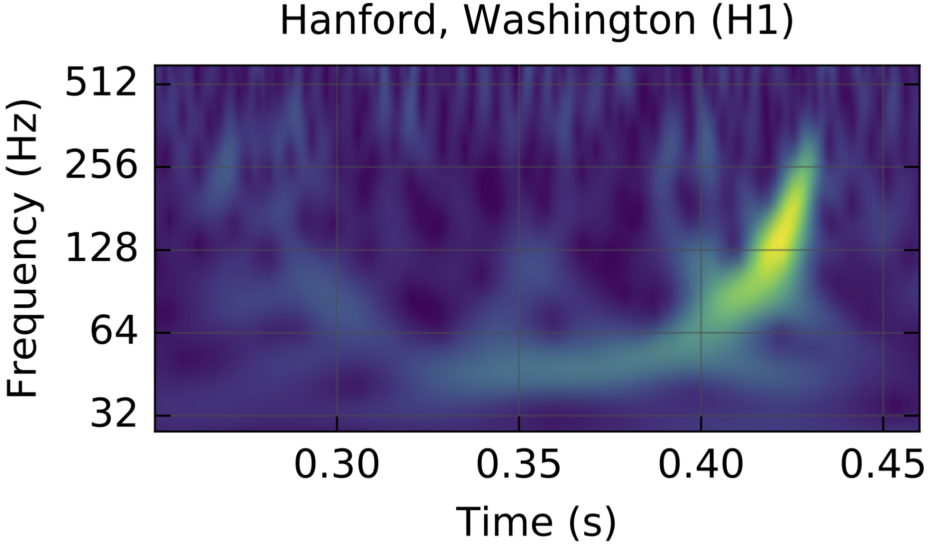
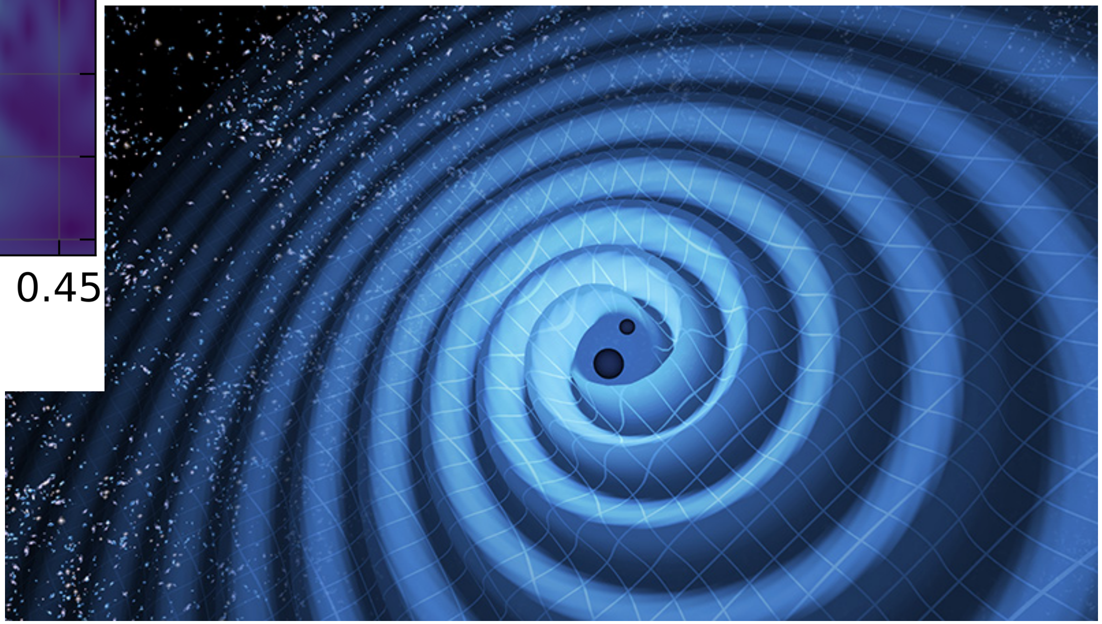
Michela Mapelli

Lecture 1.
Gravitational waves for dummies
& Observational facts





Innsbruck, October 17 2017

* What do you know about observations of gravitational waves (GWs)?

* What do you know about GWs?

* What do you know about black holes (BHs) and neutron stars (NSs)?

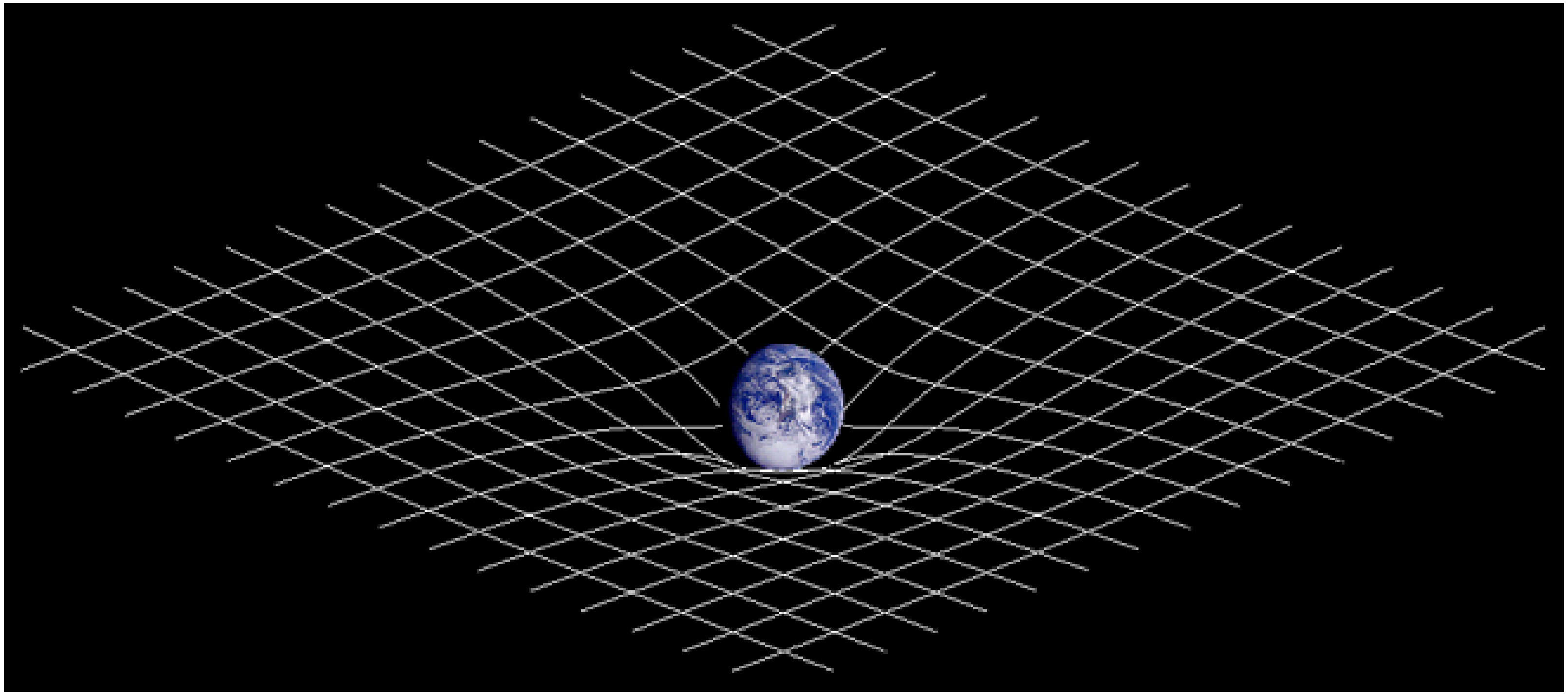
* What do you know about massive star evolution?

* What do you know about stellar binaries?

