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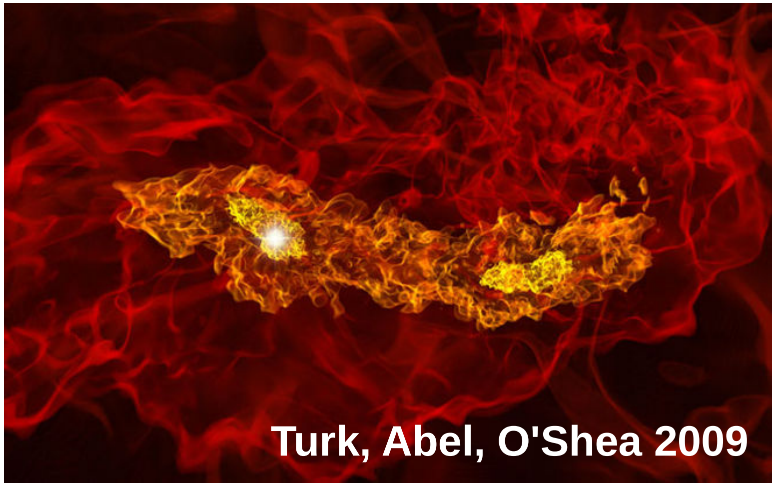
Binary evolution processes II

Innsbruck, December 12 2017

Formation of black hole and neutron star binaries



PRIMORDIAL BINARIES:

Two stars form from same cloud and evolve into two BHs gravitationally bound

NOT SO EASY:

Many evolutionary processes can affect the binary

SN kick	tidal evolution
wind mass transfer	magnetic braking
Roche lobe mass transfer	orbital evolution
common envelope	gravitational wave decay

Tidal evolution

1. What are tidal forces?

2. Stars rotate while in binaries:
   formation of a tidal bulge and lag

3. tidal equilibrium only if corotation, coplanarity and
   circularization

4. physical mechanisms driving the tides: equilibrium tides
   (turbulent viscosity) and radiative damping

Tidal evolution

1. What are tidal forces?

Tidal evolution

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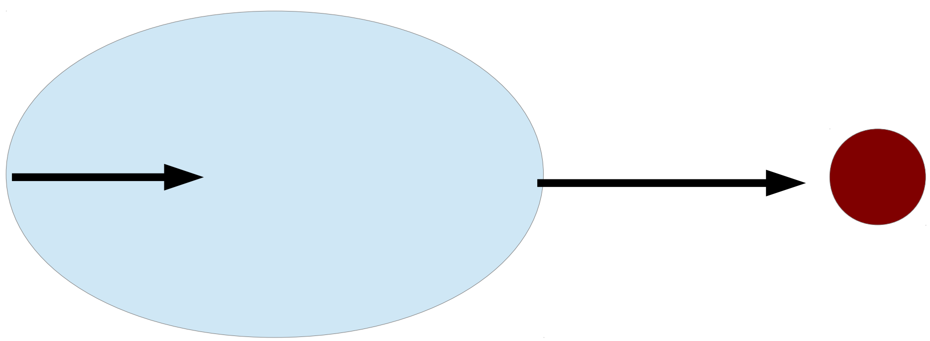
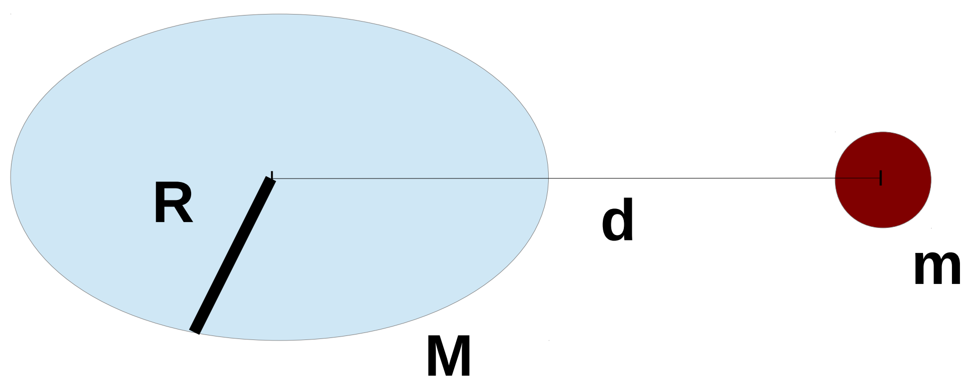


tidally perturbed body is not spherical: forms a tidal <u>bulge</u>

Tidal evolution

1. What are tidal forces?

Tidal amplitude parameter $\text{tid} = (m/M)\,(R/d)^3$



measure of the ratio, at the surface of body 1, of the tidal gravity due to body 2 to the gravity of body 1 itself

