Michela Mapelli

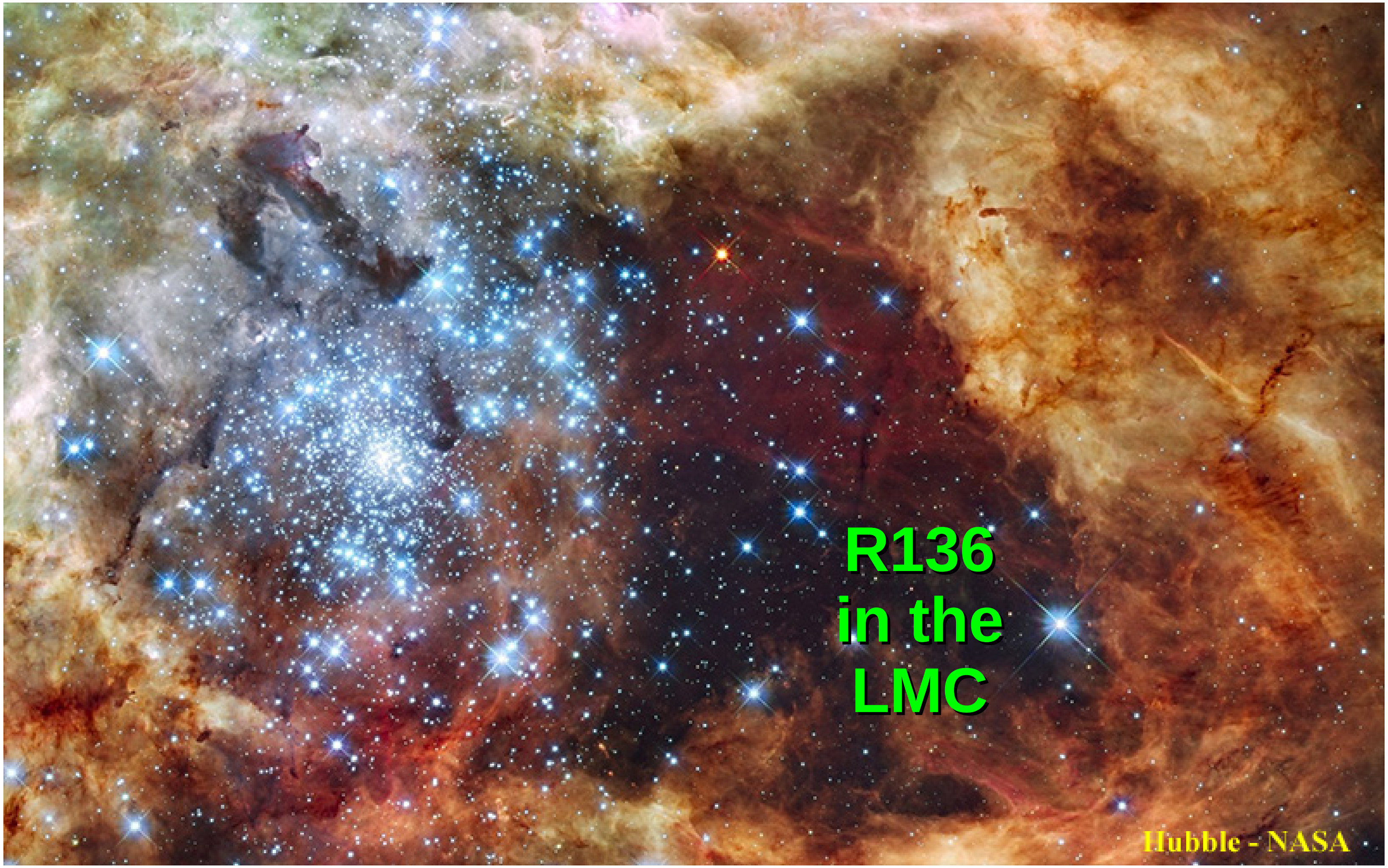
Dynamics of Stars and Black Holes in Dense Stellar Systems I

Innsbruck, January 9 2018

WHY should we care about DYNAMICS when studying BH binaries?

WHY DYNAMICS???????

Massive stars (BH progenitors) form in STAR CLUSTERS: dynamically 'ACTIVE' places (Lada & Lada 2003)



WHY DYNAMICS???????

Massive stars (BH progenitors) form in STAR CLUSTERS

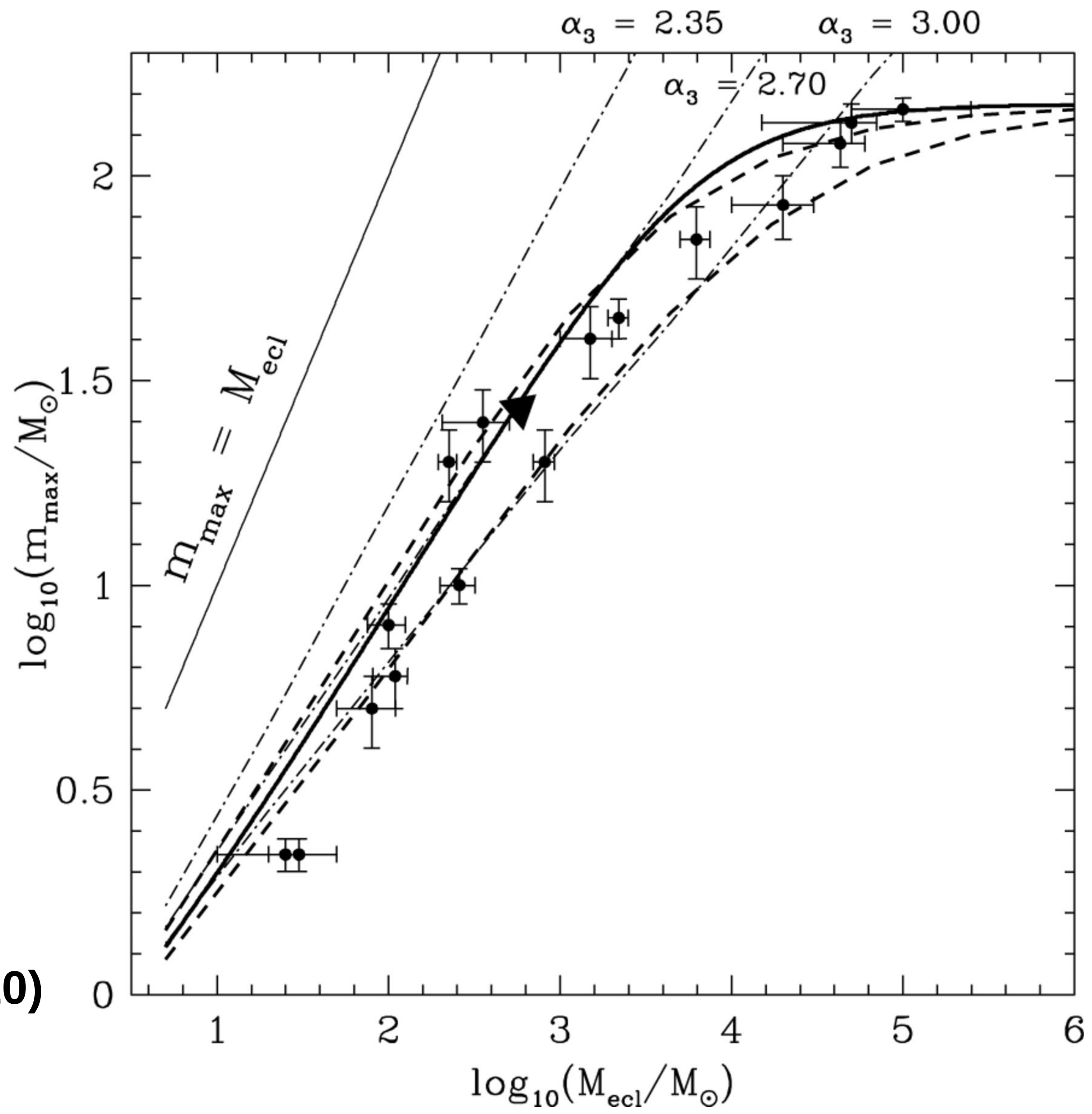


Figure from Weidner & Kroupa (2006)

Data points: observed star clusters

Lines: theoretical fits

See also Weidner, Kroupa & Bonnell (2010)