Gravitational waves for dummies

Gravitational waves for dummies

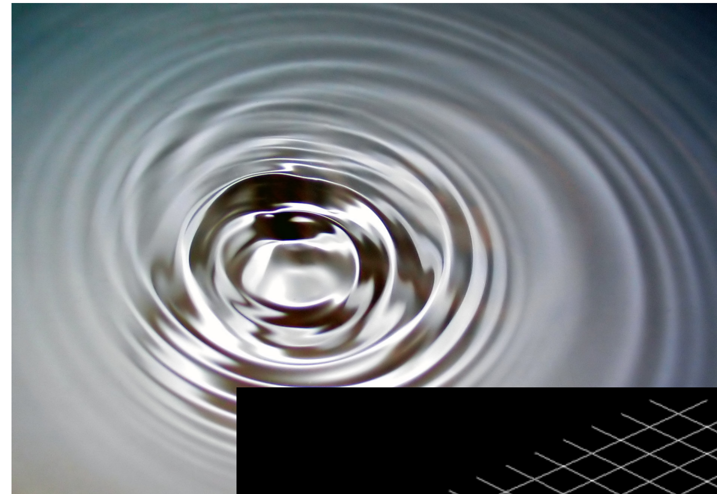
Browsing wikipedia `gravitational waves are ripples in the curvature of spacetime which propagate as waves, travelling outward from the source'

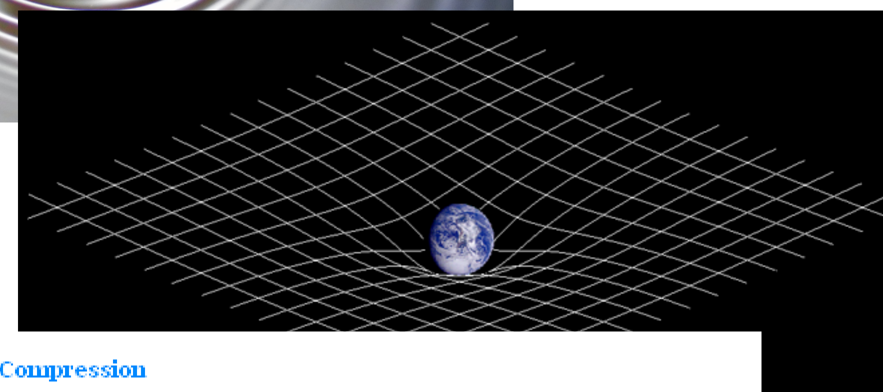


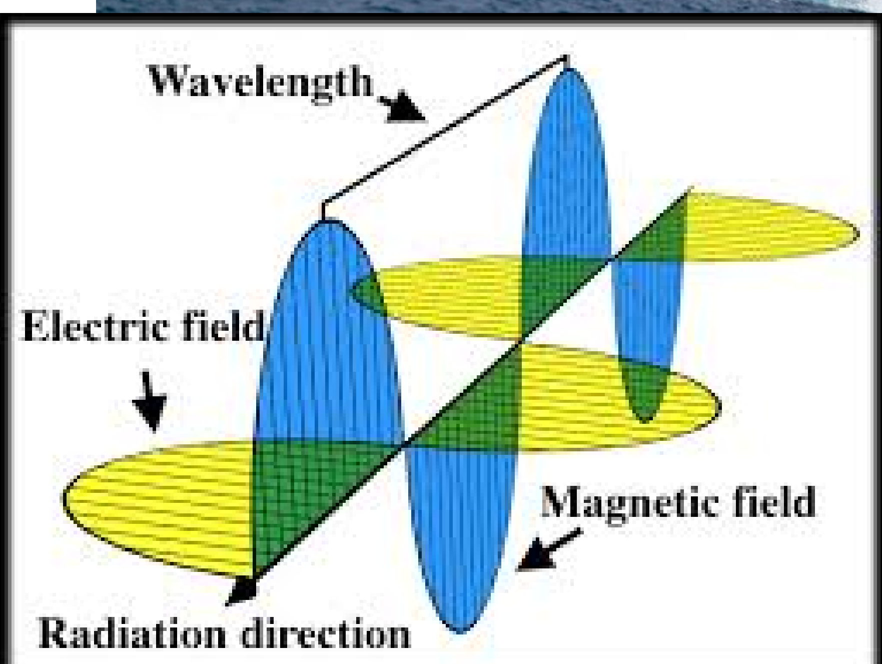
A mass deforms space time

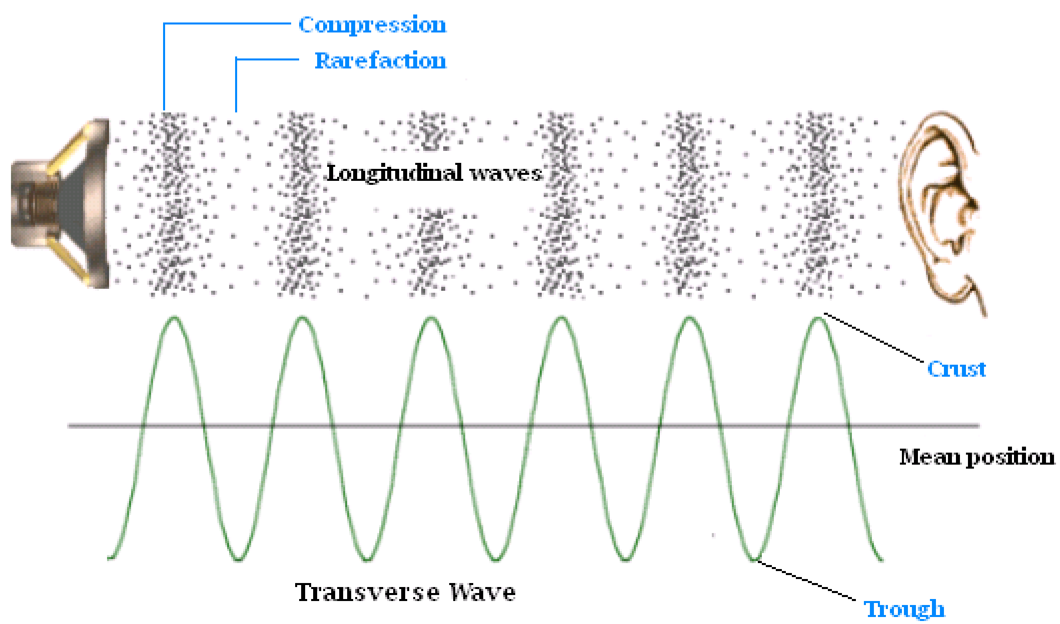
If mass moves/oscillates it produces "waves" in the space time

Nature is full of waves...









Analogy with electromagnetic field

- an accelerating charge produces a perturbation in electromagnetic field that propagates as wave

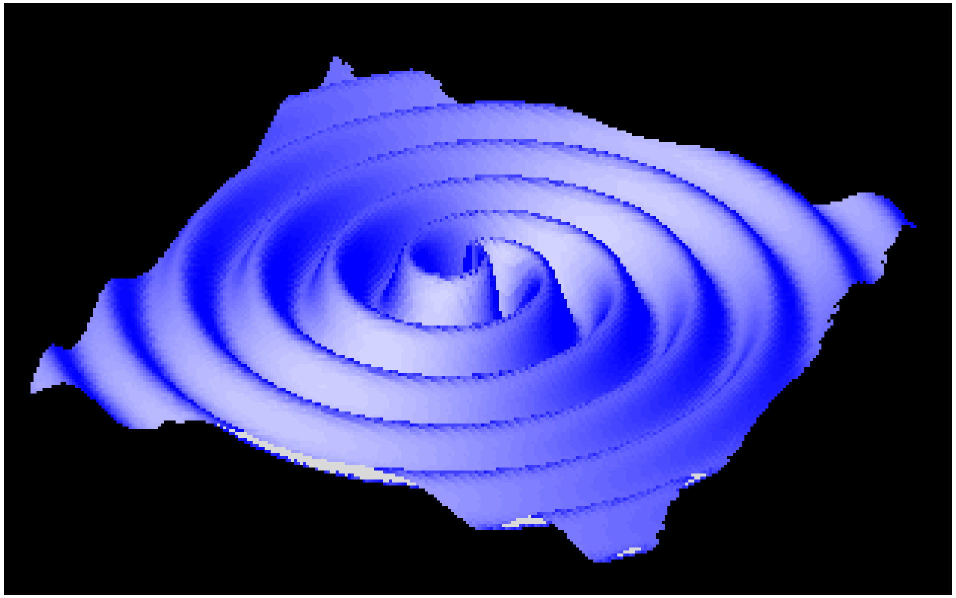
Analogy with electromagnetic field

a mass is source of gravitational field as a charge is source of electromagnetic field