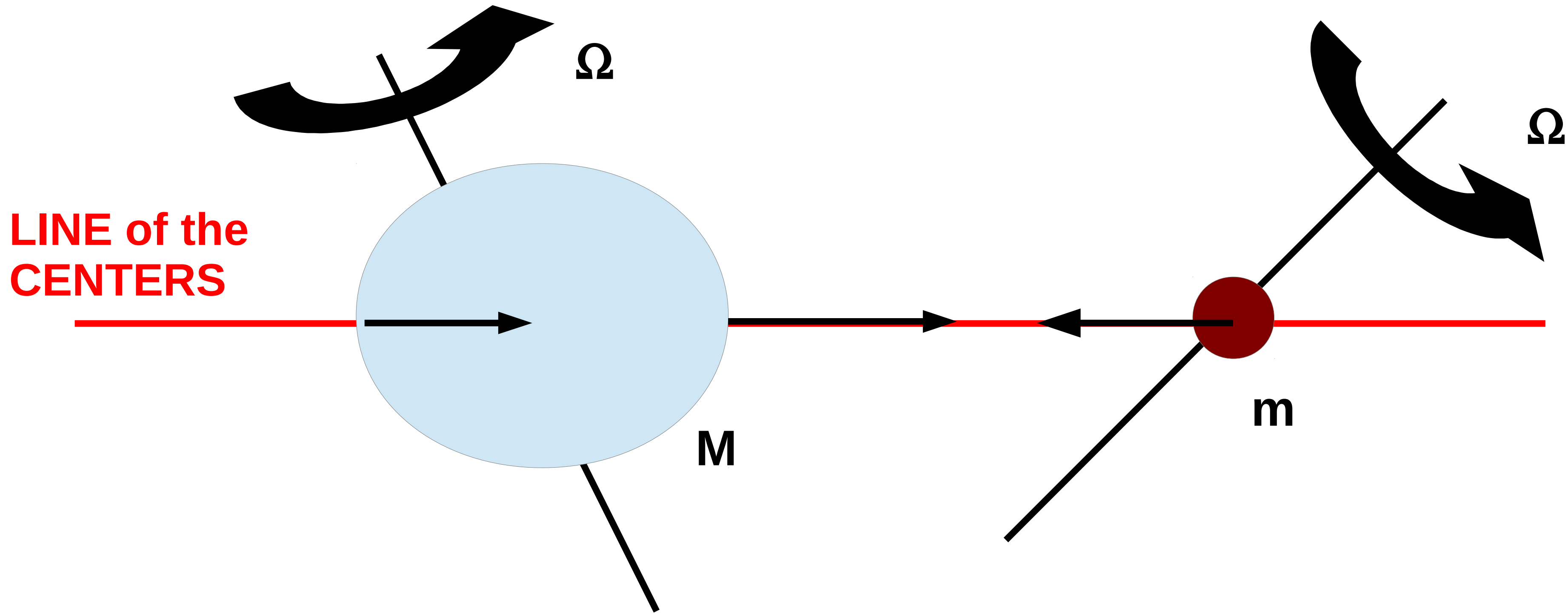
$F = G\,m\,R\,/\,d^3$

$F = G\,M\,/\,R^2$

Ogilvie 2014, https://arxiv.org/pdf/1406.2207.pdf

Tidal evolution

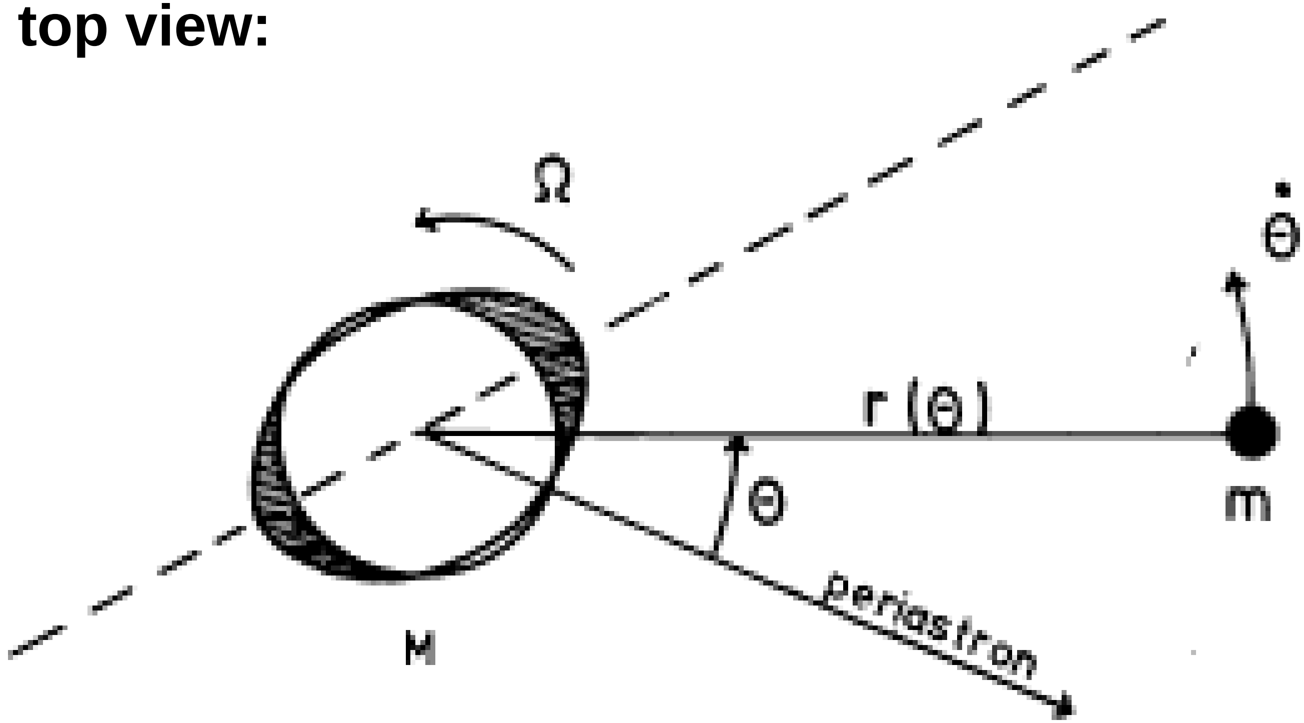
2. Stars rotate while orbiting each other



stellar rotation misplaces the tidal bulge from the line of the centers (they can lag or lead)

Tidal evolution

From top view:



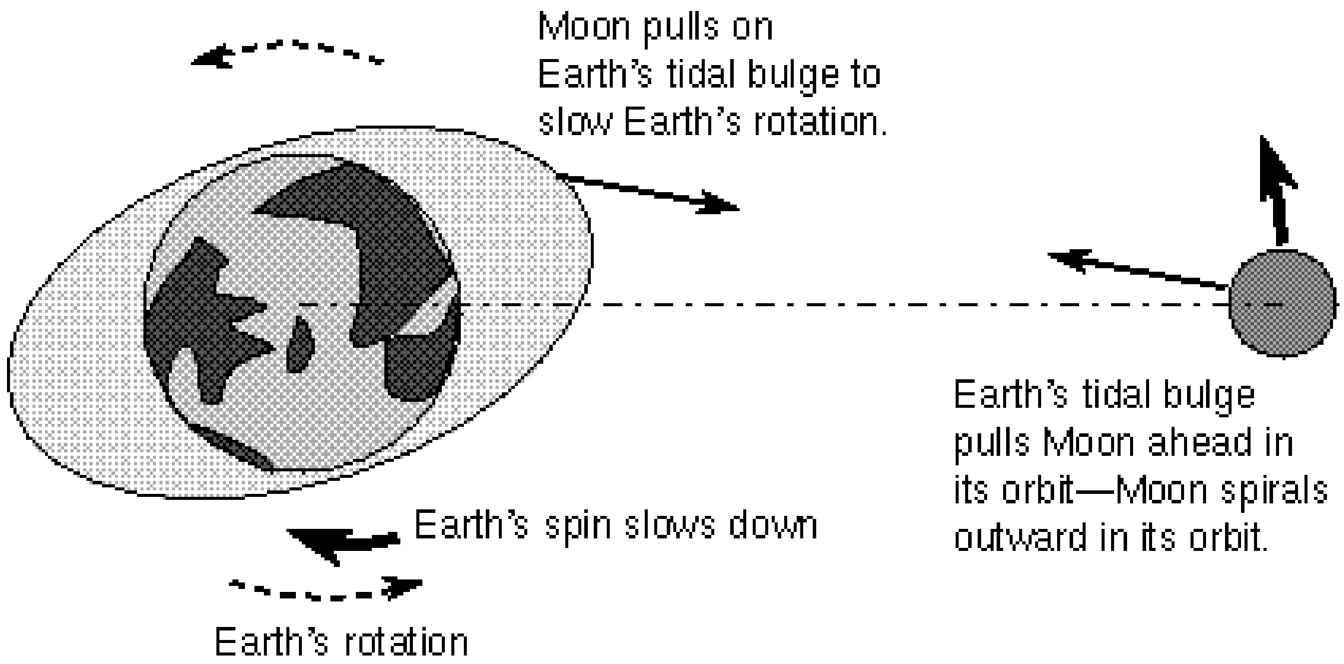
stellar rotation misplaces the tidal bulge from the line of the centers

- **bulge leads if Ω > ω (where ω is orbital angular frequency)**
- **bulge lags if Ω < ω (where ω is orbital angular frequency)**

Hut 1981

Tidal evolution

Same as Earth – Moon system:



stellar rotation misplaces the tidal bulge from the line of the centers

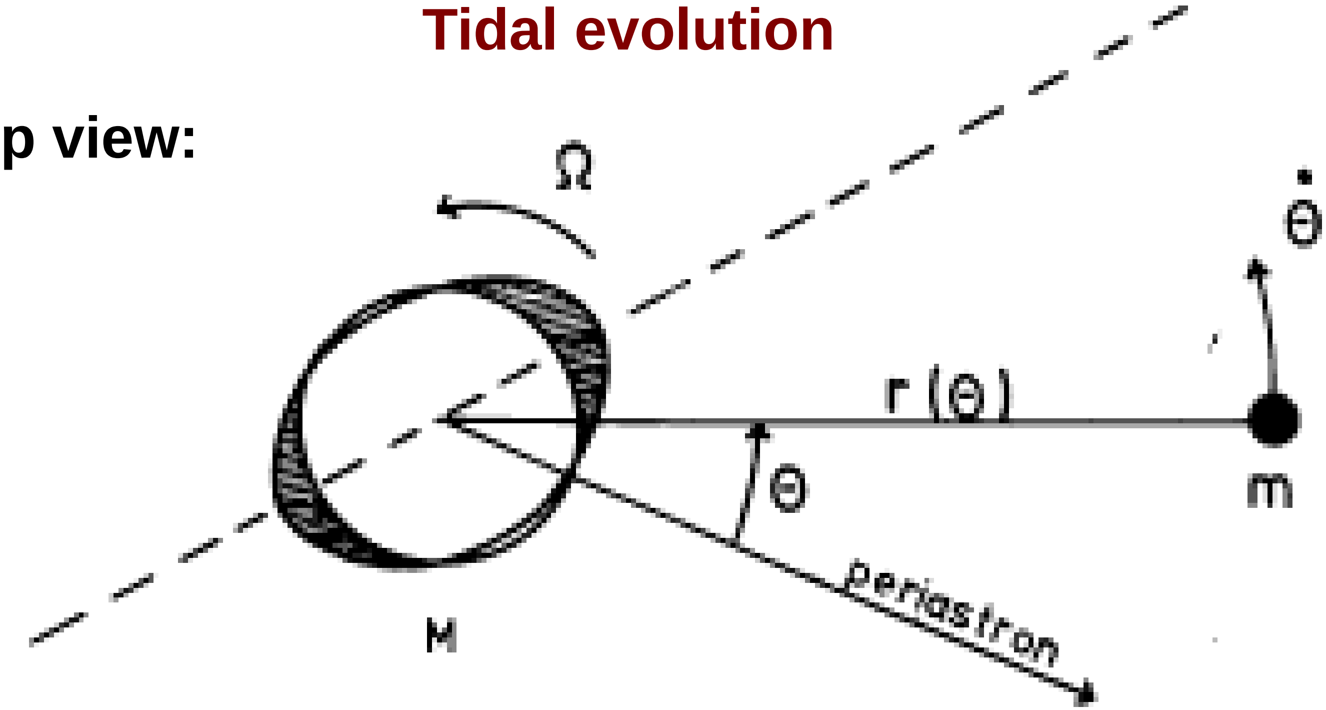
*** bulge leads if $\Omega > \omega$ (where ω is orbital angular frequency)**

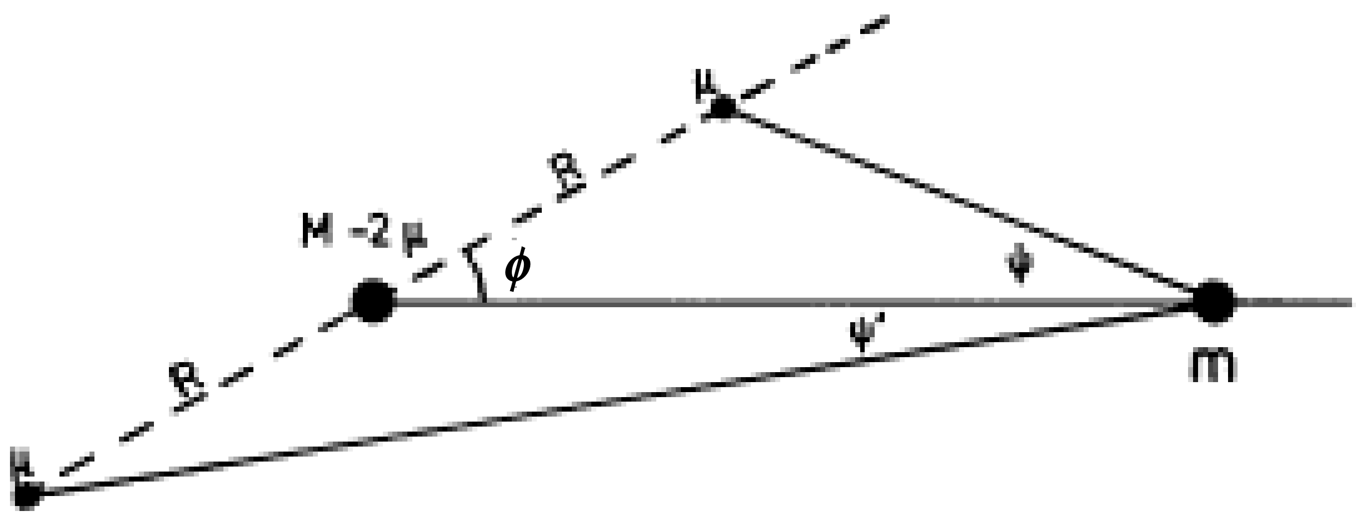
*** bulge lags if $\Omega < \omega$ (where ω is orbital angular frequency)**