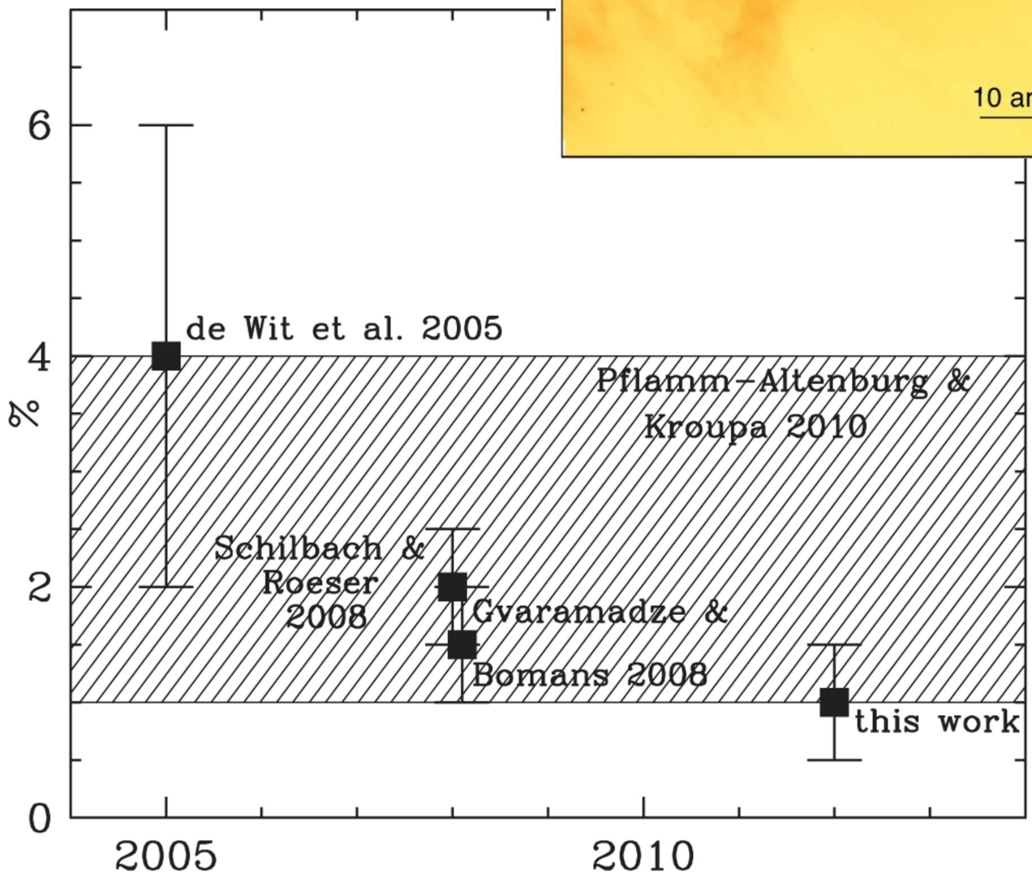
WHY DYNAMICS???????

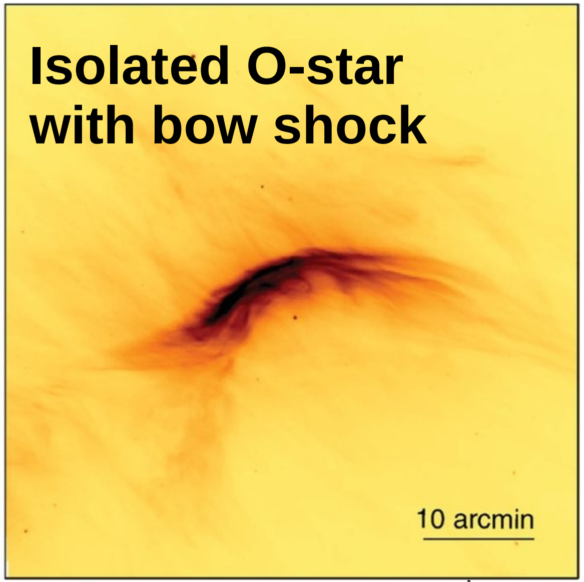
O-type stars in the field are mostly RUNAWAY from star clusters (as we see from bow shocks)

Figures from Gvaramadze et al. (2012)

See also
De Wit et al. (2004, 2005)
Schilbach & Roeser (2008)

Percentage of genuine field O stars





FIELD:

* NO dynamics
(density in solar
neighborhood
<1 star pc^-3)

GLOBULAR CLUSTERS:

* dynamics
* long-lived
(12 Gyr)
* < 1 % baryonic
mass of
the Universe

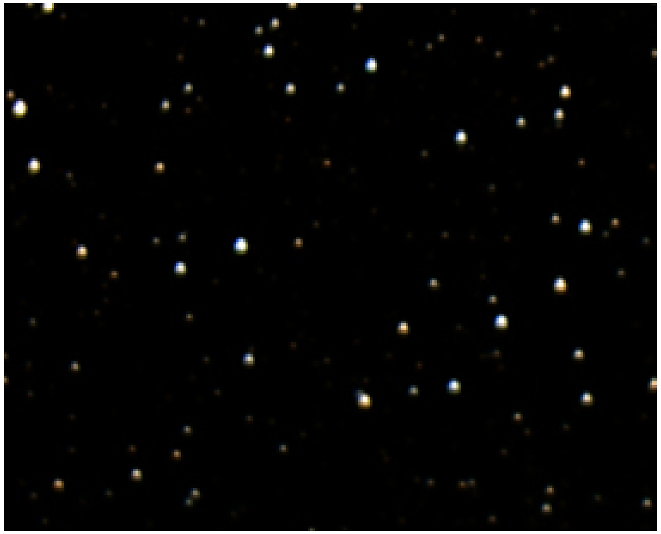


Image credit: Jim Mazur's Astrophotography, via http://www.skyledge.net/.

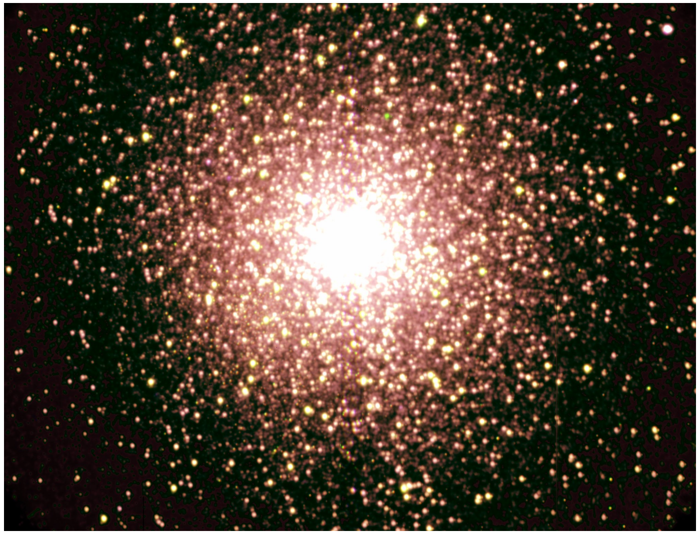


Image credit: HST

FIELD:

* NO dynamics (density in solar neighborhood <1 star pc^-3)

YOUNG STAR CLUSTERS and OPEN CLUSTERS:

* dynamics
* short-lived (0.01 – 1 Gyr)
* cradle of massive stars (80% star formation)

GLOBULAR CLUSTERS:

* dynamics
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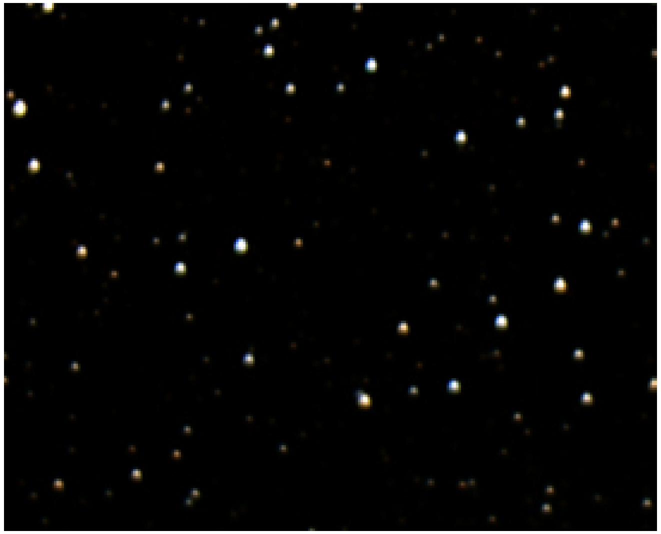

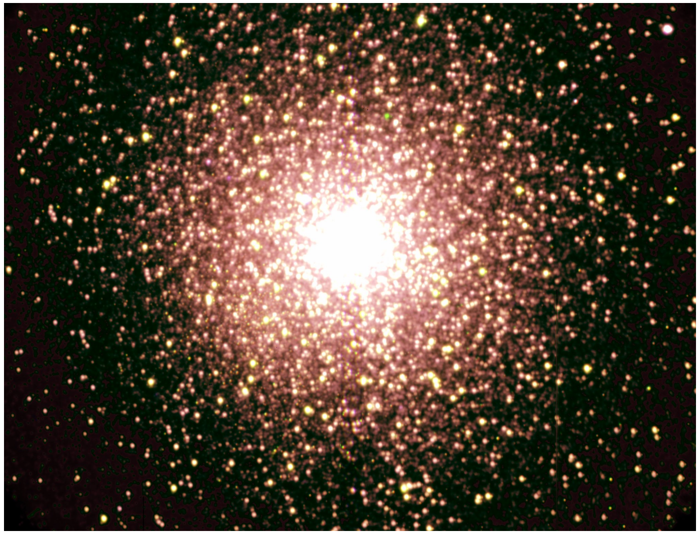

Image credit: Jim Mazur’s Astrophotography, via http://www.skyledge.net/.

