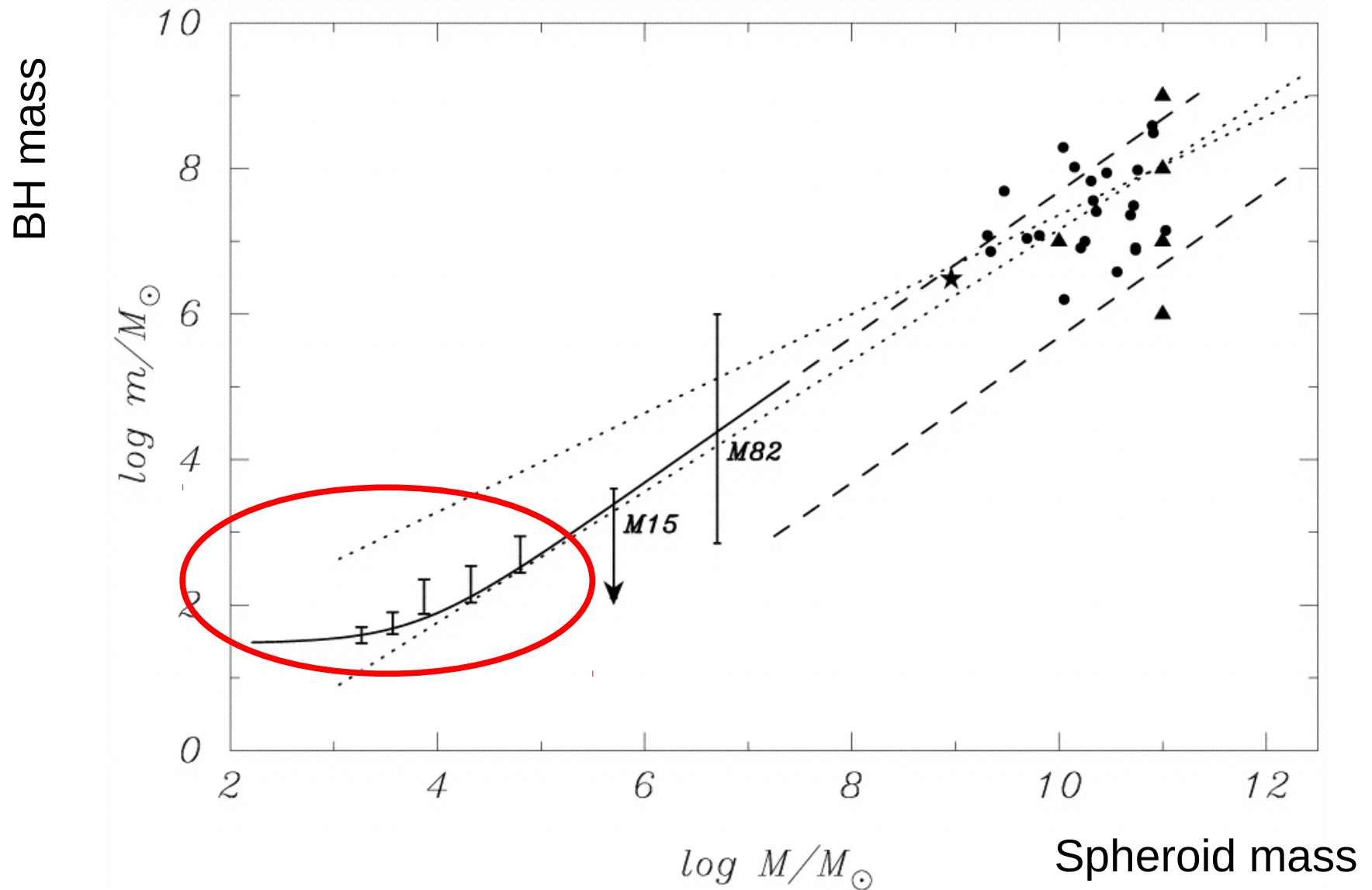
+ **encounter rate sufficiently high**

→ massive stars collide, merge and form a super-massive star, which collapses to a BH



5.1 IMBHs from Runaway collisions

“Analytic” *formalism by Portegies Zwart & McMillan 2002, ApJ, 576,899*
confirmed by their simulations