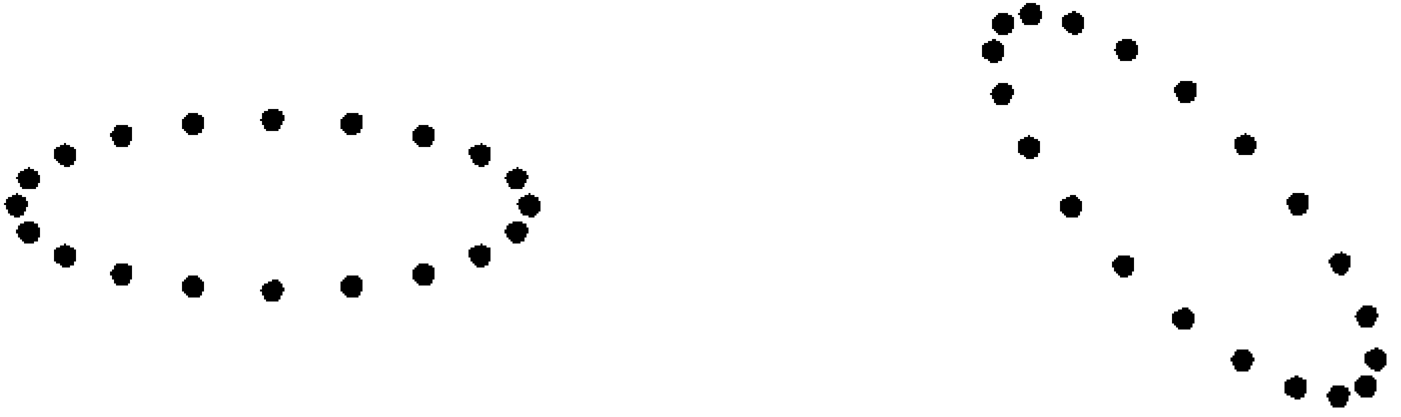
→ an accelerating mass should produce perturbations in gravitational field,

i.e. intrinsic perturbations of space-time that propagate as waves:

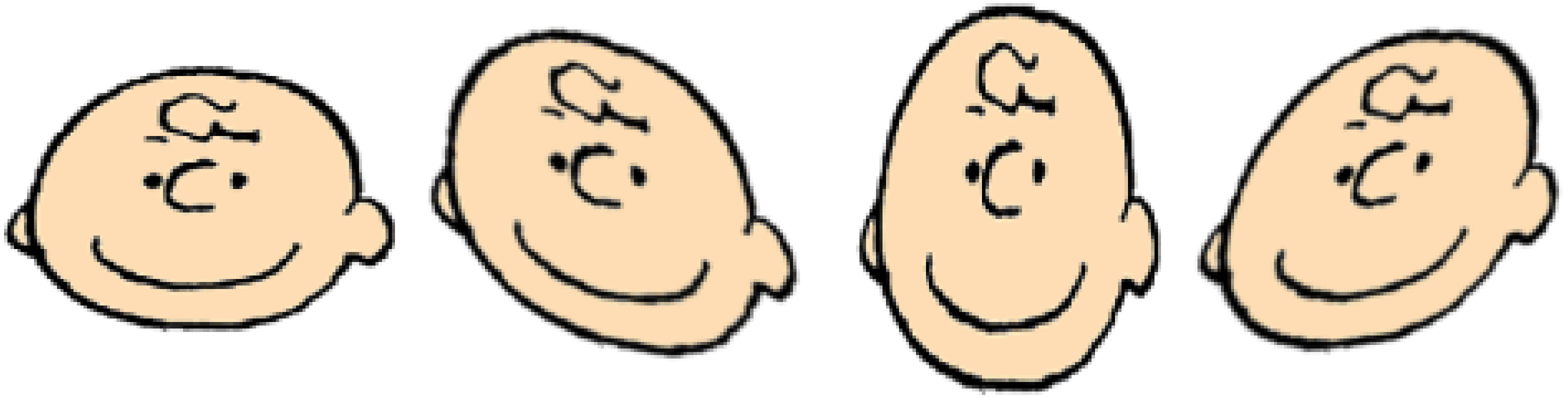
do not move in space-time but MOVE SPACE-TIME at speed of light (i.e. lead deformation in space time – squeeze stretch)



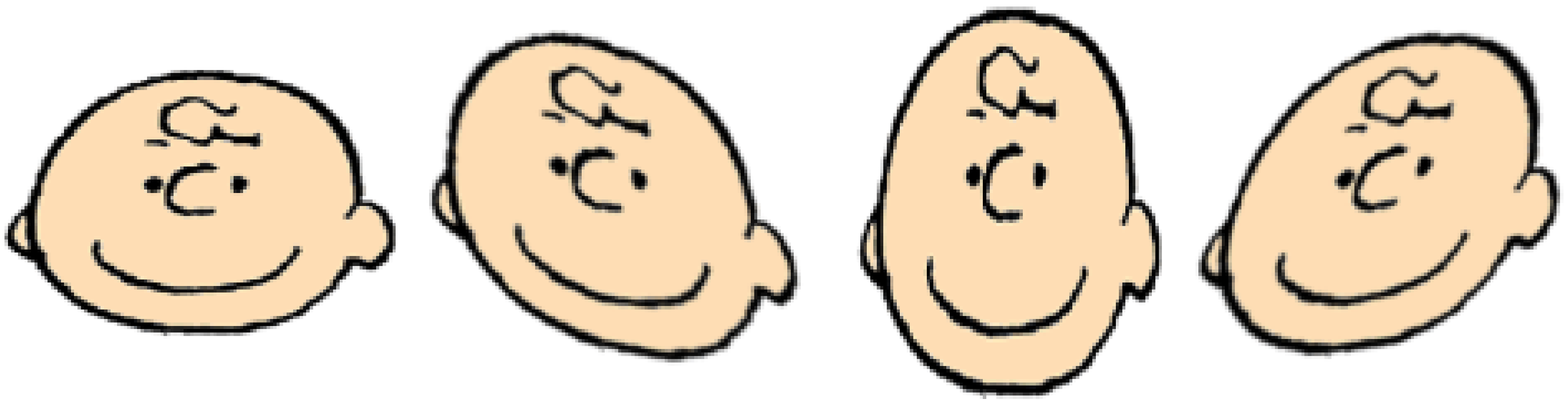
When GW passes through space deforms it



When GW passes through space deforms it



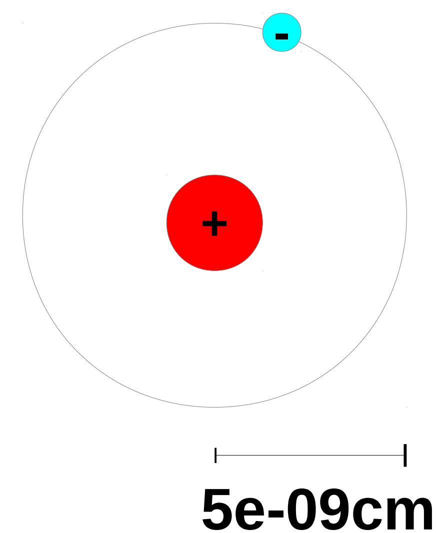
When GW passes through space deforms it



But deformations are very small:
strain=relative deformation

$$h = DL/L \sim 1e\text{-}21$$

For $L_{\text{Sun-Earth}} \sim 1.5e13$ cm
$h\ L_{Sun\text{-}Earth} \sim$ 1e-21 x 1.5e13 ~ 1.5e-08cm
size of H atom at distance Sun-Earth



Some math:

Consider Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Consider a small perturbation of the flat Cartesian metric
Weak field (far from source)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad with\,|h_{\mu\nu}| \ll 1$$

Using gauge invariance and assuming vacuum (*T*=0 no mass no energy)

Equation of WAVES!!

$$\Box h = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$$

If you want to know more about GR formalism:
https://arxiv.org/pdf/1607.04202.pdf

By integrating equation $$\Box h = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

$$h^{ij}(t, \vec{x}) \sim \frac{2\,G}{r\,c^4}\frac{d^2}{dt^2} I^{ij}(t - r/c)$$

Distance source-observer

Moment of inertia, or second mass moment, or quadrupole moment of mass $I^{ij} = \int dx^3 \, \rho(t, \vec{x}) \, x^i \, x^j$

Retarded time

→ **not all accelerating masses do this job but only those with** **QUADRUPOLE**

If you do calculation, monopole and dipole disappear

→ **for a gravitational wave to form, there must be an** **ASYMMETRY IN MASS DISTRIBUTION**