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provide stars (and compact objects) to the field

share dynamical properties with globular clusters

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GLOBULAR CLUSTERS:

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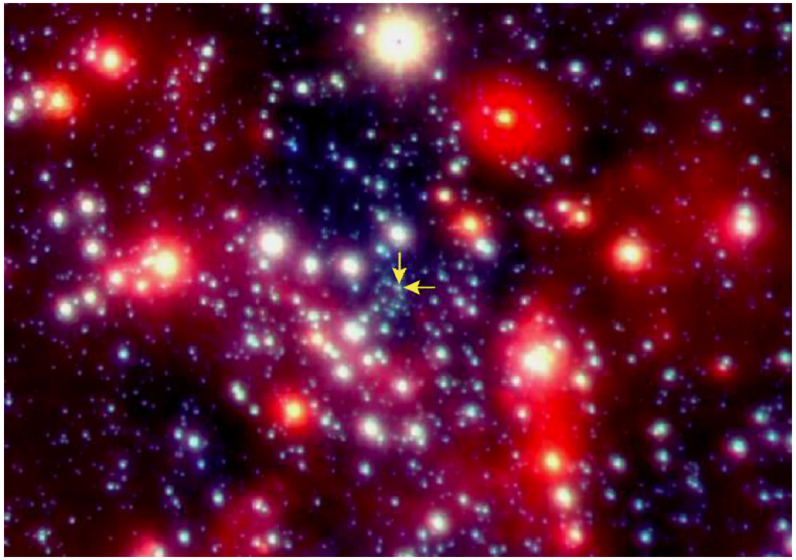
NUCLEAR STAR CLUSTERS:

* dynamics
* long-lived (12 Gyr)
* host SUPER-MASSIVE BHs

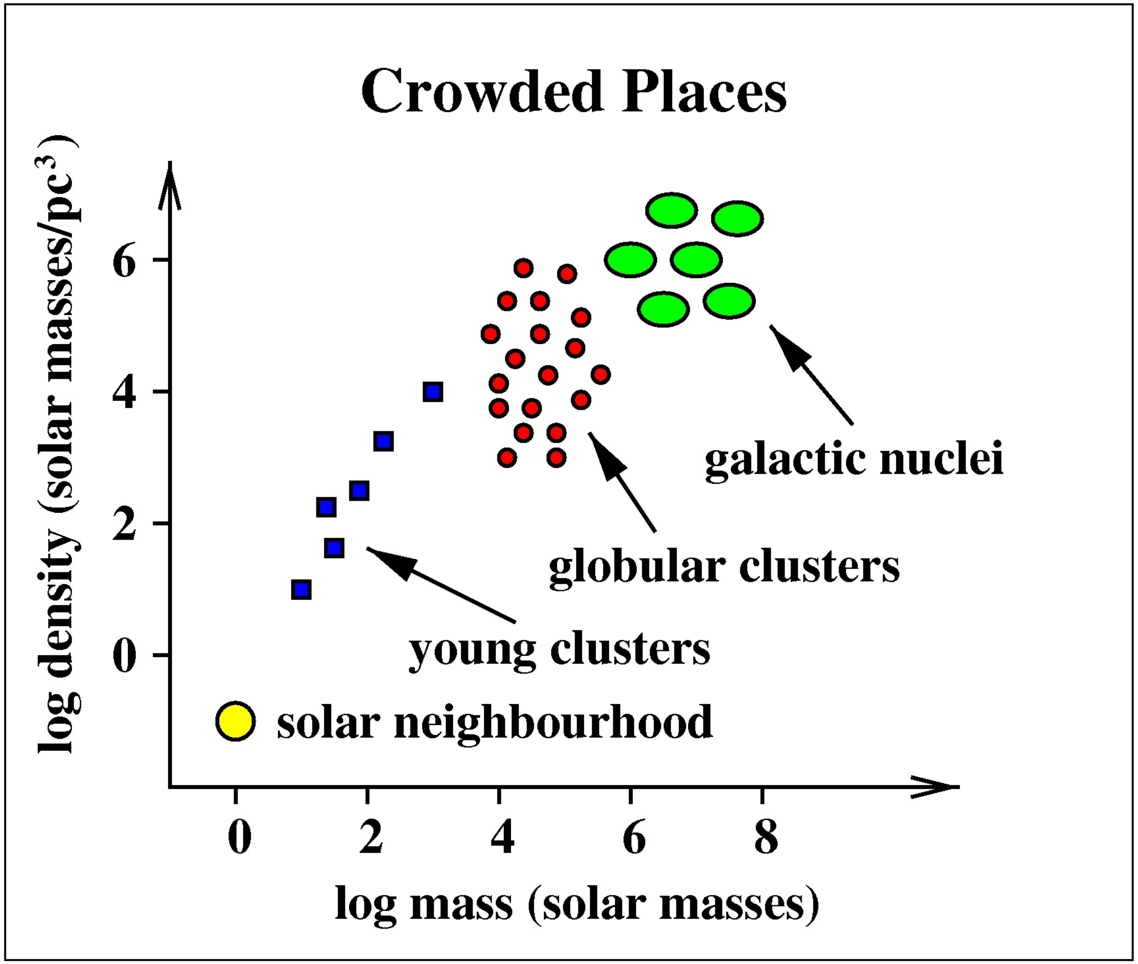


Schoedel et al. 2002, Nature, 419, 694

WHAT KIND OF DYNAMICS?

Star clusters: high density systems (>1000 Msun pc^-3) with small dispersion velocity (~ 10 – 30 km/s)

→ close encounters between stars are likely



M. B. Davies, 2002, astroph/0110466

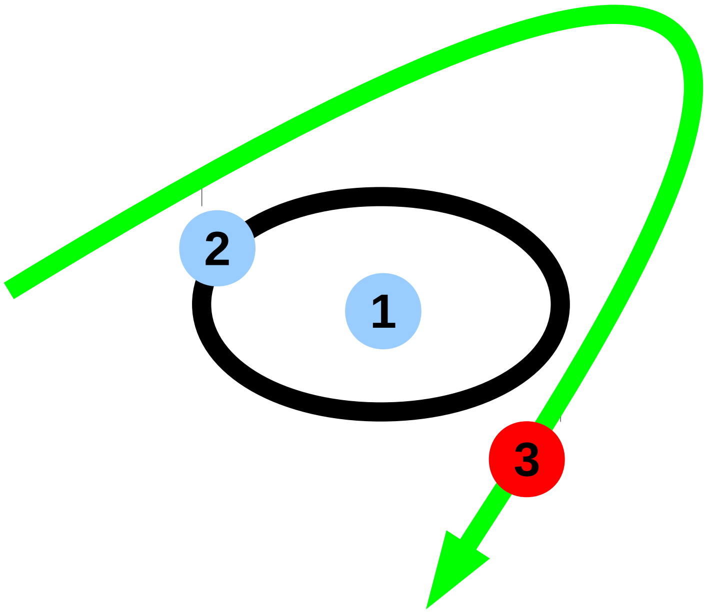
Binaries have a energy reservoir (internal energy)

$$E_{int} = \frac{1}{2}\,\mu\,v^2 - \frac{G\,m_1\,m_2}{r}$$

where m_1 and m_2 are the mass of the primary and secondary member of the binary,
μ is the reduced mass (:= $m_1\, m_2/(m_1+m_2)$), r and v are the relative separation and velocity.

$$E_{int} = -\frac{G\,m_1\,m_2}{2\,a} = -E_b$$

THE ENERGY RESERVOIR of BINARIES can be EXCHANGED with stars during a 3-BODY INTERACTION, i.e. an interaction between a binary and a single star