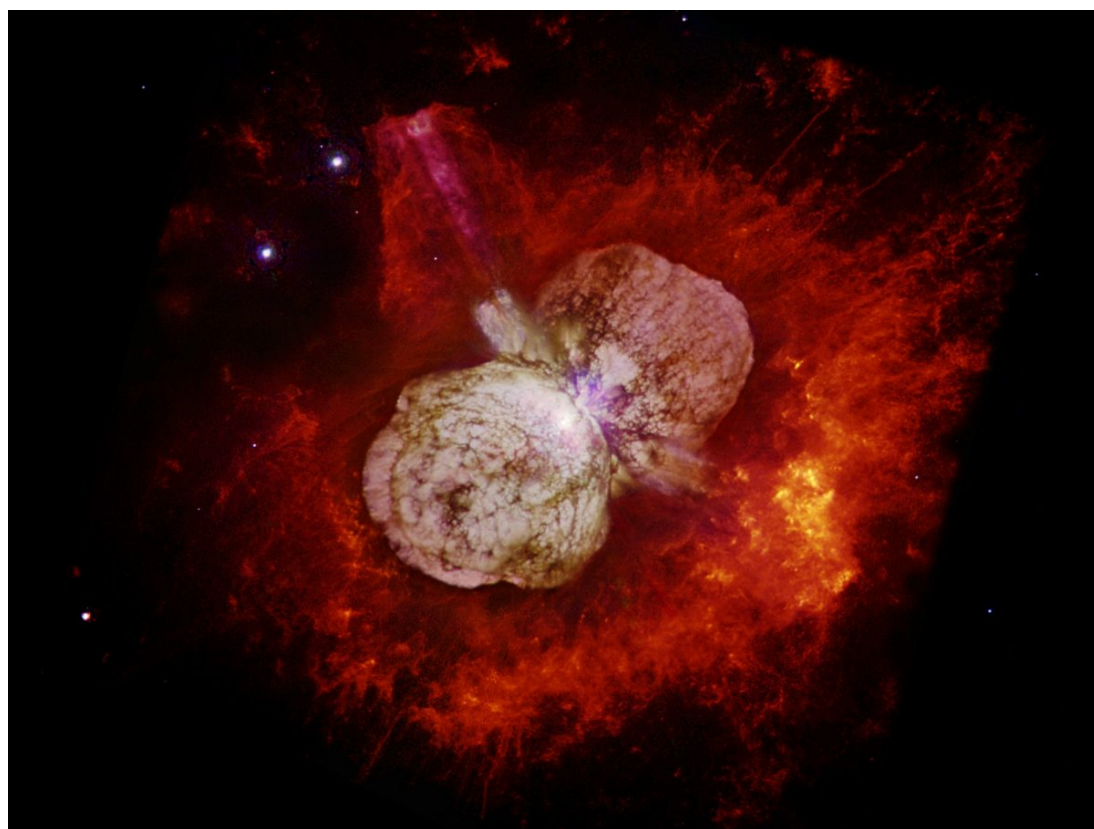


5.1 IMBHs from Runaway collisions

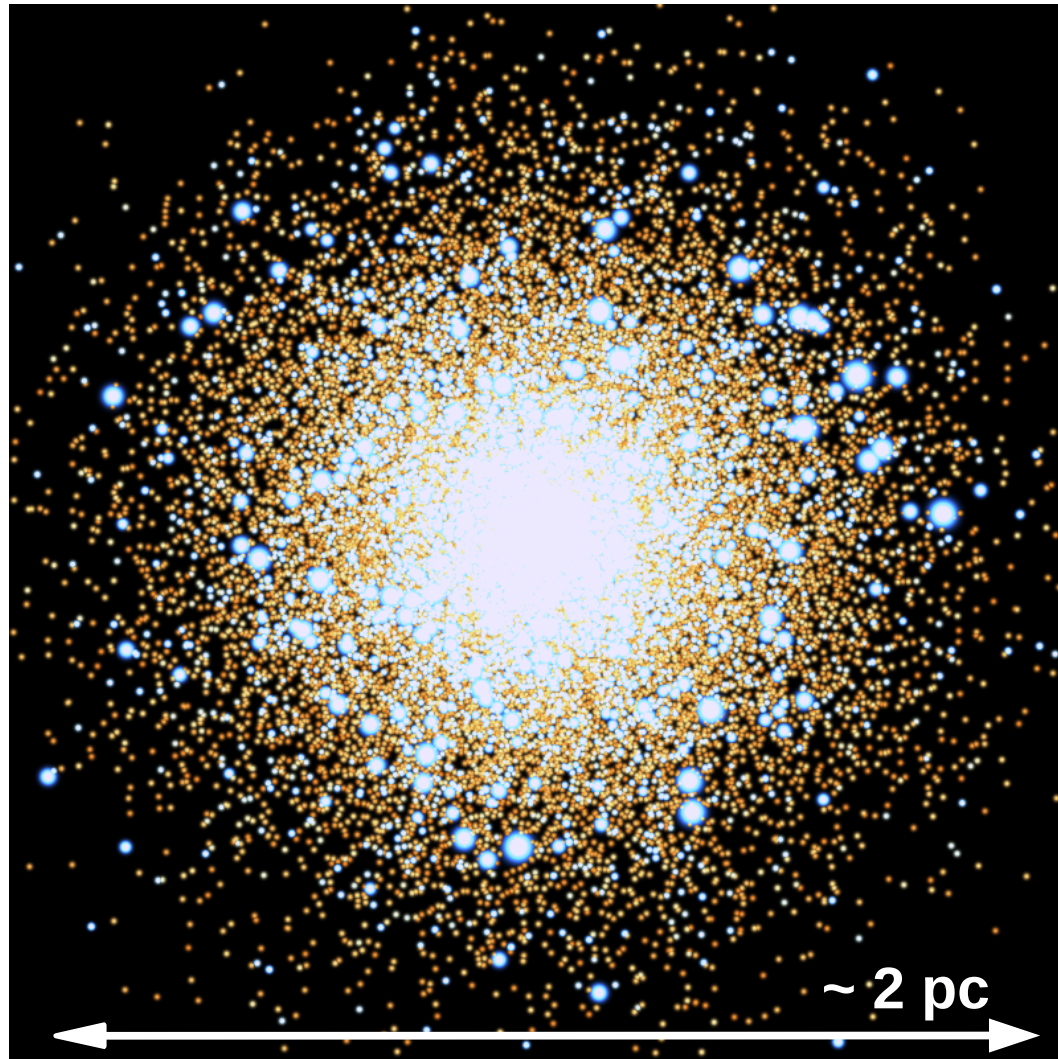
MAIN ISSUE: MASS LOSS!!!

(1) during merger
simulations show mass loss
up to 25% of total mass
(Gaburov et al. 2010, MNRAS, 402, 105)

(2) after merger, by stellar winds
the super-massive star