Hut 1981

Tidal evolution

From top view:



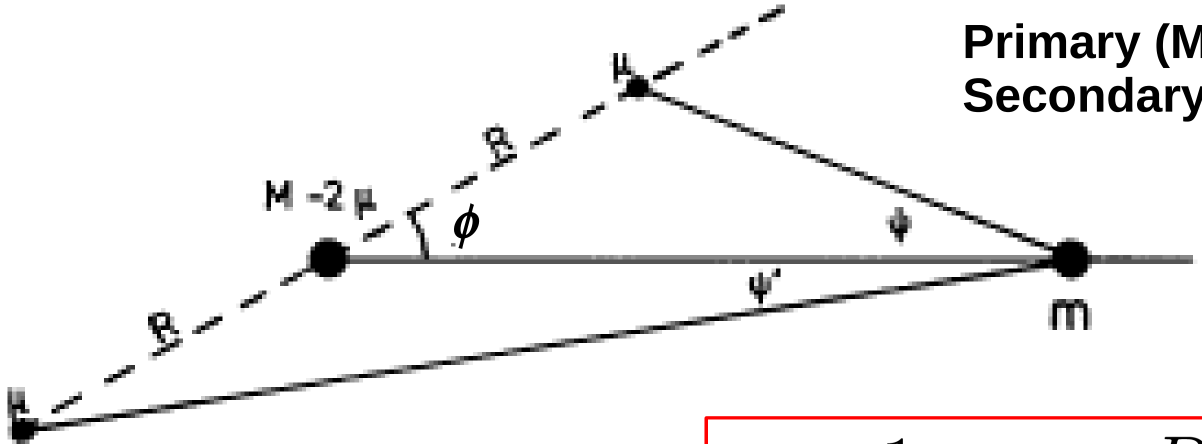


Bulge motion can be modelled as two additional masses μ

Hut 1981

Tidal evolution

Bulge motion can be modelled as two additional masses μ



Primary (M): star which is tidally deformed
Secondary (m): star which induces tide

If no dissipation ϕ = 0, $$\boxed{\mu = \frac{1}{2} k\, m \left(\frac{R}{r}\right)^3}$$

radius of the primary

distance between primary and secondary

apsidal motion constant of the primary (how concentrated is the primary ~0.01 – 0.2)

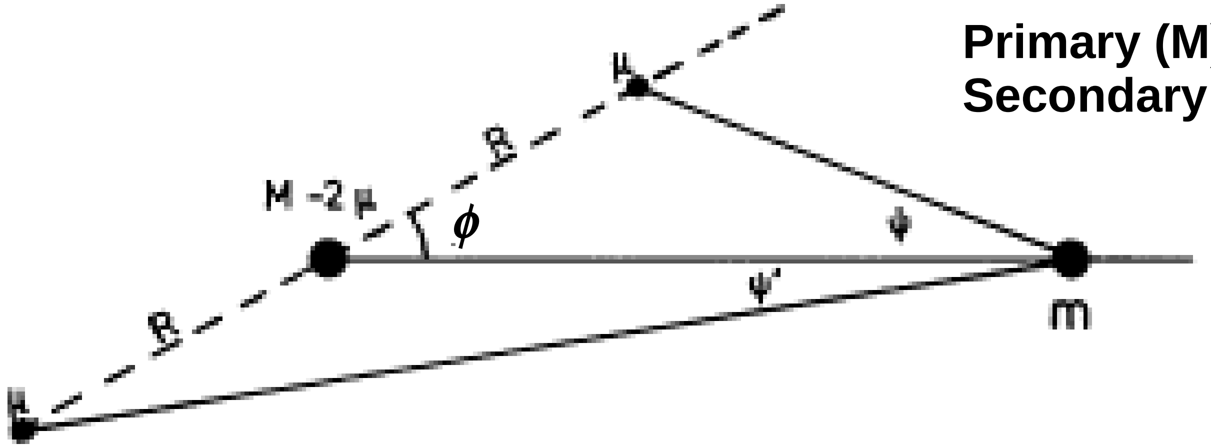
mass of the secondary

$$\mu \propto \frac{m}{r^3}$$ **because of nature of tidal forces**

Hut 1981

Tidal evolution

Bulge motion can be modelled as two additional masses μ



Primary (M): star which is tidally deformed
Secondary (m): star which induces tide

If dissipation ϕ != 0,

$$\mu(t) = \frac{1}{2}\, k\, m \left[\frac{R}{r(t-\tau)}\right]^3$$

$$\longrightarrow \mu(t) \sim \frac{1}{2}\, k\, m \left(\frac{R}{r}\right)^3 \left(1 + 3\,\frac{\dot{r}}{r}\tau\right)$$

$$\phi = (\Omega - \dot{\theta})\,\tau$$

where τ is a small constant time lag

Hut 1981

Tidal evolution

From μ we derive force due to tides (assuming orbital plane and equatorial plane of the primary are the same):

$$\vec{F} = \frac{M\,m}{M+m}\,\ddot{r} = -G\frac{(M-2\,\mu)\,m}{r^2}\hat{r} - G\frac{\mu\,m}{r^2+R^2-2r\,R\,\cos\phi}\left[\cos\psi\hat{r} - \sin\psi\hat{\theta}\right]$$
$$+G\frac{\mu\,m}{r^2+R^2+2r\,R\,\cos\phi}\left[\cos\psi'\hat{r} - \sin\psi'\hat{\theta}\right]$$