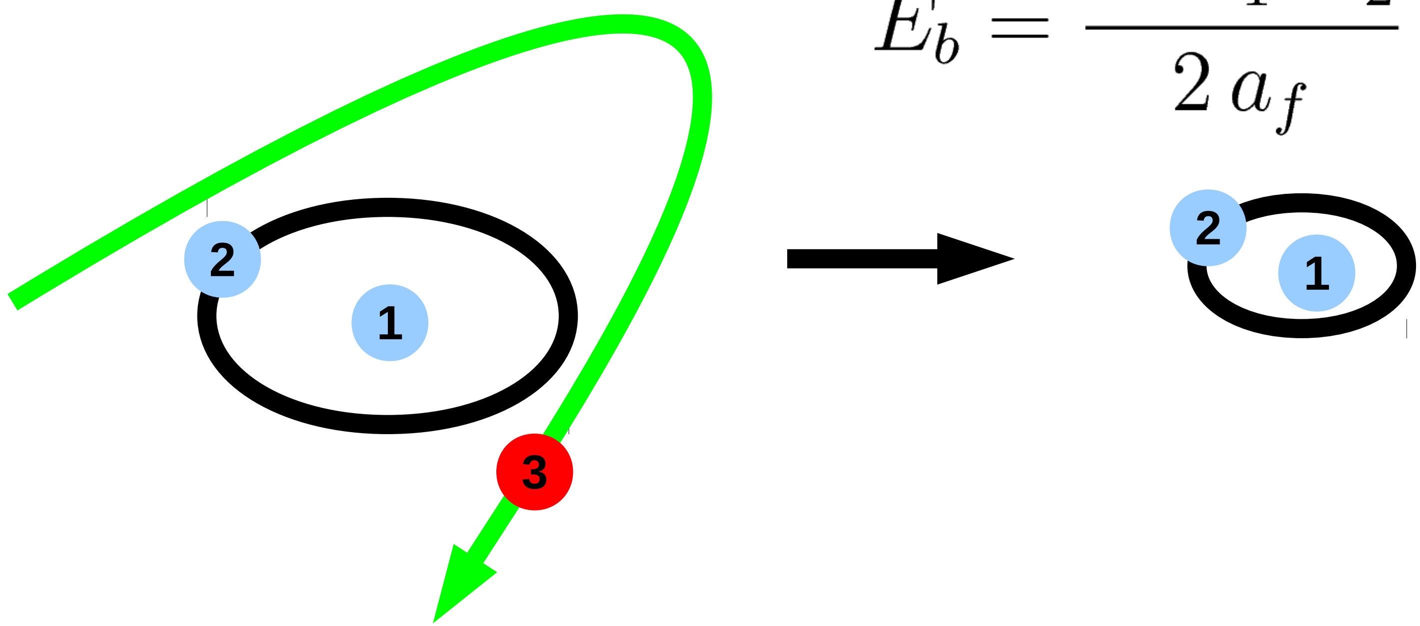


If the star extracts E_{int} from the binary, its final kinetic energy (K_f) is higher than the initial kinetic energy (K_i).
To better say: K_f of the centres-of-mass of the single star and of the binary is higher than their K_i.
We say that the STAR and the BINARY acquire **RECOIL VELOCITY**.

E_{int} becomes more negative, i.e. E_b higher: the binary becomes more bound (e.g. a decreases or m_1 and m_2 change).

$a_f < a_i$

$$E_b = \frac{G\, m_1\, m_2}{2\, a_f} > \frac{G\, m_1\, m_2}{2\, a_i}$$



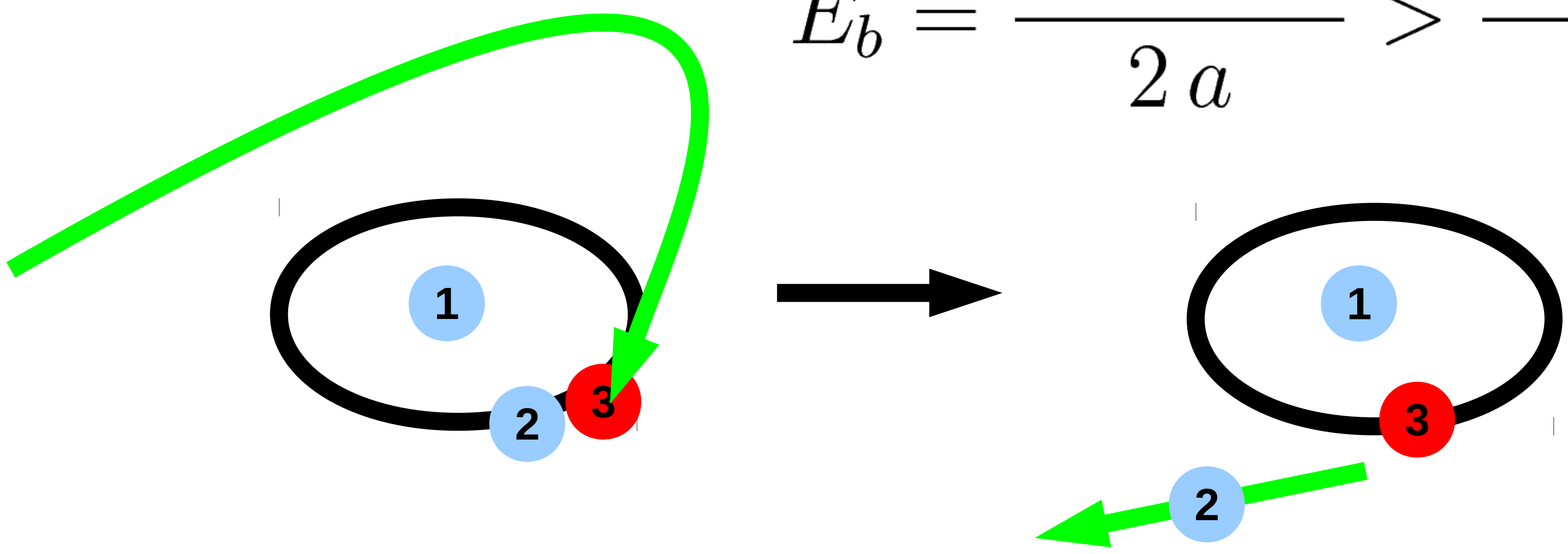
CARTOON of a FLYBY ENCOUNTER where $a_f < a_i \rightarrow E_b$ increases

An alternative way for a binary to transfer internal energy to field stars and increase its binding energy E_b is an **EXCHANGE**:
the single star replaces one of the former members of the binary.

An exchange interaction is favored when the mass of the single star m_3 is HIGHER than the mass of one of the members of the binary so that the new E_b of the binary is higher than the former:

$m_3 > m_2$

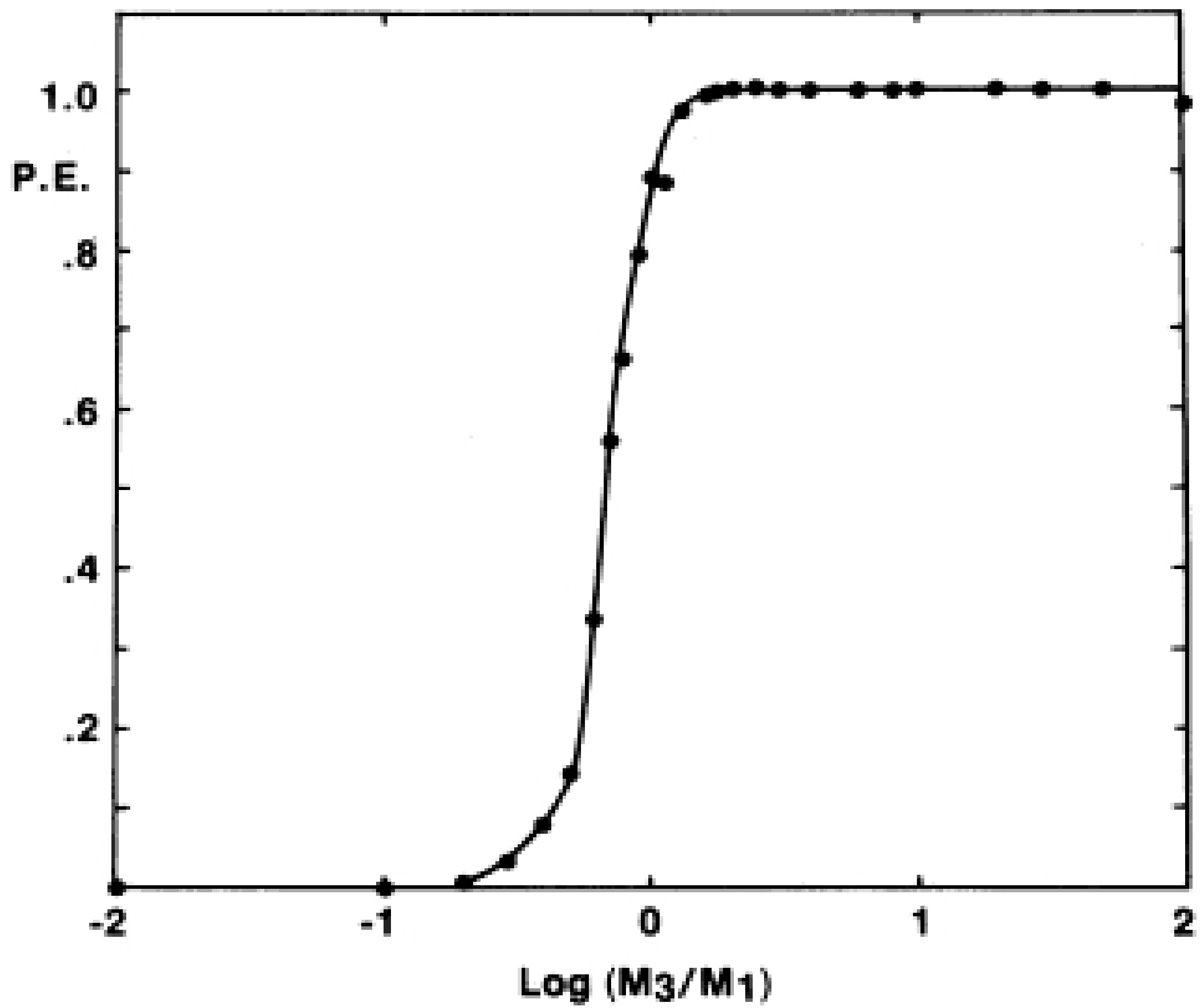
$$E_b = \frac{G\,m_1\,m_3}{2\,a} > \frac{G\,m_1\,m_2}{2\,a}$$



CARTOON of a EXCHANGE ENCOUNTER where $m_3 > m_2 \rightarrow E_b$ increases

EXCHANGE PROBABILITY:

Probability increases dramatically if $m_3 \geq m_1$



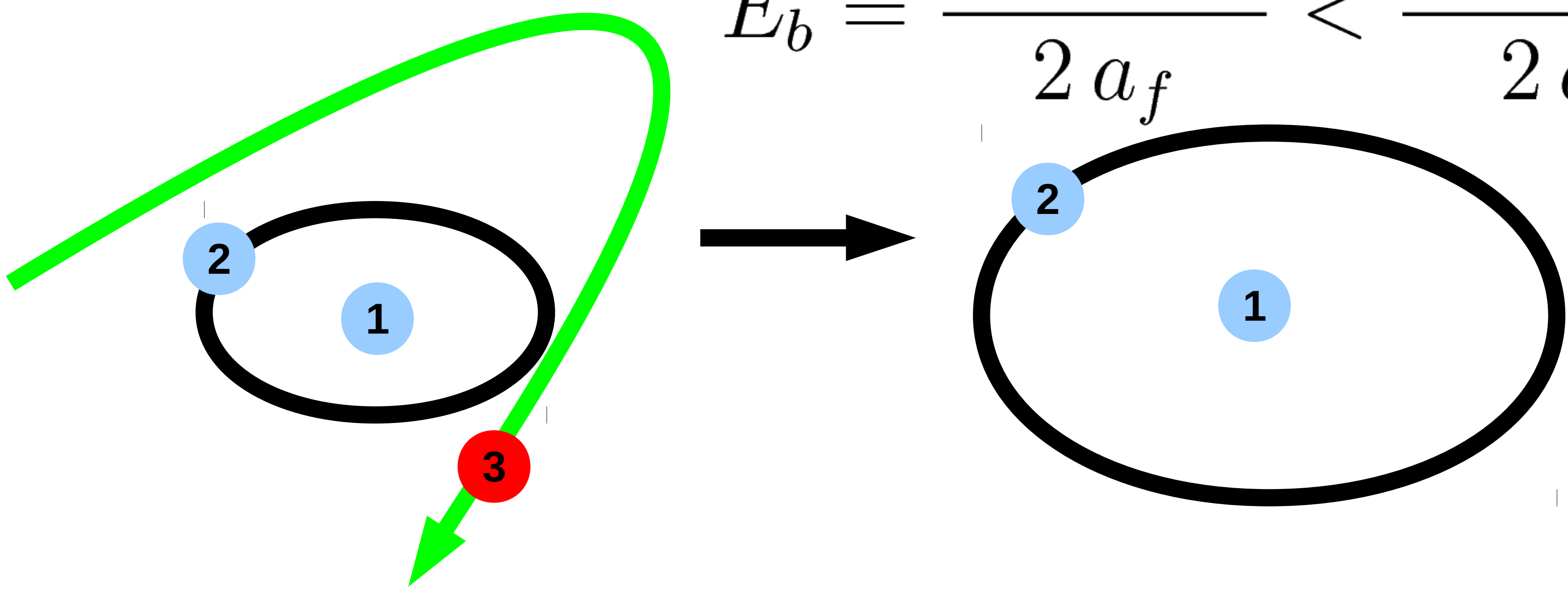
Hills & Fullerton 1980, AJ, 85, 1281

If the star transfers kinetic energy to the binary, its final kinetic energy (K_f) is obviously lower than the initial kinetic energy (K_i). To better say: K_f of the centres-of-mass of the single star and of the binary is lower than their K_i.

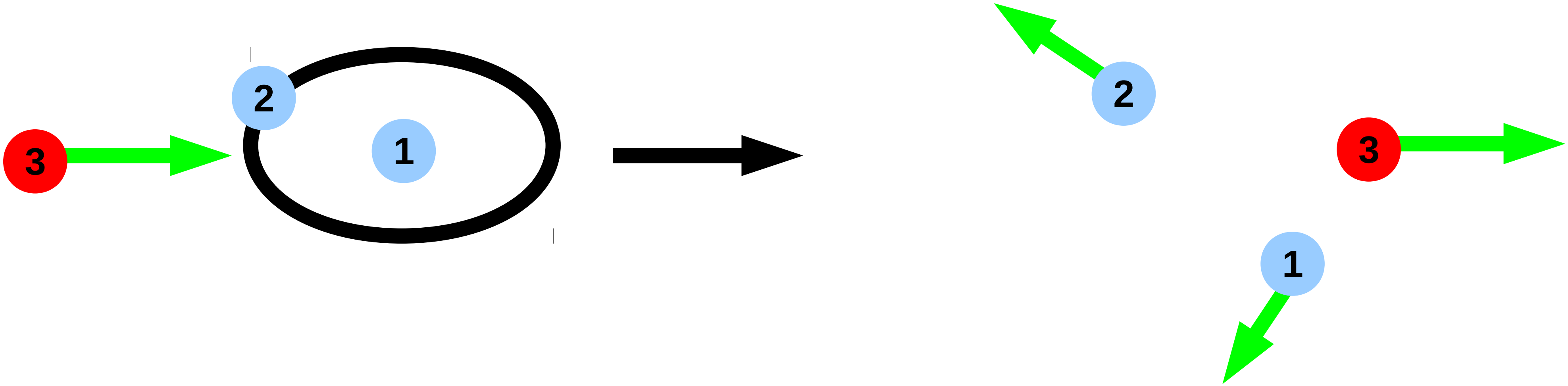
E_{int} becomes less negative, i.e. E_b smaller: the binary becomes less bound (e.g. a increases) or is even **IONIZED** **(:= becomes UNBOUND)**.