will be very unstable
(radiation pressure dominated)
e.g. MM 2016, MNRAS, 459, 3432



5.1 IMBHs from Runaway collisions



N-body simulations
of collisional systems
(direct summation N-body)

+

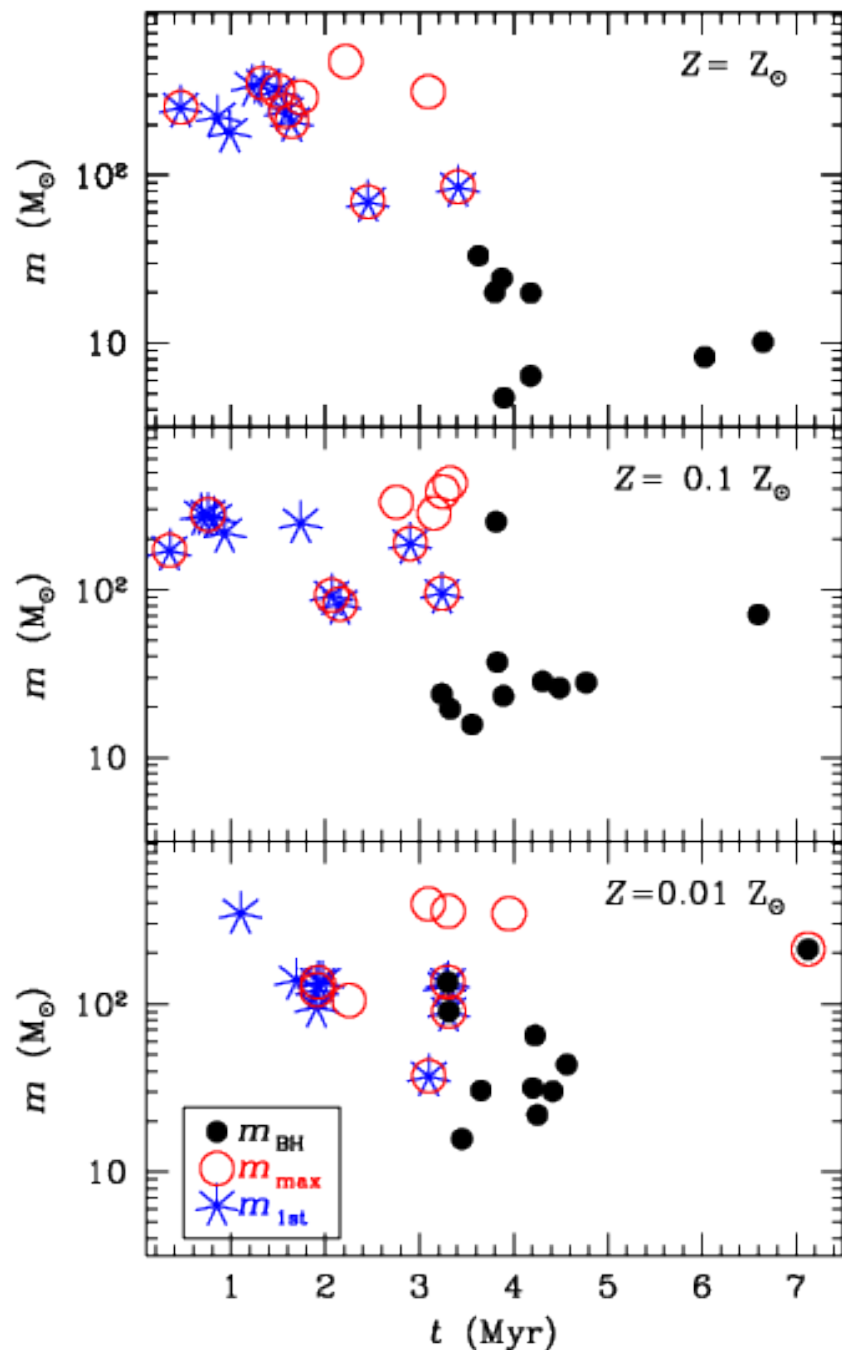
stellar and binary evolution
(population synthesis)
embedded in N-body