where

$$\sin\psi = \frac{R\,\sin\phi}{(r^2+R^2-2\,r\,R\,\cos\phi)^{1/2}}$$
$$\sin\psi' = \frac{R\,\sin\phi}{(r^2+R^2+2\,r\,R\,\cos\phi)^{1/2}}$$

developing to 5th order in *R/r*

$$\boxed{\underbrace{\vec{F} = -G\frac{M\,m}{r^2}}_{\text{Newton}}\left\{\hat{r} + 3\,\frac{m}{M}\,\left(\frac{R}{r}\right)^5\,\underbrace{k\,\left[\left(1+3\,\frac{\dot{r}}{r}\,\tau\right)\hat{r} - (\Omega-\dot{\theta})\,\tau\,\hat{\theta}\right]}_{\text{Tidal perturbation}}\right\}}$$

Newton **Tidal perturbation**

Hut 1981

Tidal evolution

From expression of tidal force + angular mom. conservation + energy dissipation, we derive expression for tidal change of a, e, Ω

$$\frac{da}{dt} = -6\,\frac{k}{T}\,q\,(1+q)\,\left(\frac{R}{a}\right)^8\,\frac{a}{(1-e^2)^{15/2}}\,\left[f_1-(1-e^2)^{3/2}\,f_2\,\frac{\Omega}{\omega}\right]$$

$$\frac{de}{dt} = -27\,\frac{k}{T}\,q\,(1+q)\,\left(\frac{R}{a}\right)^8\,\frac{e}{(1-e^2)^{13/2}}\,\left[f_3-\frac{11}{18}\,(1-e^2)^{3/2}\,f_4\,\frac{\Omega}{\omega}\right]$$

$$\frac{d\Omega}{dt} = 3\,\frac{k}{T}\,\frac{q^2}{r_g^2}\,\left(\frac{R}{a}\right)^6\,\frac{\omega}{(1-e^2)^6}\,\left[f_2-(1-e^2)^{3/2}\,f_5\,\frac{\Omega}{\omega}\right]$$

$$T=\frac{R^3}{G\,M\,\tau}$$

timescale for tidal dissipation

$$I = M\,(r_g\,R)^2$$

gyration radius

$$q=\frac{m}{M}$$

$$f_1 = 1+\frac{31}{2}e^2+\frac{255}{8}e^4+\frac{185}{16}e^6+\frac{25}{64}e^8$$

$$f_2 = 1+\frac{15}{2}e^2+\frac{45}{8}e^4+\frac{5}{16}e^6$$

$$f_3 = 1+\frac{15}{4}e^2+\frac{15}{8}e^4+\frac{5}{64}e^6$$

$$f_4 = 1+\frac{3}{2}e^2+\frac{1}{8}e^4$$

$$f_5 = 1+3\,e^2+\frac{3}{8}e^4$$

Hut 1981

Tidal evolution

Tidal evolution is

* orbital energy and rotational energy dissipation
  → two stars (even if do not fill Roche lobes) can spiral in by tidal evolution

* angular momentum exchange between stellar rotation and orbit
  → a star transfers rotational ang. mom. to orbital ang. mom. and slows down
  → stars in binaries generally spin lower

Hut 1981

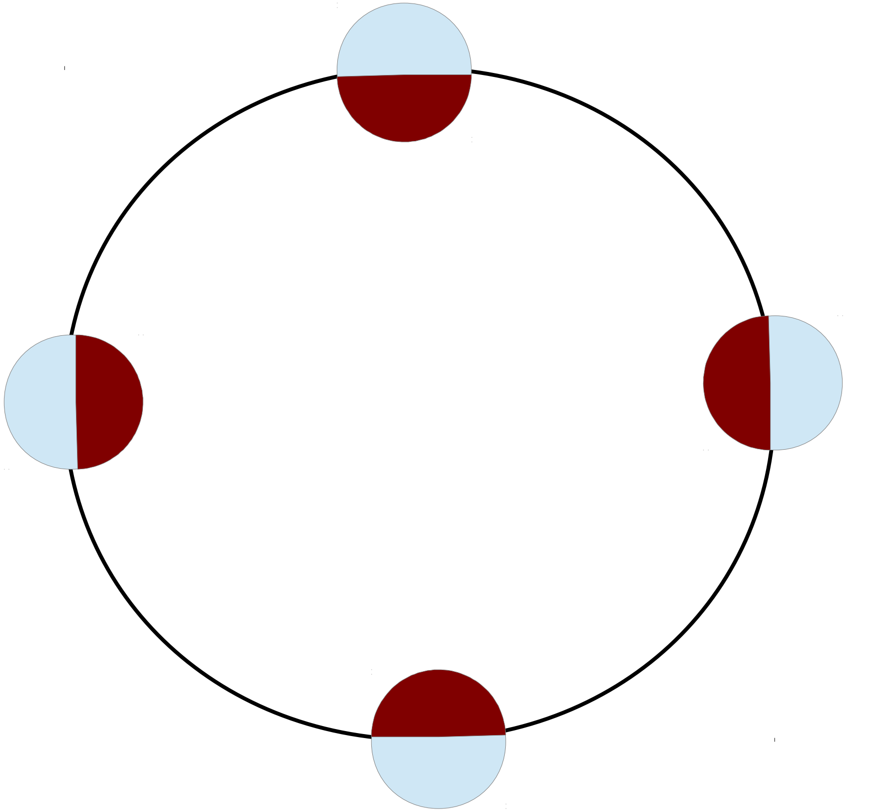
Tidal evolution

3. It has been shown (Hut 1980) that tides tend to lead to

Synchronization (Corotation)