Mass monopole:

$$\int \rho(\vec{x}) d^3\vec{x}$$

Mass energy - conserved

Mass dipole:

$$\int \rho(\vec{x})\, \vec{x}\, d^3\vec{x}$$

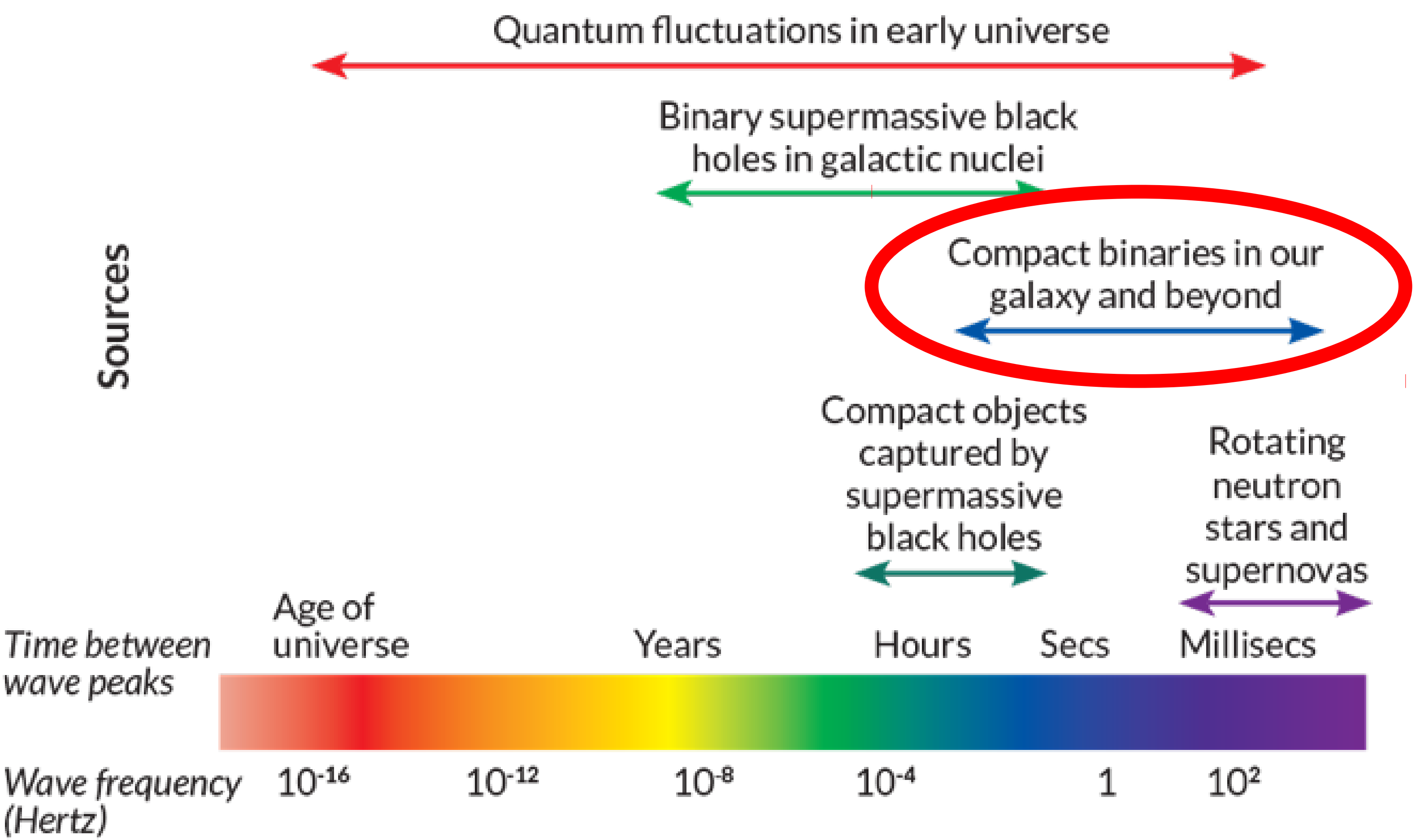
centre of mass energy - conserved

Mass quadrupole:

$$I^{ij} = \int \rho(\vec{x})\, \vec{x^i}\, \vec{x^j}\, d^3\vec{x}$$

Moment of inertia – not conserved

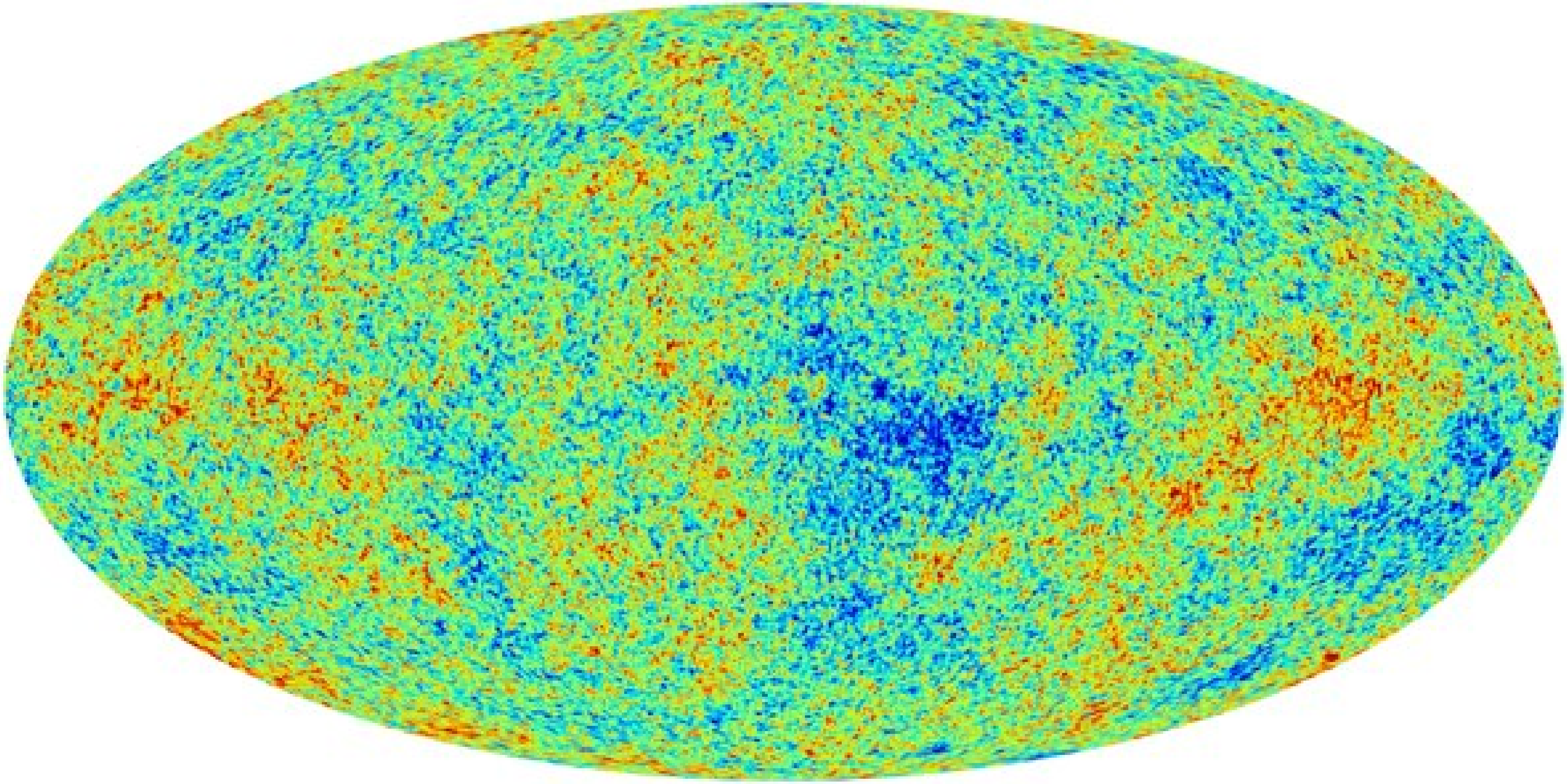
What are the astrophysical objects with non-zero quadrupole?



Compact binaries will be the focus of this course!