=

can be used to study
IMBH formation accounting
for mass loss



5.1 IMBHs from Runaway collisions



Mass loss by stellar winds prevents formation of IMBHs from runaway collisions

UNLESS METALLICITY $< 0.1 Z_{\text{sun}}$

e.g. MM 2016, MNRAS, 459, 3432

* maximum star mass up to 500 M_{sun}

* 1/10 BH in the IMBH regime ($>100 M_{\text{sun}}$) at $Z = 0.01 - 0.1 Z_{\text{sun}}$

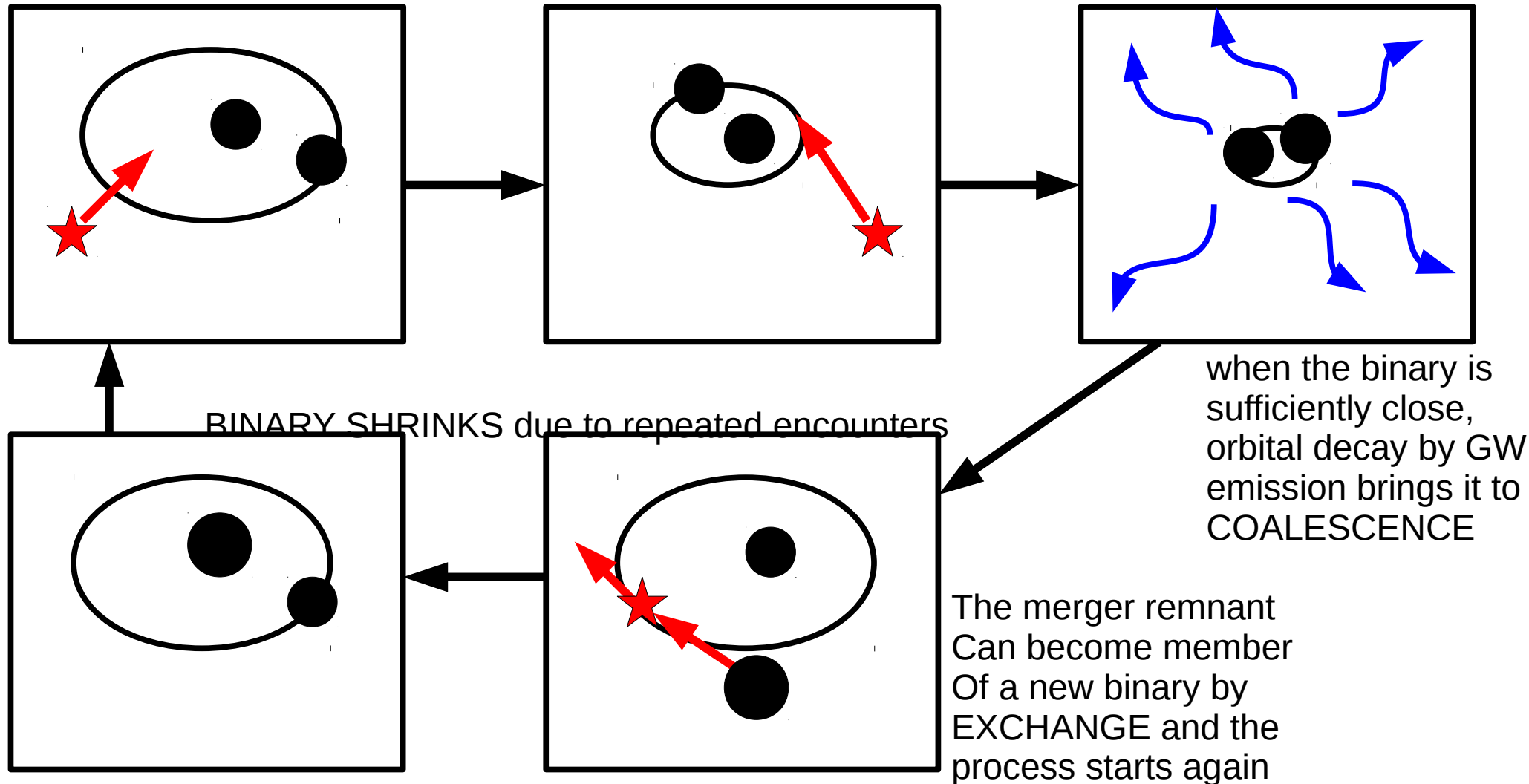
* CAVEAT 1: uncertainties in the evolution of very massive stars