Coplanarity

Circularization



if these are reached then
tides reach equilibrium
if not two stars spiral in

Tidal evolution

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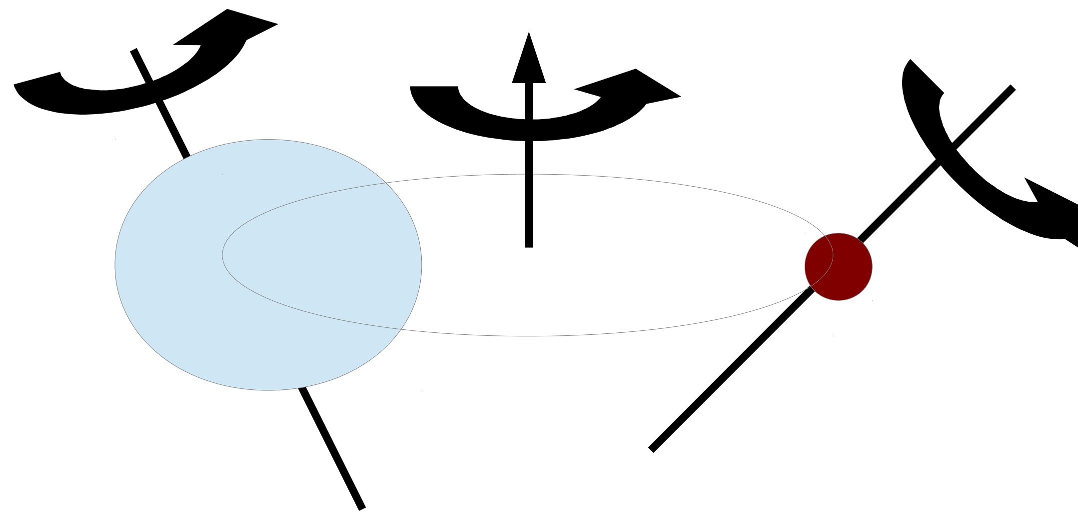
Synchronization

Coplanarity

Circularization



if these are reached then tides reach equilibrium
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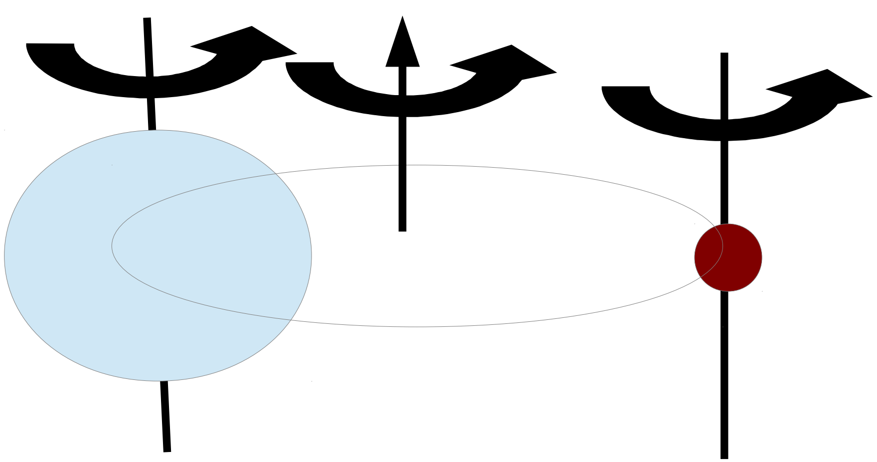
alignment of stellar spins with orbital angular momentum



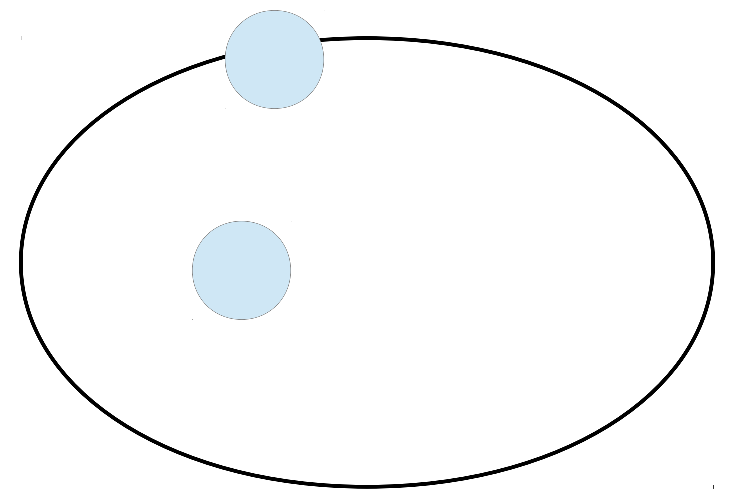
Tidal evolution

3. It has been shown (Hut 1980) that tides tend to lead to

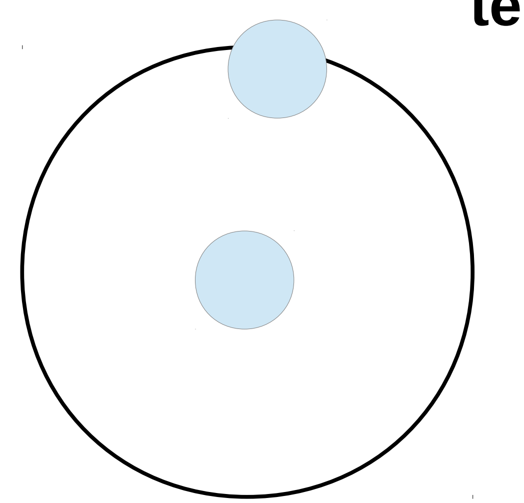
Synchronization

Coplanarity

Circularization



eccentricity tends to zero



if these are reached then tides reach equilibrium if not two stars spiral in

Tidal evolution

Corotation

EQUILIBRIUM SPIN:
spin of synchronisation between star rotation and orbit

NO further angular momentum exchange is possible

$$\Omega_{\rm eq} = f_2\,\omega\,\left[\frac{1}{f_5\,(1-e^2)^{3/2}}\right]$$

SYNCHRONIZATION TIMESCALE:
time for reaching corotation

$$\frac{1}{\tau_{\rm sync}} = \frac{\dot{\Omega}}{\Omega-\omega} \sim 3\,\left(\frac{k}{T}\right)\,\frac{q^2}{r_g^2}\,\left(\frac{R}{a}\right)^6$$

Circularization

CIRCULARIZATION TIMESCALE:
time for reaching eccentricity = 0

$$\frac{1}{\tau_{\rm circ}} = \frac{21}{2}\,\left(\frac{k}{T}\right)\,q\,(1+q)\,\left(\frac{R}{a}\right)^8$$