Primordial gravitational waves / Quantum fluctuations:

<< 1 sec from Big Bang, due to INFLATION of the Universe
Freq. ~ 10^-16 – 10^2 Hz
extremely “faint” (small amplitude)



Mergers of super-massive black holes (SMBHs, >10^5 Msun):

Black holes at centre of galaxies might form Keplerian binaries and might merge

Freq. ~ 10^-10 – 0.1 Hz

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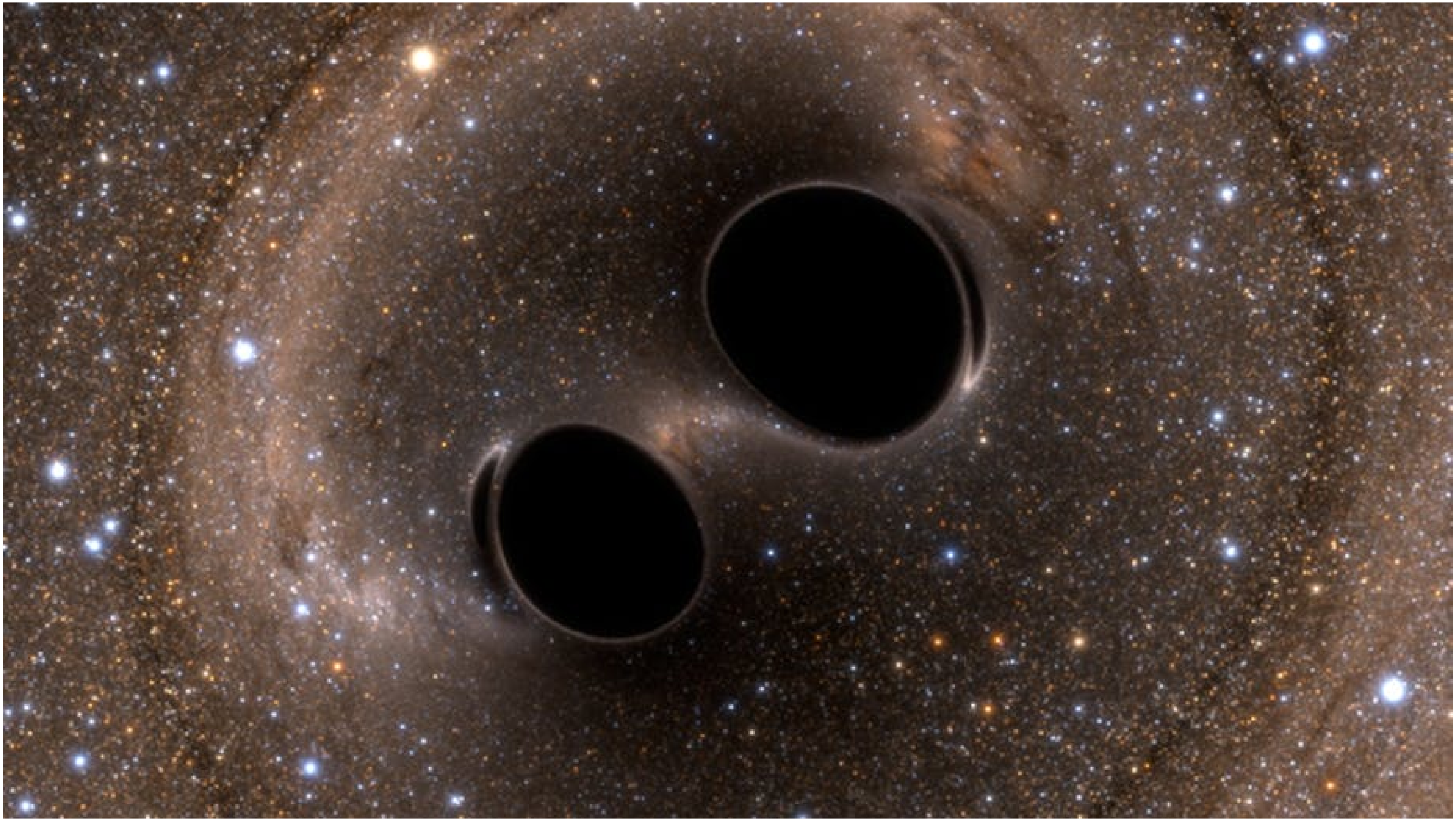
Black holes at centre of galaxies might form Keplerian binaries and might merge
Freq. ~ 10^-10 – 0.1 Hz

Mergers of compact object binaries (black holes <10^5 Msun, neutron stars):

Black holes (BHs) and neutron stars (NSs) born from stars might merge
Freq. ~ 10^-4 – 10^3 Hz



Mergers of SMBHs and stellar-mass BHs:

Small BHs might orbit SMBHs and be captured by them

Freq. ~ 10^-4 – 0.1 Hz

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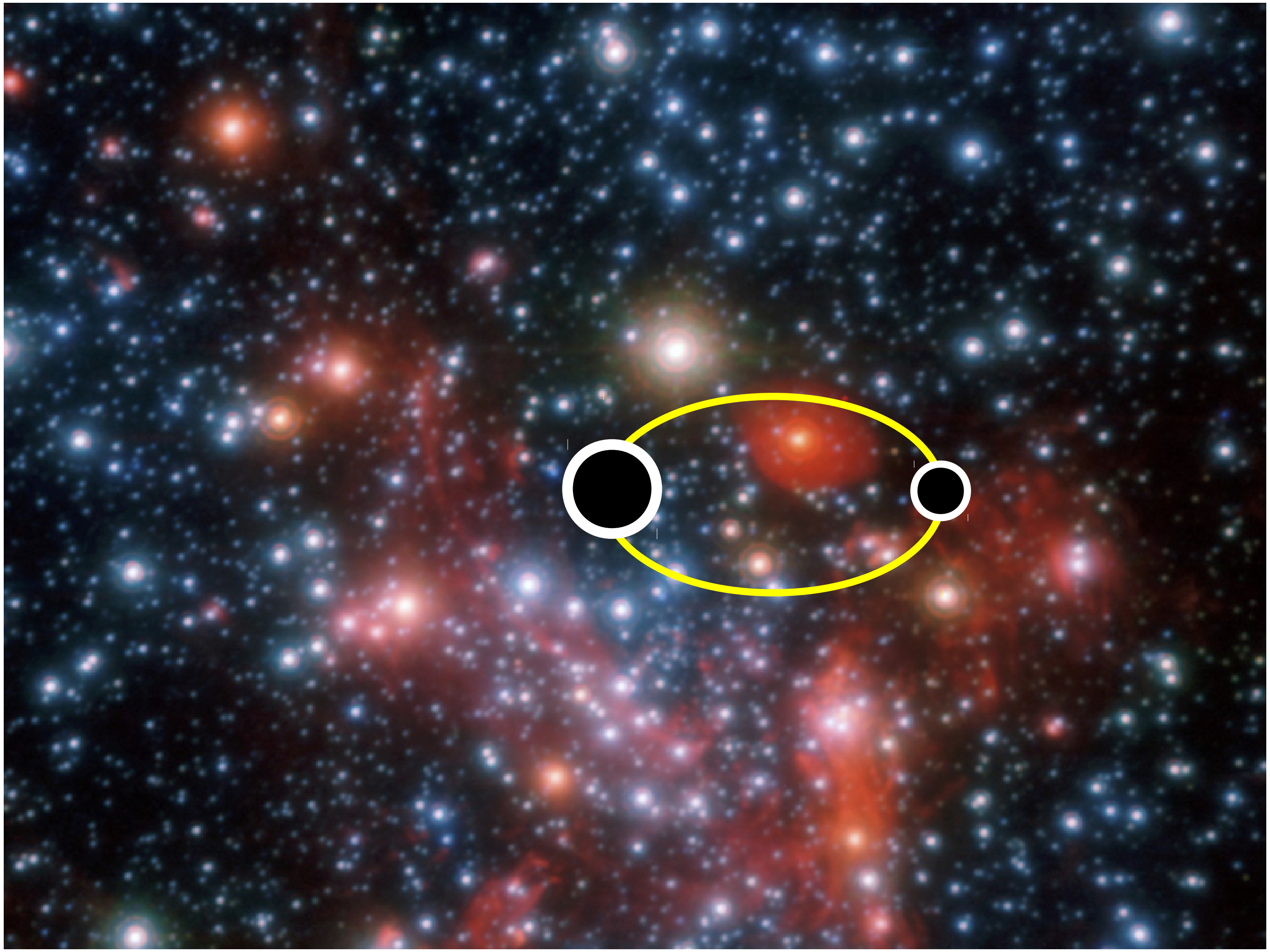
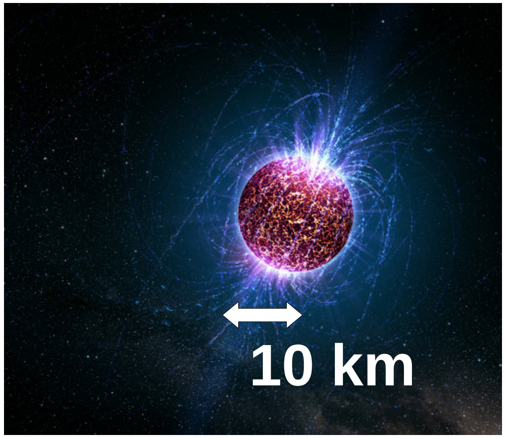
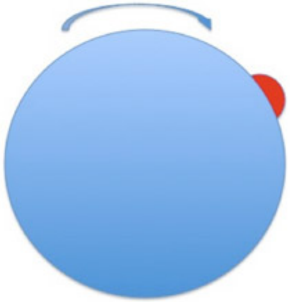
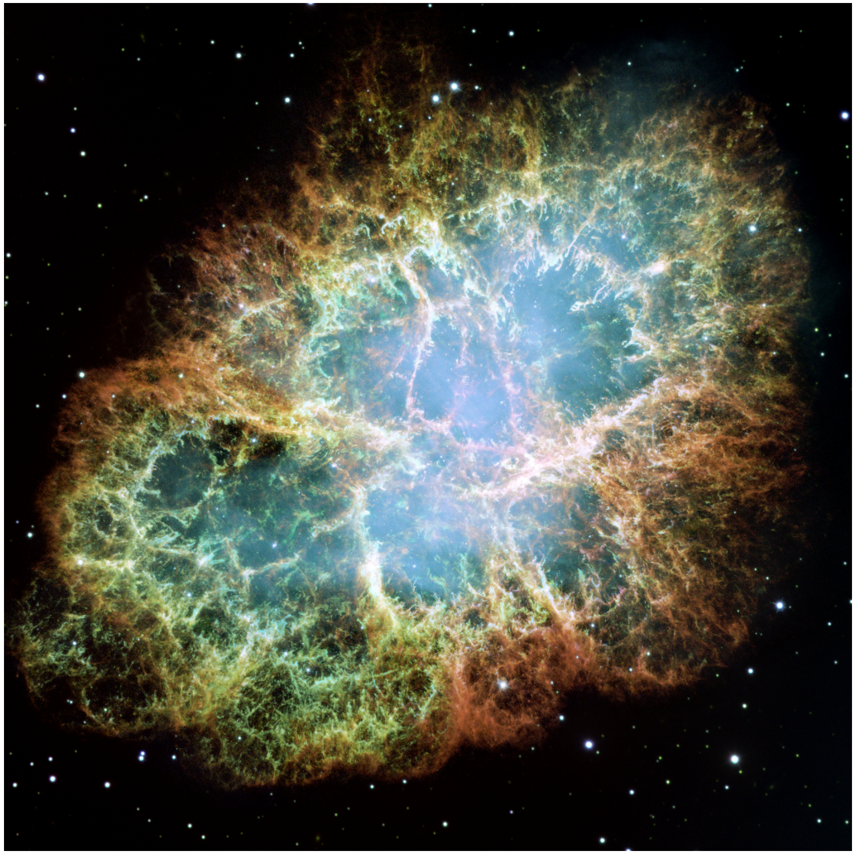