* CAVEAT 2: uncertainties in mass-loss during/after collisions

5.2 IMBHs from Repeated mergers

Formalism by Miller & Hamilton (2002)

In a old cluster stellar BHs can grow in mass because of repeated mergers with the companion triggered by 3-body encounters



5.2 IMBHs from Repeated mergers

MAIN PROBLEM: seed BH must avoid ejection before merger

$$v_{rec} = \frac{m_3}{m_T} \sqrt{\frac{m_3 (m_1 + m_2)}{m_e (m_a + m_b)} v_\infty^2 + \frac{2 m_T}{m_e (m_a + m_b)} \Delta E_b} \sim \frac{m_3}{m_T} \sqrt{\frac{2 m_T}{m_3 (m_1 + m_2)} \Delta E_b}$$

$\boxed{m_1 + m_2 \gg m_3}$

$$\sim \frac{m_3}{m_T} \sqrt{\frac{2 m_T}{m_3 (m_1 + m_2)} \frac{\xi m_3}{(m_1 + m_2)} E_b} \sim \frac{m_3}{m_1 + m_2} \sqrt{\frac{2 \xi}{m_T} E_b}$$

Find the minimum binding energy for EJECTION ($E_{b,min}$) by imposing $v_{rec} = v_{esc}$

$$\Rightarrow E_{b,min} \sim \frac{(m_1 + m_2)^3}{2 \xi m_3^2} v_{esc}^2$$

where we assumed $m_1 + m_2 \sim m_T$

$$E_{b,min} \sim 2 \times 10^{50} \text{ erg} \left(\frac{m_1}{50 M_\odot} \right)^3 \left(\frac{m_3}{10 M_\odot} \right)^{-2} \left(\frac{\xi}{0.2} \right)^{-1} \left(\frac{v_{esc}}{50 \text{ km s}^{-1}} \right)^2$$

5.2 IMBHs from Repeated mergers

Orbital separation in gravitational wave merger regime:

$$a_{GW} \sim 3 \times 10^{11} \text{ cm} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{1/4} \left(\frac{m_1}{50 M_\odot} \right)^{1/2} \left(\frac{m_2}{10 M_\odot} \right)^{1/4}$$

Binding energy in merger regime:

$$E_{b, \text{merg}} = \frac{G m_1 m_2}{2 a_{GW}} \sim 2 \times 10^{50} \text{ erg} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{-1/4} \left(\frac{m_1}{50 M_\odot} \right)^{1/2} \left(\frac{m_2}{10 M_\odot} \right)^{3/4}$$

COMPARING $E_{b, \text{min}}$ with $E_{b, \text{merg}}$:

$$x = \frac{E_{b, \text{min}}}{E_{b, \text{merg}}} \sim \left(\frac{m_1}{50 M_\odot} \right)^{5/2} \left(\frac{m_2}{10 M_\odot} \right)^{-11/4} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{1/4}$$

If $x > 1$ BINARY MERGES BEFORE EJECTION

If $x < 1$ BINARY IS EJECTED BEFORE MERGER

5.2 IMBHs from Repeated mergers

ADDITIONAL PROBLEM: INEFFICIENT!