$a_f > a_i$

$$E_b = \frac{G\,m_1\,m_2}{2\,a_f} < \frac{G\,m_1\,m_2}{2\,a_i}$$



CARTOON of a FLYBY ENCOUNTER where $a_f > a_i \rightarrow E_b$ decreases



A single star can IONIZE the binary only if its velocity at infinity (=when it is far from the binary, thus unperturbed by the binary) exceeds the CRITICAL VELOCITY (Hut & Bahcall 1983, ApJ, 268, 319)

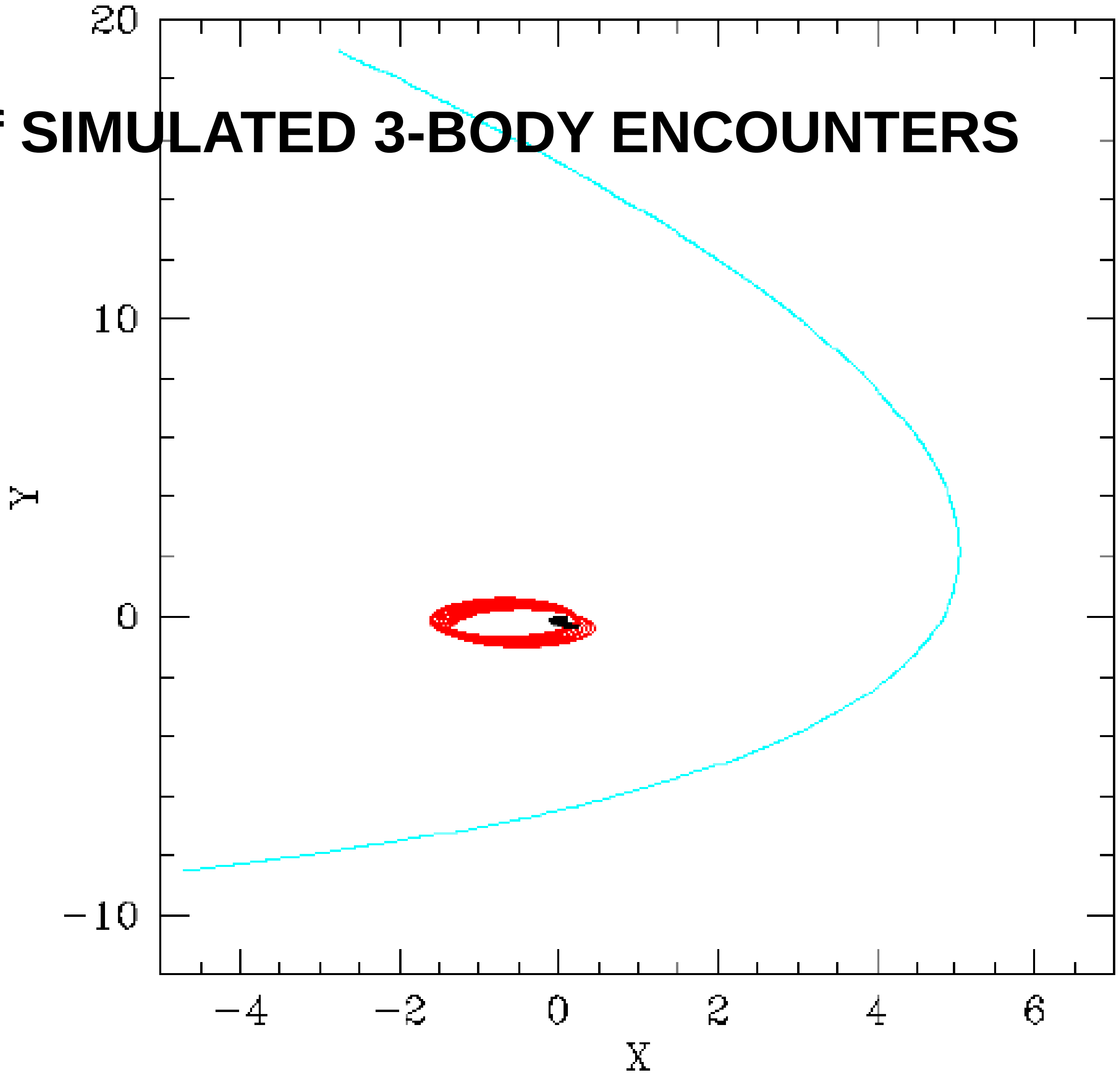
$$v_c = \sqrt{\frac{G\, m_1\, m_2\, (m_1 + m_2 + m_3)}{m_3\, (m_1 + m_2)\, a}}$$

This critical velocity was derived by imposing that the K of the reduced particle of the 3-body system is equal to E_b:

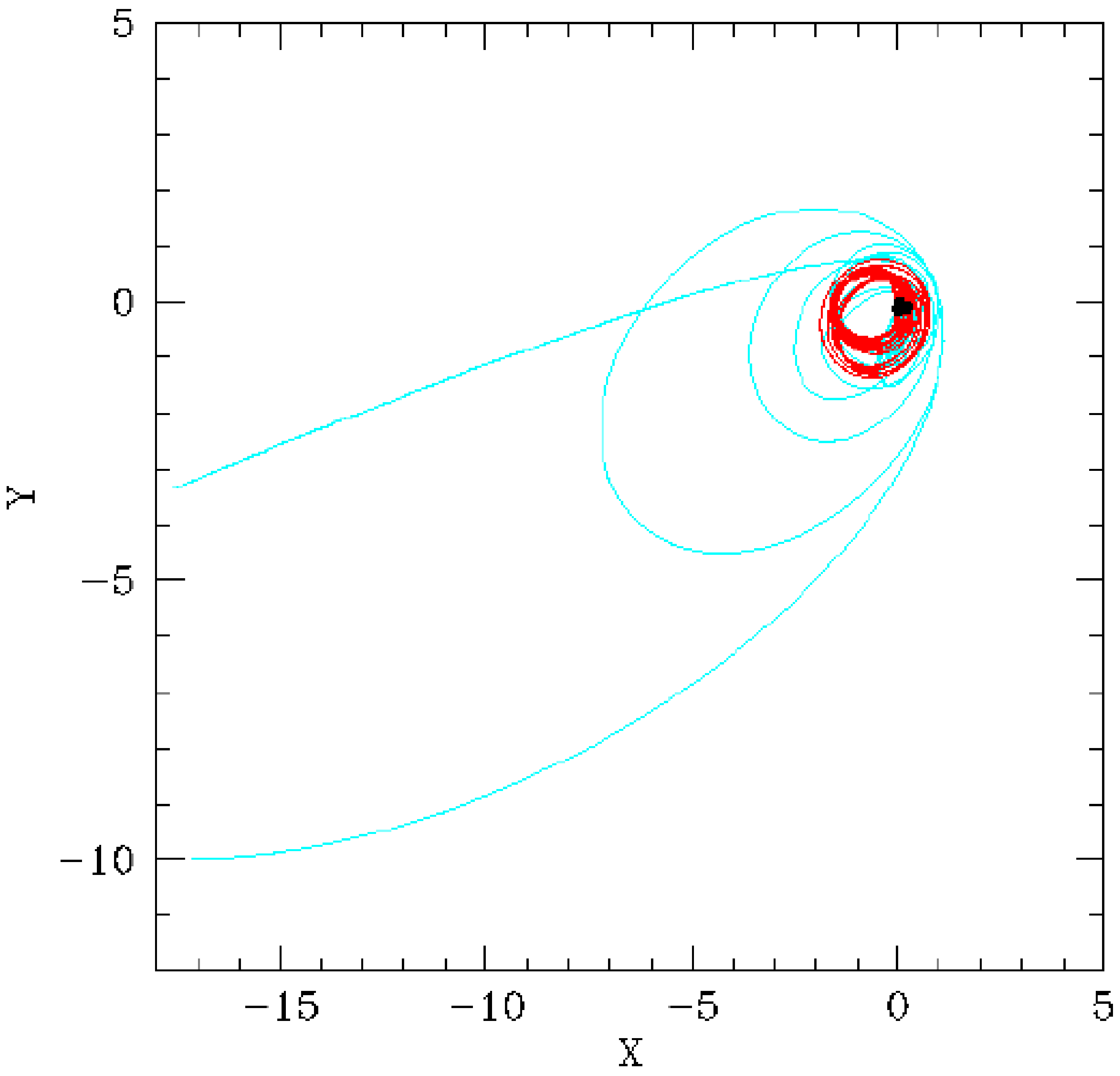
$$\frac{1}{2}\,\frac{m_3\,(m_1+m_2)}{(m_1+m_2+m_3)}\,v_c^2 = \frac{G\,m_1\,m_2}{2\,a}$$