Neutron stars with crustal asymmetries (~ few cm mountains):

Freq. ~ 10 – 10^2 Hz







Asymmetric supernova explosions:

Freq. ~ 10 – 10^2 Hz

Primordial gravitational waves / Quantum fluctuations

Mergers of super-massive black holes (SMBHs, >10^5 Msun)

Mergers of compact object binaries

Only GWs observed so far

Mergers of SMBHs and BHs

Neutron stars with crustal asymmetries

Asymmetric supernova explosions

Some essential math about GWs from BINARIES:

It can be shown that $$h^{ij}(t,\vec{x}) \sim \frac{2\,G}{r\,c^4}\frac{d^2}{dt^2}\,I^{ij}(t-r/c)$$

can be expressed in spherical coordinates (*r, ϕ, θ*)
for a KEPLERIAN BINARY with reduced mass μ = *m1 m2/(m1+m2)*
with semi-major axis *a*, with orbital frequency ω_{orb}
and eccentricity *e* = 0 as

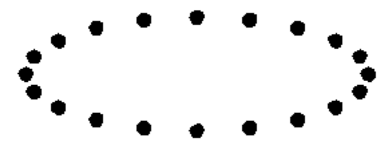
$$h_+(t,\theta,\phi,r) = \frac{1}{r}\,\frac{4\,G\,\mu\,\omega_{orb}^2\,a^2}{c^4}\,\frac{1+\cos^2\theta}{2}\,\cos\left(2\,\omega_{orb}\,t_{ret}+\phi\right)$$

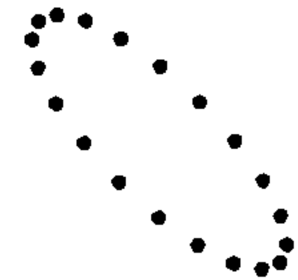
$$h_x(t,\theta,\phi,r) = \frac{1}{r}\,\frac{4\,G\,\mu\,\omega_{orb}^2\,a^2}{c^4}\,\cos\theta\,\sin\left(2\,\omega_{orb}\,t_{ret}+\phi\right)$$

where $$t_{ret} = t - r/c \qquad \omega_{orb}^2 = \frac{G\,(m_1+m_2)}{a^3}$$

This equation tells us:

– **GWs are POLARIZED (h_+ , h_x)**





– **FREQUENCY TERM DEPENDS only ON 2 ω_{orb}**

→ **frequency of GWs** $\omega_{GW} = 2\,\omega_{orb}$

(true for most evolution)

– **AMPLITUDE of GWs:**

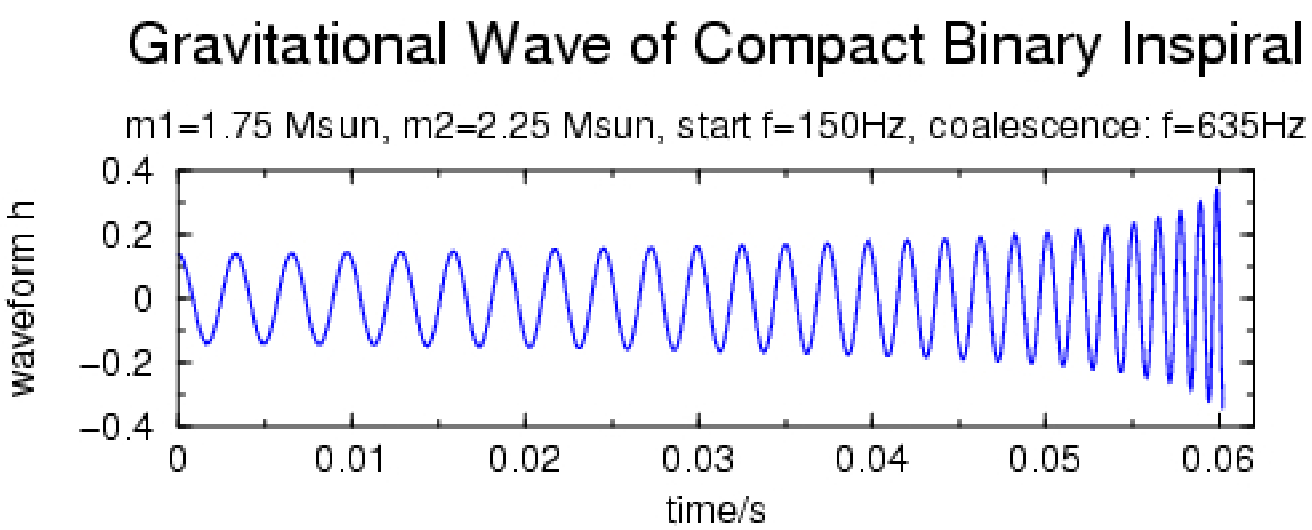
$$h = \frac{1}{2}\sqrt{h_+^2 + h_x^2} = \frac{2\,G\,\mu\,\omega_{orb}^2\,a^2}{c^4}\,\frac{1}{r}\,\sqrt{\frac{(1+\cos^2\theta)^2}{4} + cos^2\theta}$$

$$h = \frac{1}{2}\sqrt{h_+^2 + h_x^2} = \frac{2\,G^2\,m_1\,m_2}{a\,c^4}\,\frac{1}{r}\,\sqrt{\frac{(1+\cos^2\theta)^2}{4} + cos^2\theta}$$