Number of 3-body encounters for a BH to merge with its companion
(from lecture 3):

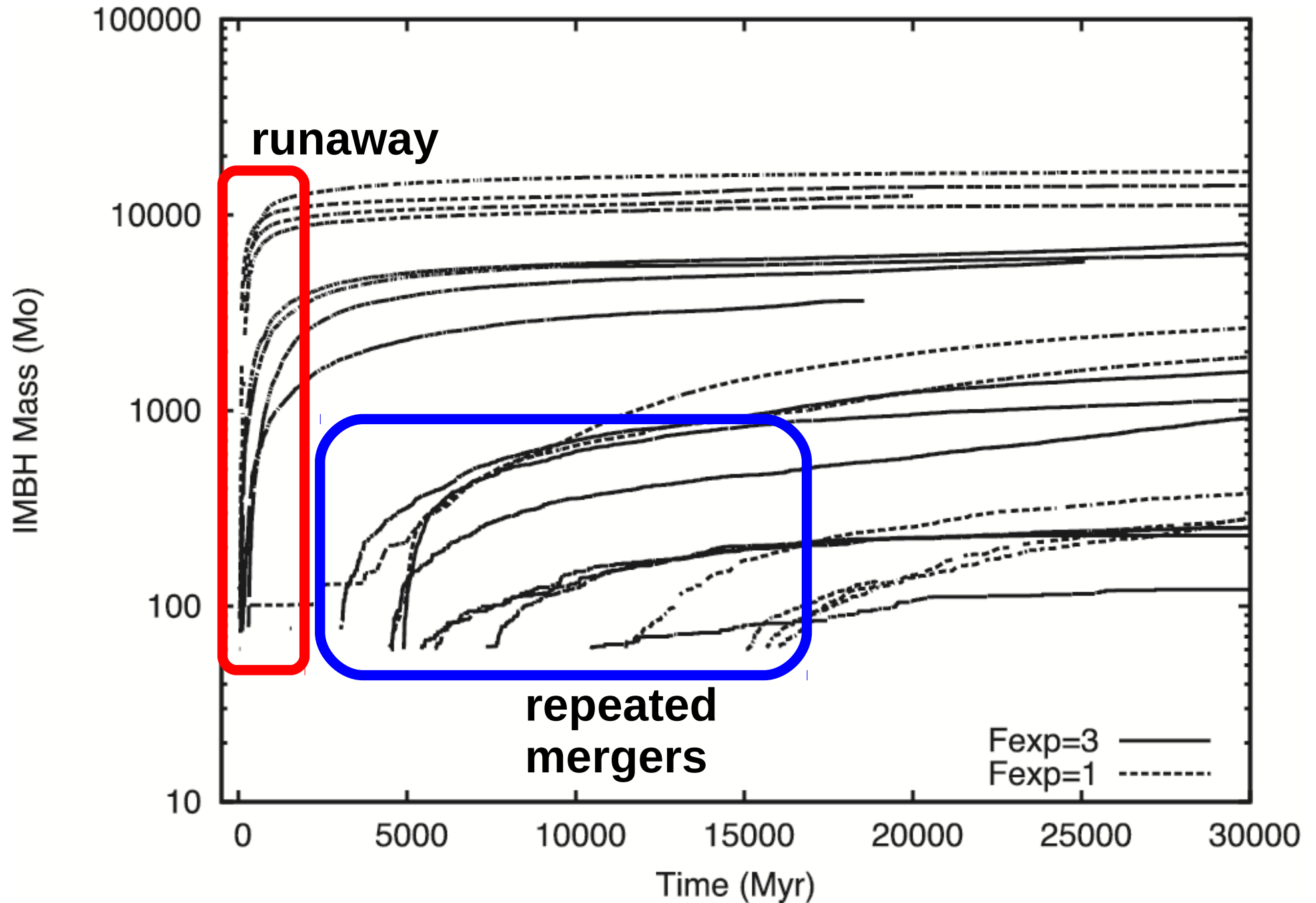
$$N_{merg} = \frac{1}{\xi} \frac{m_T}{\langle m \rangle} \ln \left(\frac{a_0}{a_{GW}} \right)$$

Time required for 1 merger:

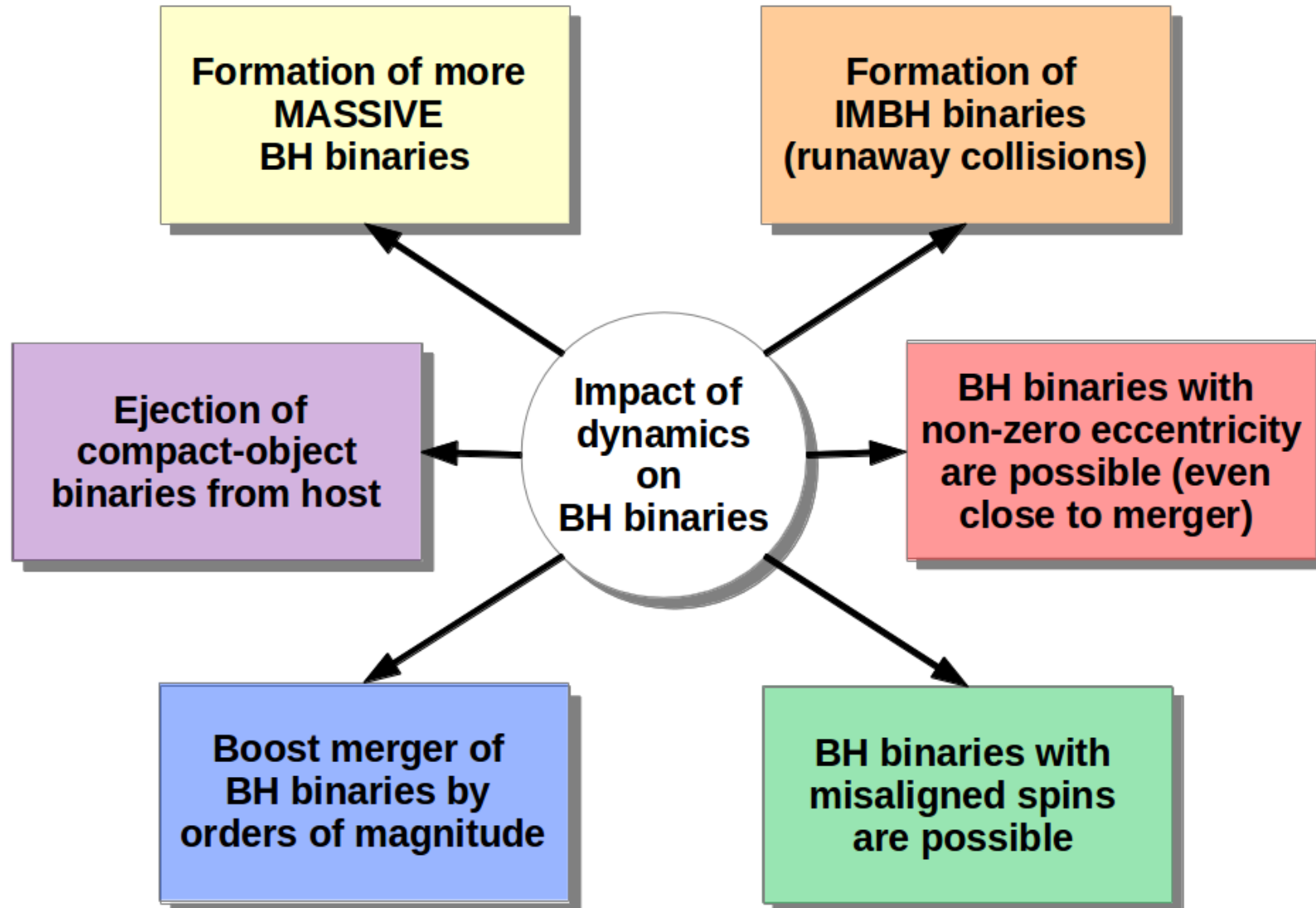
$$dt = -\frac{\sigma}{2\pi G \xi \rho} \frac{da}{a^2} \longrightarrow \int_0^{t_{GW}} dt = -\frac{\sigma}{2\pi G \xi \rho} \int_{a_0}^{a_{GW}} \frac{da}{a^2}$$

$$t_{GW} = \frac{\sigma}{2\pi G \xi \rho} \left(\frac{1}{a_{GW}} - \frac{1}{a_0} \right)$$