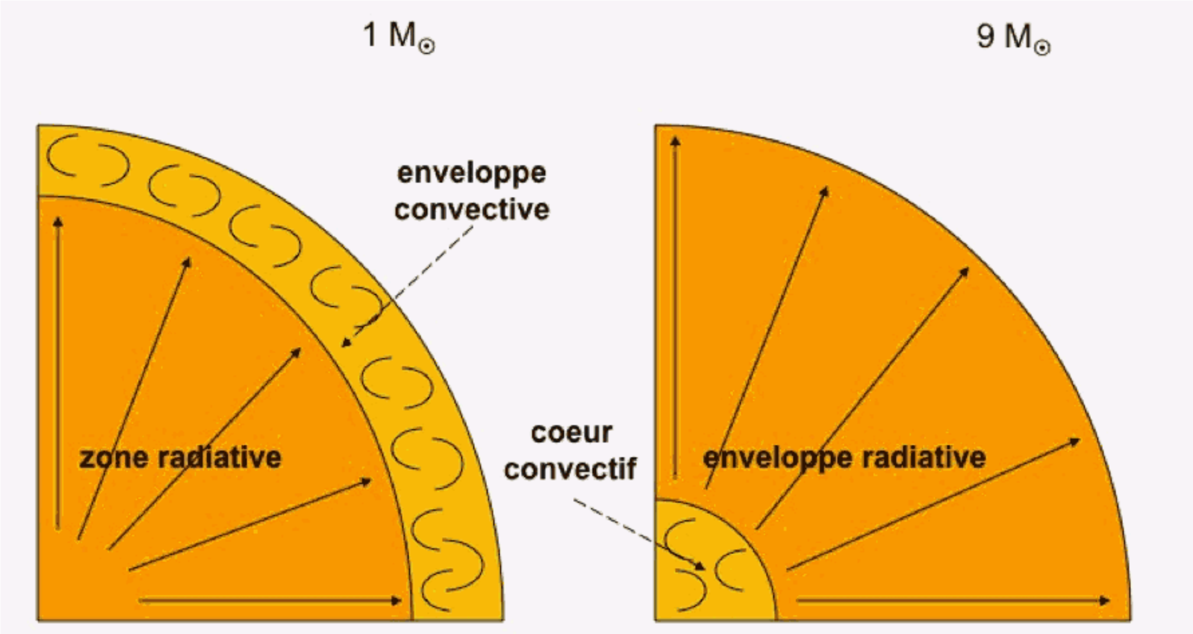
Tidal evolution

4. physical mechanisms driving the tides

* so far we made no assumption on what drives the tide
* at least two possible mechanisms:

turbulent viscosity in a convective envelope

radiative damping in a radiative envelope



Tidal evolution

4.1 Equilibrium tides with convective damping

Turbulent viscosity in a convective envelope excites tides which are close to equilibrium

→ assumption of star in hydrostatic equilibrium

same formulas as discussed before

4.2 Dynamical tides with radiative damping

If radiative envelope turbulence is not efficient

Stellar oscillations driven by tidal forces: DYNAMICAL TIDE

(excited through GRAVITY WAVES – note: not gravitational waves)

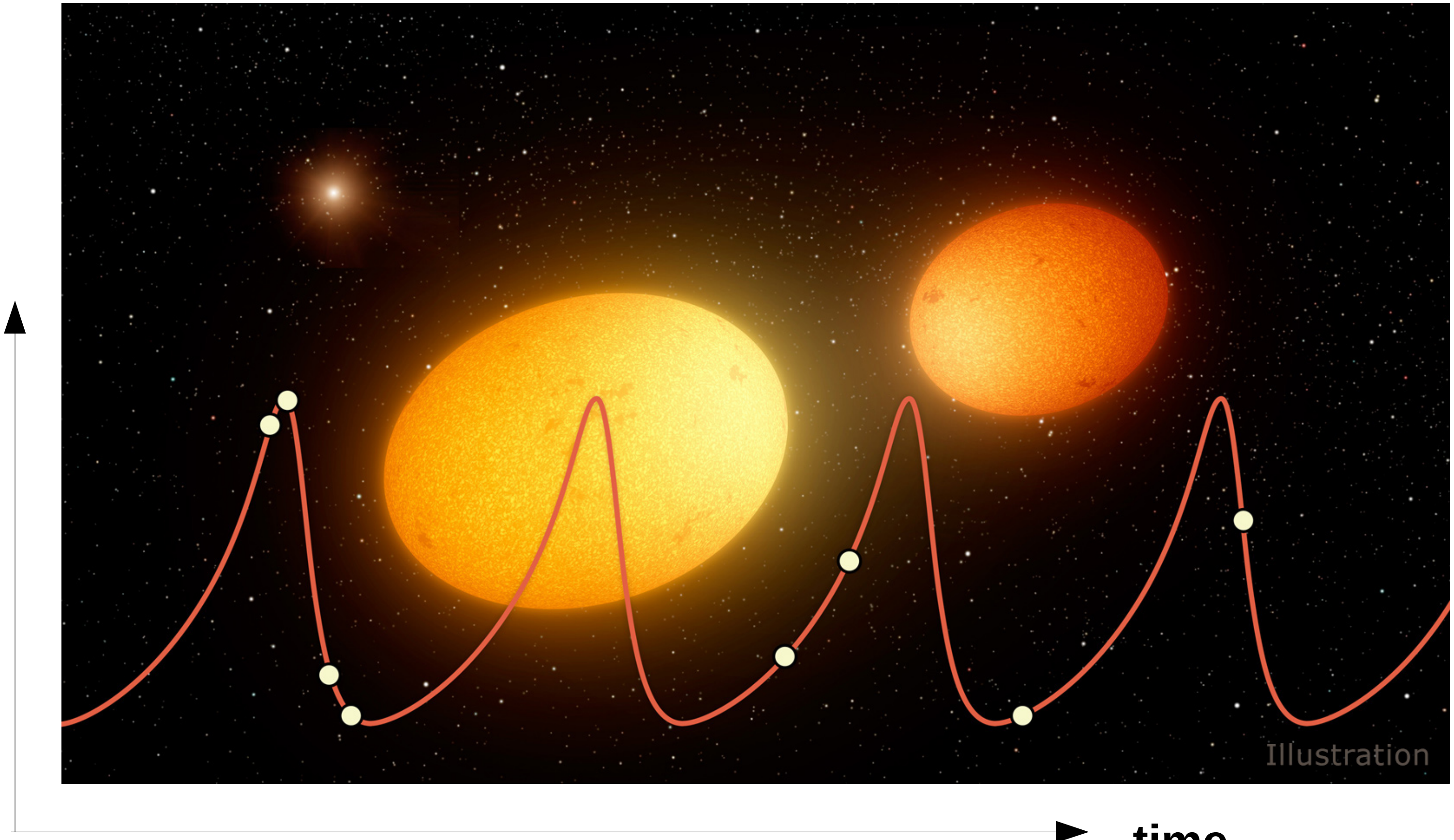
Different equations (Zahn 1975, 1977, Ogilvie 2014)