– AMPLITUDE of GWs:

$$h = \frac{1}{2}\sqrt{h_+^2 + h_x^2} = \frac{2\,G^2\,m_1\,m_2}{a\,c^4}\,\frac{1}{r}\,\sqrt{\frac{(1+\cos^2\theta)^2}{4} + cos^2\theta}$$

* the bigger the amplitude (strain), the easier the detection
* the farther the binary, the smaller the amplitude
* the larger the masses, the larger the amplitude
* the smaller the semi-major axis, the larger the amplitude



– **EMISSION of GWs implies LOSS of ORBITAL ENERGY:**

$$E_{orb} = -\frac{G\, m_1\, m_2}{2\, a}$$

THE BINARY SHRINKS WHILE EMITTING GWs TILL IT MERGES

https://www.youtube.com/watch?v=g8s81MzzJ5c

Credits: NASA

– EMISSION of GWs implies LOSS of ORBITAL ENERGY:

$$E_{orb} = -\frac{G\,m_1\,m_2}{2\,a}$$

THE BINARY SHRINKS WHILE EMITTING GWs TILL IT MERGES

– If the binary shrinks ($a \rightarrow 0$), frequency becomes higher

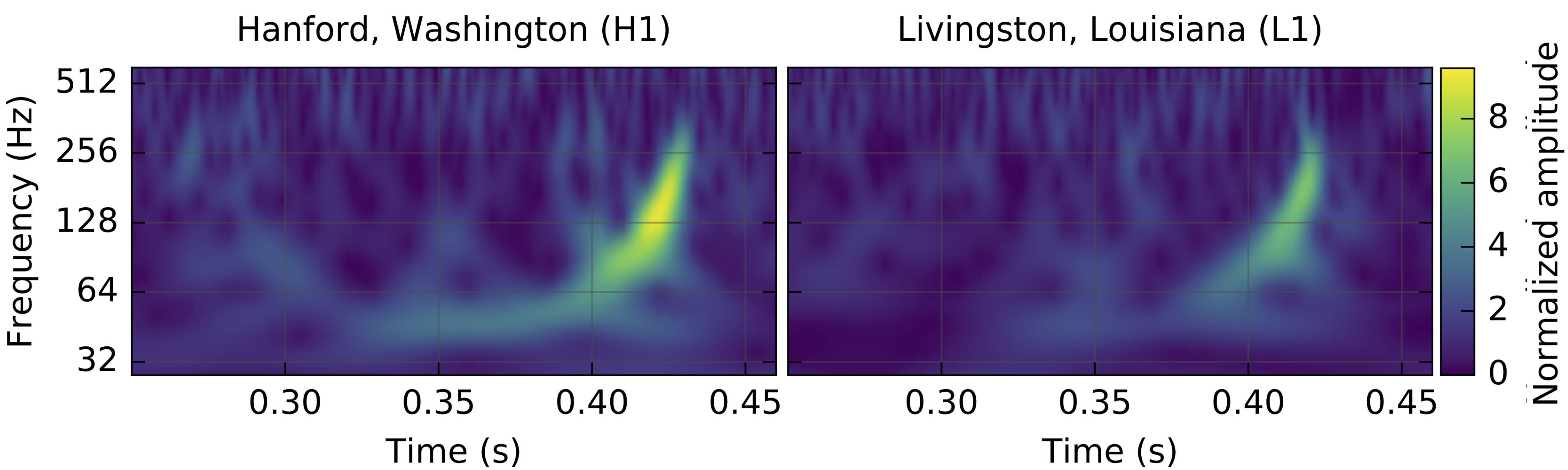
$$\omega_{GW} = 2\,\omega_{orb} = 2\,\sqrt{\frac{G\,(m_1 + m_2)}{a^3}}$$

– If the binary shrinks amplitude increases

$$h \propto \frac{1}{a}$$

– **EMISSION of GWs implies LOSS of ORBITAL ENERGY: THE BINARY SHRINKS WHILE EMITTING GWs TILL IT MERGES**

– **If the binary shrinks ($a \rightarrow 0$), frequency becomes higher**



Abbott et al. 2016

– **EMISSION of GWs implies LOSS of ORBITAL ENERGY: THE BINARY SHRINKS WHILE EMITTING GWs TILL IT MERGES**

– **If the binary shrinks ($a \to 0$), frequency becomes higher**

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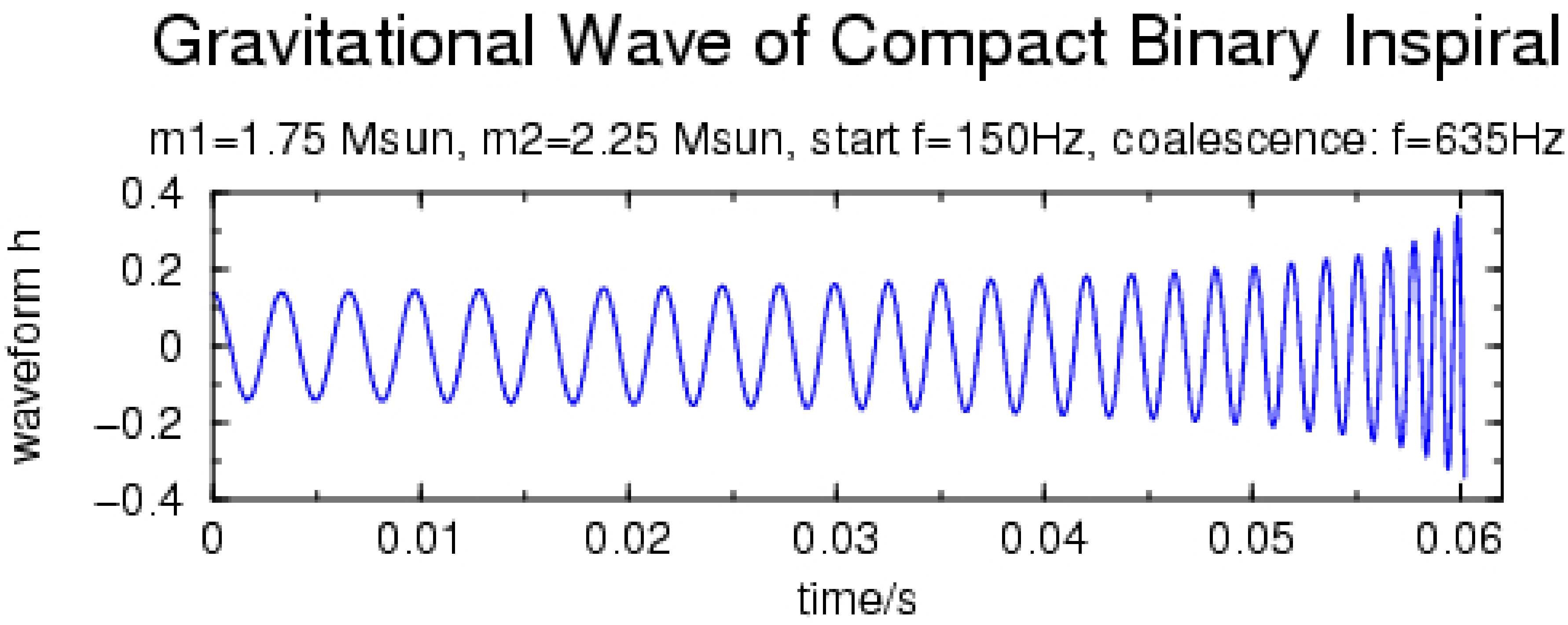


– EMISSION of GWs implies LOSS of ORBITAL ENERGY:

Power radiated by GWs:

From GR

$$P_{GW} = \frac{32}{5}\,\frac{G^4}{c^5}\,\frac{1}{a^5}\,m_1^2\,m_2^2\,(m_1+m_2)$$

$$P_{GW} = \frac{dE_{orb}}{dt} = \frac{G\,m_1\,m_2}{2\,a^2}\,\frac{da}{dt}$$

From Kepler and Newton

$$\Rightarrow \quad \frac{da}{dt} = \frac{64}{5}\,\frac{G^3}{c^5}\,a^{-3}\,m_1\,m_2\,(m_1+m_2)$$

Integrating differential equation:

$$t_{GW} = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{a^4}{m_1\,m_2\,(m_1+m_2)}$$

Timescale for a system to merge by GW emission

For binaries with general eccentricity *e*

$$t_{GW} = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{a^4\,(1-e^2)^{7/2}}{m_1\,m_2\,(m_1+m_2)}$$