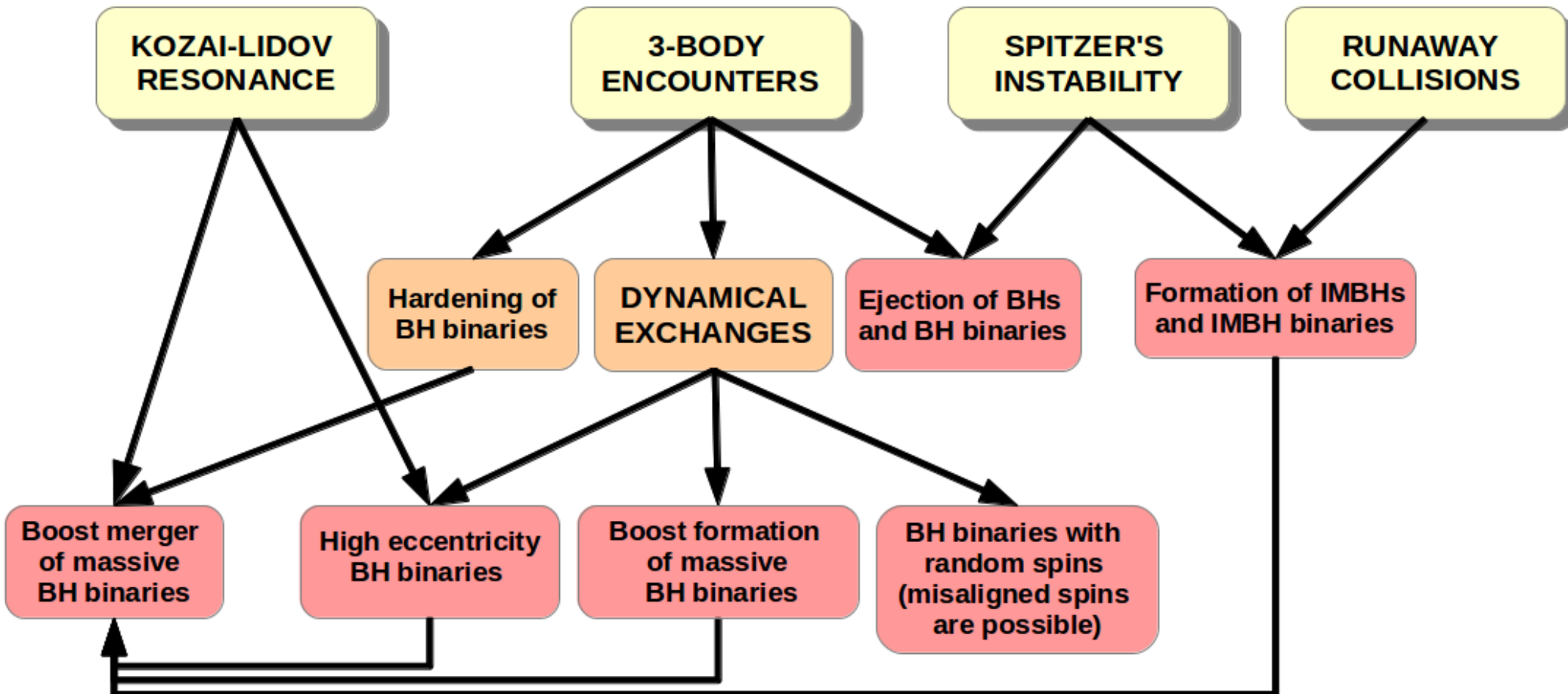
$$t_{GW} \sim 3 \times 10^8 \text{ yr} \left(\frac{\sigma}{10 \text{ km s}^{-1}} \right) \left(\frac{\xi}{0.2} \right)^{-1} \left(\frac{\rho}{10^6 M_{\odot} \text{ pc}^{-3}} \right)^{-1} \left(\frac{a_{GW}}{1 \text{ AU}} \right)^{-1}$$



SUMMARY of EFFECTs of DYNAMICS on BH binaries:



SUMMARY of EFFECTs of DYNAMICS on BH binaries:



References:

- * **Portegies Zwart & McMillan, 2002, ApJ, 576, 899**
- * **Miller & Hamilton, 2002, MNRAS, 330, 232**
- * Heger et al. 2003, arXiv:astro-ph/0211062
- * Kulkarni, Hut & McMillan 1993, Nature 364, 421
- * Sigurdsson & Hernquist 1993, Nature 364, 42
- * Giersz et al. 2015, MNRAS, 454, 3150
- * Mapelli 2016, MNRAS, 459, 3432
- * Spera, Mapelli, Bressan 2015, MNRAS, 451, 4086

THANK YOU