Tidal evolution

4.2 Dynamical tides with radiative damping

Example: heartbeat stars

Infrared luminosity



time

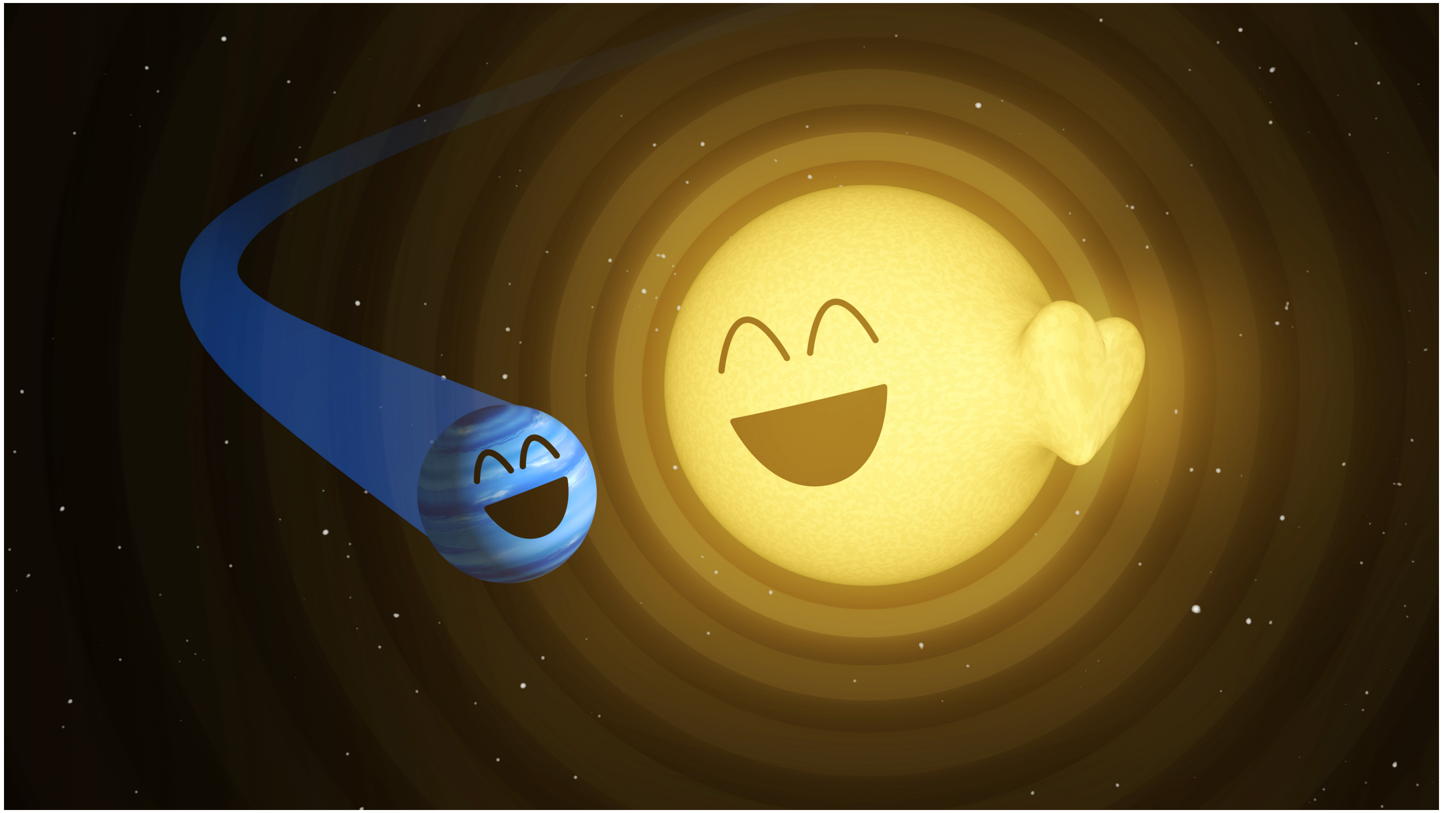
NASA

"You can think about the stars as bells, and once every orbital revolution, when the stars reach their closest approach, it's as if they hit each other with a hammer" Avi Shporer

Tidal evolution

4.2 Dynamical tides with radiative damping

Example: heartbeat stars



NASA

"You can think about the stars as bells, and once every orbital revolution, when the stars reach their closest approach, it's as if they hit each other with a hammer" Avi Shporer

Tidal evolution

4.2 Dynamical tides with radiative damping (Zahn 1975)

SYNCHRONIZATION TIMESCALE:
time for reaching corotation

$$\frac{1}{\tau_{\rm synch}} \propto \left(\frac{G\,M}{R^3}\right)^{1/2} q^2\,(1+q)^{5/6}\left(\frac{R}{a}\right)^{17/2}$$

CIRCULARIZATION TIMESCALE (Zahn 1977):
time for reaching eccentricity = 0

$$\frac{1}{\tau_{\rm circ}} \propto \left(\frac{G\,M}{R^3}\right)^{1/2} q\,(q)^{11/6}\left(\frac{R}{a}\right)^{21/2}$$

Gravitational wave decay

Implemented in the simplest possible way with Peters 1964 formalism (see lecture 1)

produces both circularization and orbital decay

$$\frac{da}{dt} = -\frac{64}{5}\,\frac{G^3\,M\,m\,(M+m)}{c^5\,a^3\,(1-e^2)^{7/2}}\,\left(1+\frac{73}{24}\,e^2+\frac{37}{96}\,e^4\right)$$

$$\frac{de}{dt} = -\frac{304}{15}\,e\,\frac{G^3\,M\,m\,(M+m)}{c^5\,a^4\,(1-e^2)^{5/2}}\,\left(1+\frac{121}{304}\,e^2\right)$$

THANK YOU