EXAMPLES of SIMULATED 3-BODY ENCOUNTERS

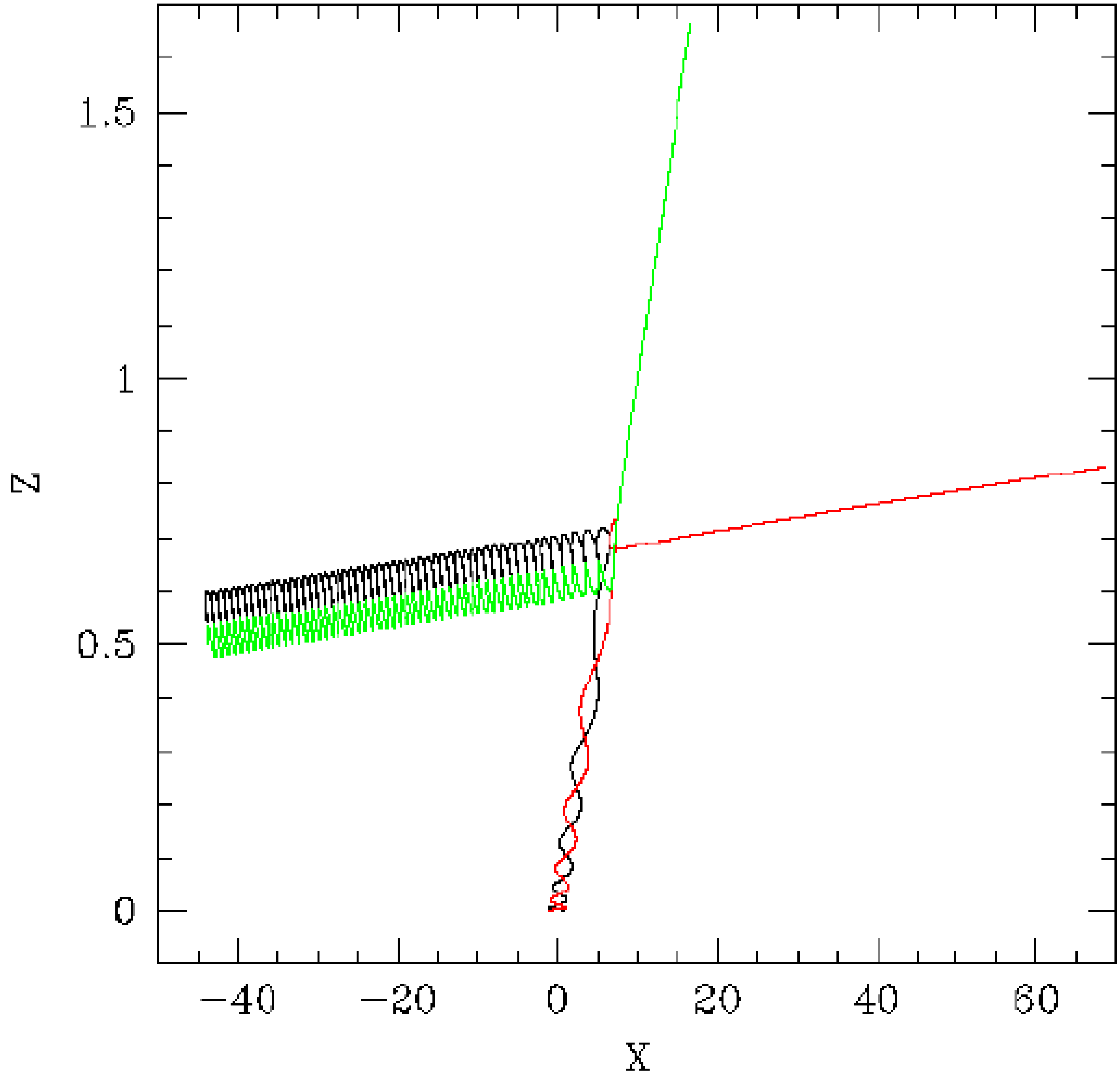
PROMPT
FLYBY:



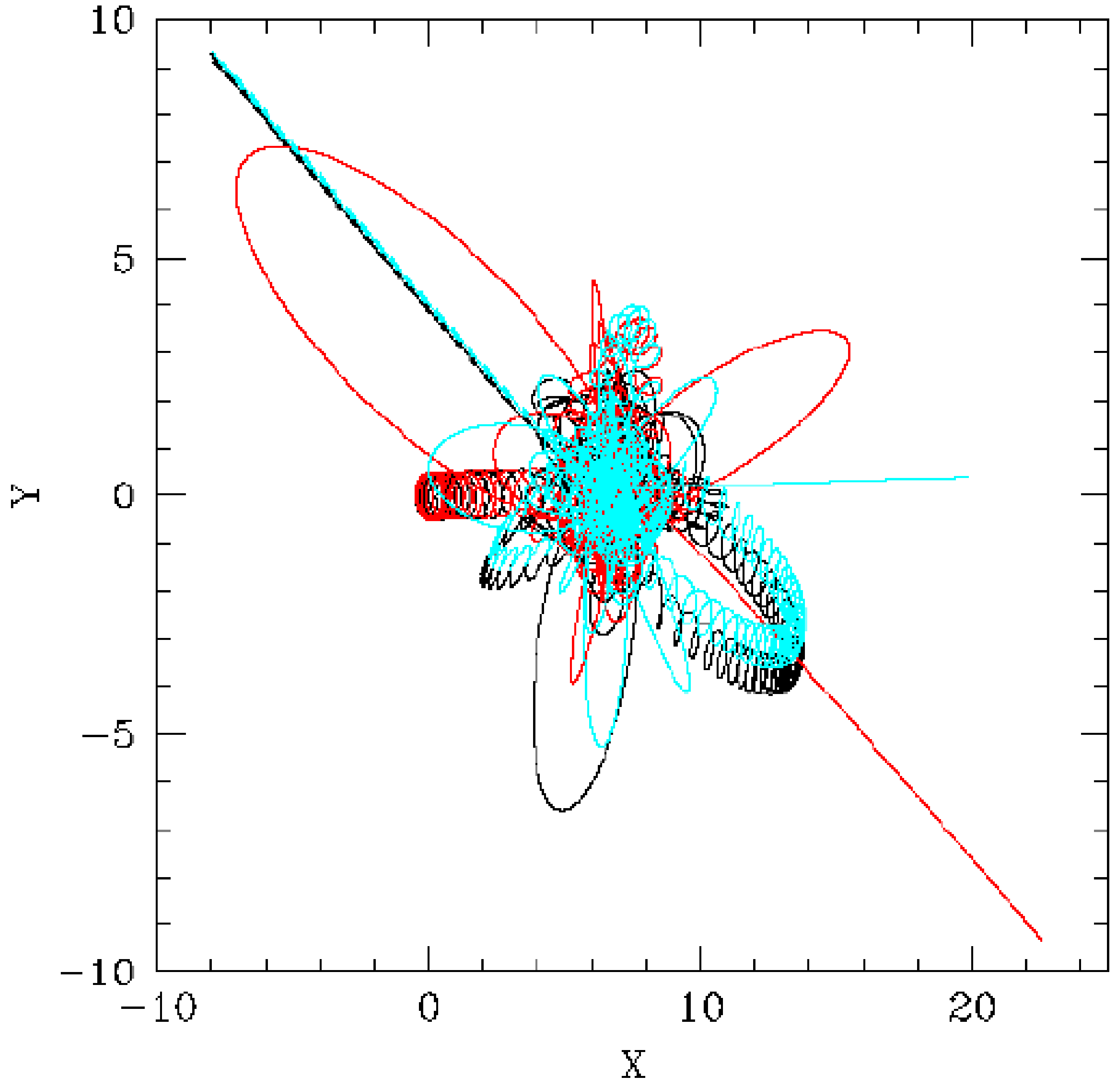
RESONANT
FLYBY:



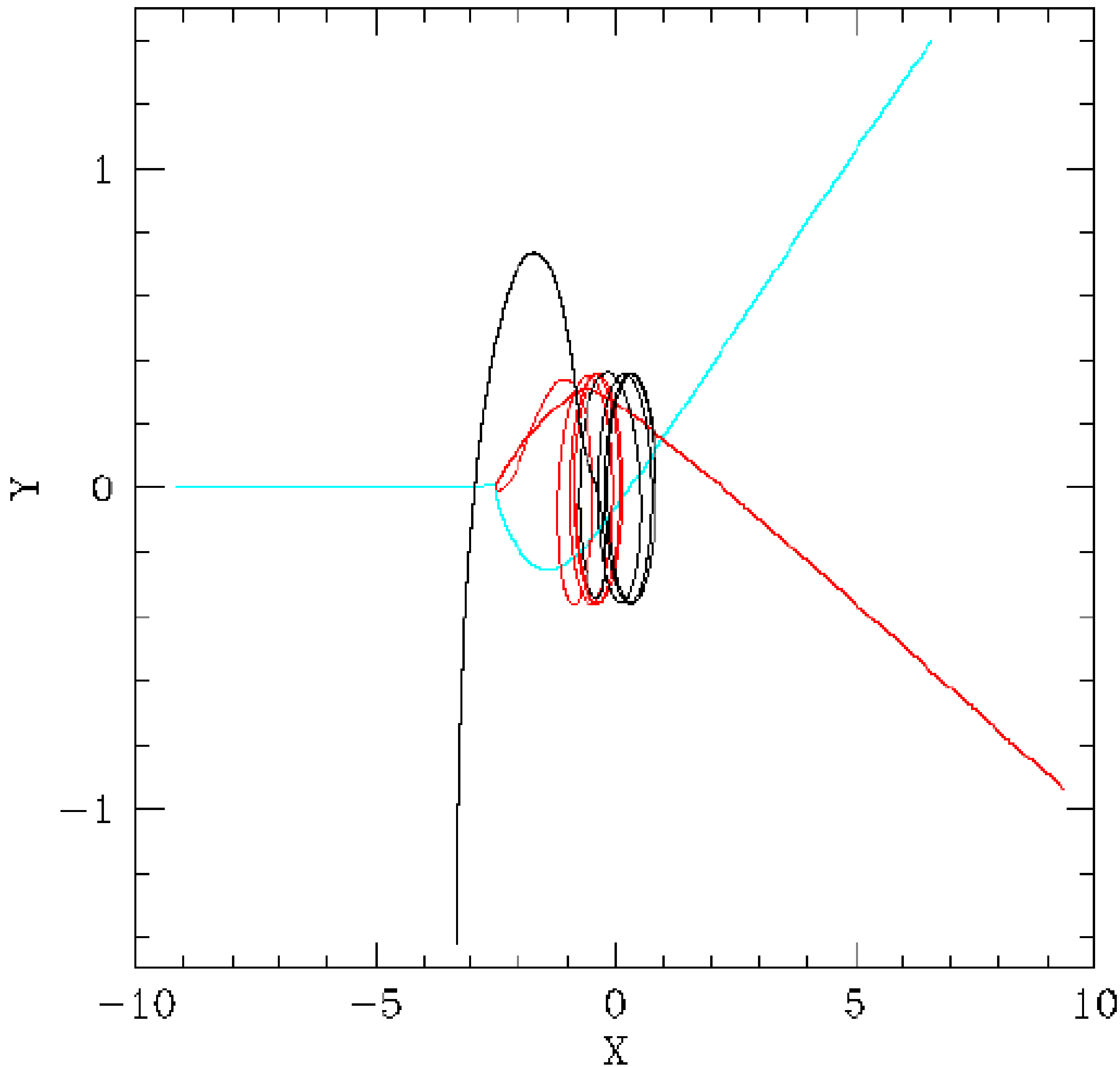
PROMPT
EXCHANGE:



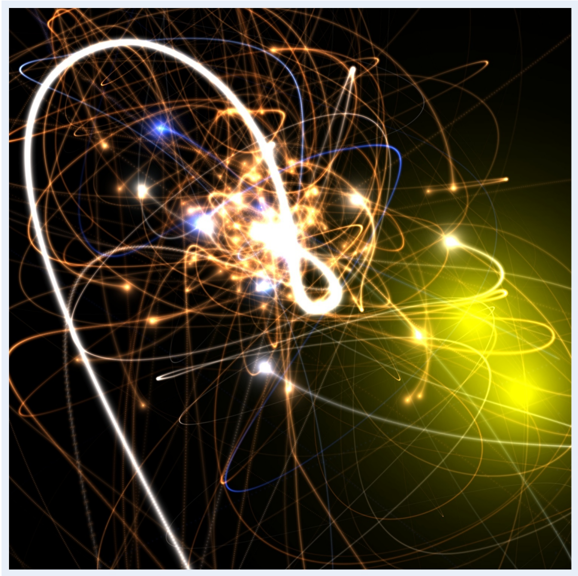
RESONANT
EXCHANGE:



IONIZATION:



Credits: Aaron Geller (@Northwestern):



Movie 2 : binary – single interaction
ciera.northwestern.edu/Research/visualizations/videos/Binary+single.mp4

Movie 3 : dynamical exchange
ciera.northwestern.edu/Research/visualizations/videos/Binary+singleex.mp4

Movie 4: 5-body interaction (leads to a COLLISION!)
ciera.northwestern.edu/Research/visualizations/videos/Triple+binary.mp4

Can we understand whether a binary will lose or acquire E_b?

YES, but ONLY in a STATISTICAL SENSE

We define **HARD BINARIES:** binaries with binding energy higher than the average kinetic energy of a star in the cluster

$$\frac{G\,m_1\,m_2}{2\,a} > \frac{1}{2}\langle m\rangle\,\sigma^2$$

$$\frac{G\,m_1\,m_2}{2\,a} < \frac{1}{2}\langle m\rangle\,\sigma^2$$

SOFT BINARIES: binaries with binding energy lower than the average kinetic energy of a star in the cluster

HEGGIE'S LAW (1975):

Hard binaries tend to become harder (i.e. increase E_b)

Soft binaries tend to become softer (i.e. decrease E_b)

as effect of three-body encounters

What is the rate of three-body encounters?

CROSS SECTION for 3-BODY ENCOUNTERS:

$$\Sigma = \pi \, b_{max}^2$$

where b_{max} is the maximum impact parameter for a non-zero energy exchange between star and binary

For most binaries $b_{\rm max}$ can be expressed as

$$\Sigma = 2\,\pi\,G\,\frac{m_T\,a}{{v_\infty}^2}$$

where m_T is the total mass of the binary, a its semi-major axis and v_∞ is the typical velocity between stars in a star cluster

As usual, an interaction rate has the form

$$R = \frac{dN}{dt} = n\,\Sigma\,v_\infty$$

where *n* is the local density of stars.
For the cross section in (*), the rate becomes

$$\boxed{R = 2\,\pi\,G\,\frac{m_T\,n\,a}{v_\infty}}$$

Note: the rate depends

(1) on the total mass of the interacting objects (more massive objects interact more),
(2) on the semi-major axis of the binary (wider binaries have a larger cross section),
(3) on the local density (denser environments have higher interaction rate),
(4) on the local velocity field (systems with smaller velocity dispersion have higher interaction rate).

How much energy is exchanged during three-body encounters?

$$\frac{\Delta E_b}{E_b} \propto \frac{m_3}{m_1 + m_2}$$

$$\xi \equiv \frac{m_1 + m_2}{m_3} \frac{\langle \Delta E_b \rangle}{E_b}$$

dimensionless factor which can be derived from simulations (~ 0.1 – few)

Hills (1983, AJ, 88, 1269)

Hardening rate:

Rate of binding energy exchange for a hard binary

$$\frac{dE_b}{dt} = \langle \Delta E_b \rangle \frac{dN}{dt} = \xi \frac{m_3}{m_1 + m_2} E_b \frac{dN}{dt}$$

Where *dN/dt* is the 3-body encounter rate (see slide 25)

$$\frac{dE_b}{dt} = \xi \frac{\langle m \rangle}{m_1 + m_2} E_b \frac{2\,\pi\,G\,(m_1 + m_2)\,n\,a}{\sigma}$$

Average star mass (because average energy exchange)

Note: **<*m*> *n* = *ρ*** (local mass density of stars)

substituting *Eb*

$$\frac{dE_b}{dt} = \pi\,\xi\,G^2\,\frac{\rho}{\sigma}\,m_1\,m_2$$

* *Depends only on cluster environment and binary mass!*
* *Constant in time, if the cluster properties do not change and if the binary members do not exchange:*

–> *'A hard binary hardens at a constant rate'* (Heggie 1975, 3-body Bible)

Hardening rate:

Expressing *a* in terms of E_b (assuming m_1 and m_2 constant, i.e. no exchange)

$$\frac{d}{dt}\left(\frac{1}{a}\right) = \frac{2}{G\, m_1\, m_2}\,\frac{dE_b}{dt} = 2\,\pi\,G\,\xi\,\frac{\rho}{\sigma}$$

Also called **HARDENING RATE**

From it we can derive the average time evolution of the semi-major axis of a hard binary:

$$\frac{da}{dt} = -2\,\pi\,G\,\xi\,\frac{\rho}{\sigma}\,a^2$$

It means that the smaller a, the more difficult is for the binary to shrink further (because cross section becomes smaller).

When binary is very hard, three body encounters are no longer efficient:
further evolution of the binary is affected by tidal forces
or by gravitational wave emission.

Hardening timescale:

Timescale for a binary to harden by 3-body encounters

$$t_h = \left|\frac{a}{\dot{a}}\right| = \frac{1}{2\,\pi\,G\,\xi}\,\frac{\sigma}{\rho}\,\frac{1}{a}$$

Depends only on

*** local velocity dispersion**
*** local mass density**

*** semi-major axis**

*** parameter for efficiency of energy exchange**

THANK YOU