Peters 1964

Timescale depends on semi-major axis, eccentricity, masses

Timescale extremely long

EXERCISE: calculate tGW for 2 neutron stars
with mass equal to the Sun mass (1 Msun)
orbiting at the distance
between Sun and Earth (1 AU)

For binaries with general eccentricity *e*

$$t_{GW} = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{a^4\,(1-e^2)^{7/2}}{m_1\,m_2\,(m_1+m_2)}$$

Peters 1964

Timescale depends on semi-major axis, eccentricity, masses

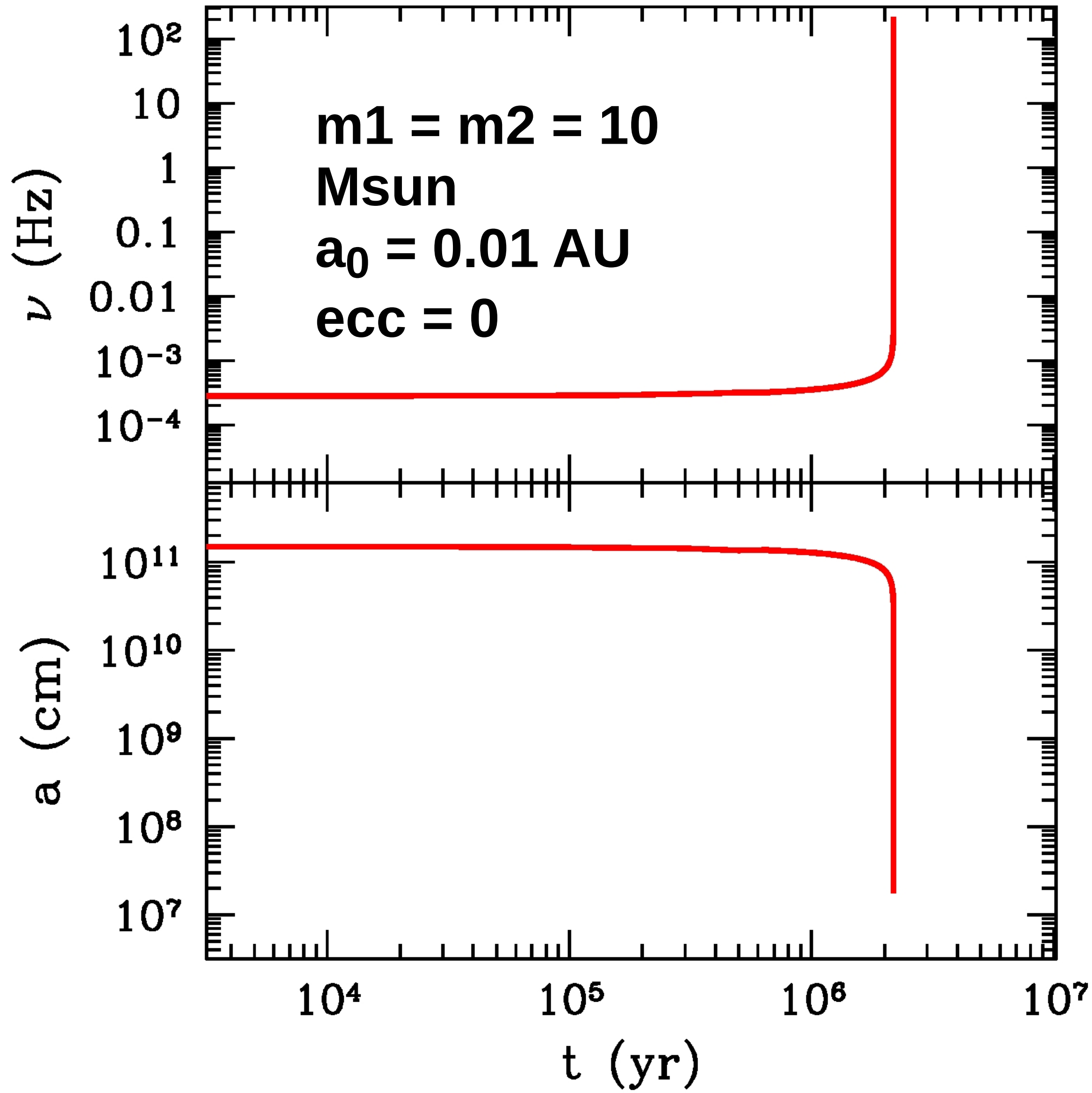
Timescale extremely long

If m1 = m2 = Msun, a = 1 AU, e = 0

→ t_{GW} ~ 2 x 10^17 yr

Life of the Universe ~ 13 x 10^9 yr

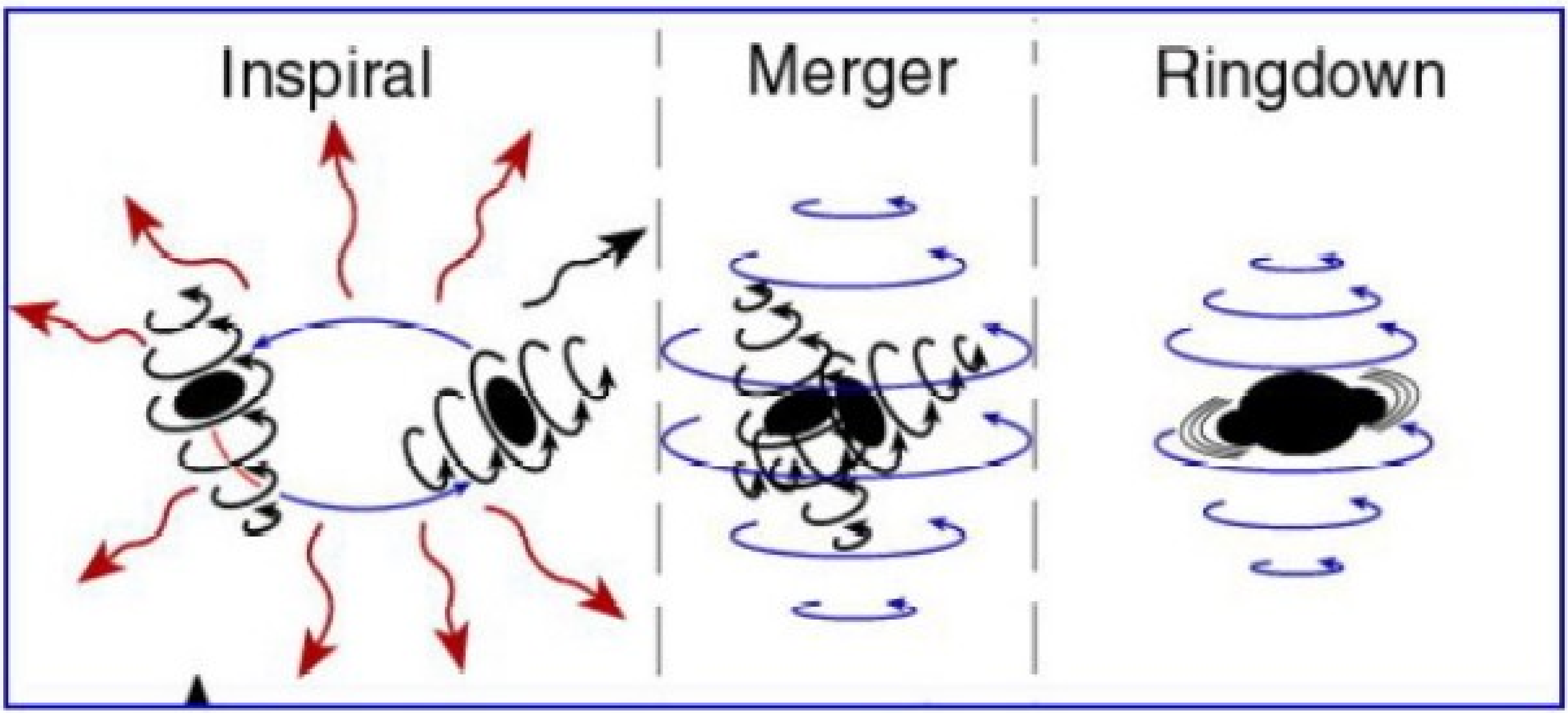
$$a(t) = a_0 \left[1 - \frac{256/5\, G^3\, m_1\, m_2\, (m_1 + m_2)\, t}{c^5\, (1-e^2)^{7/2}\, a_0^4}\right]^{1/4}$$

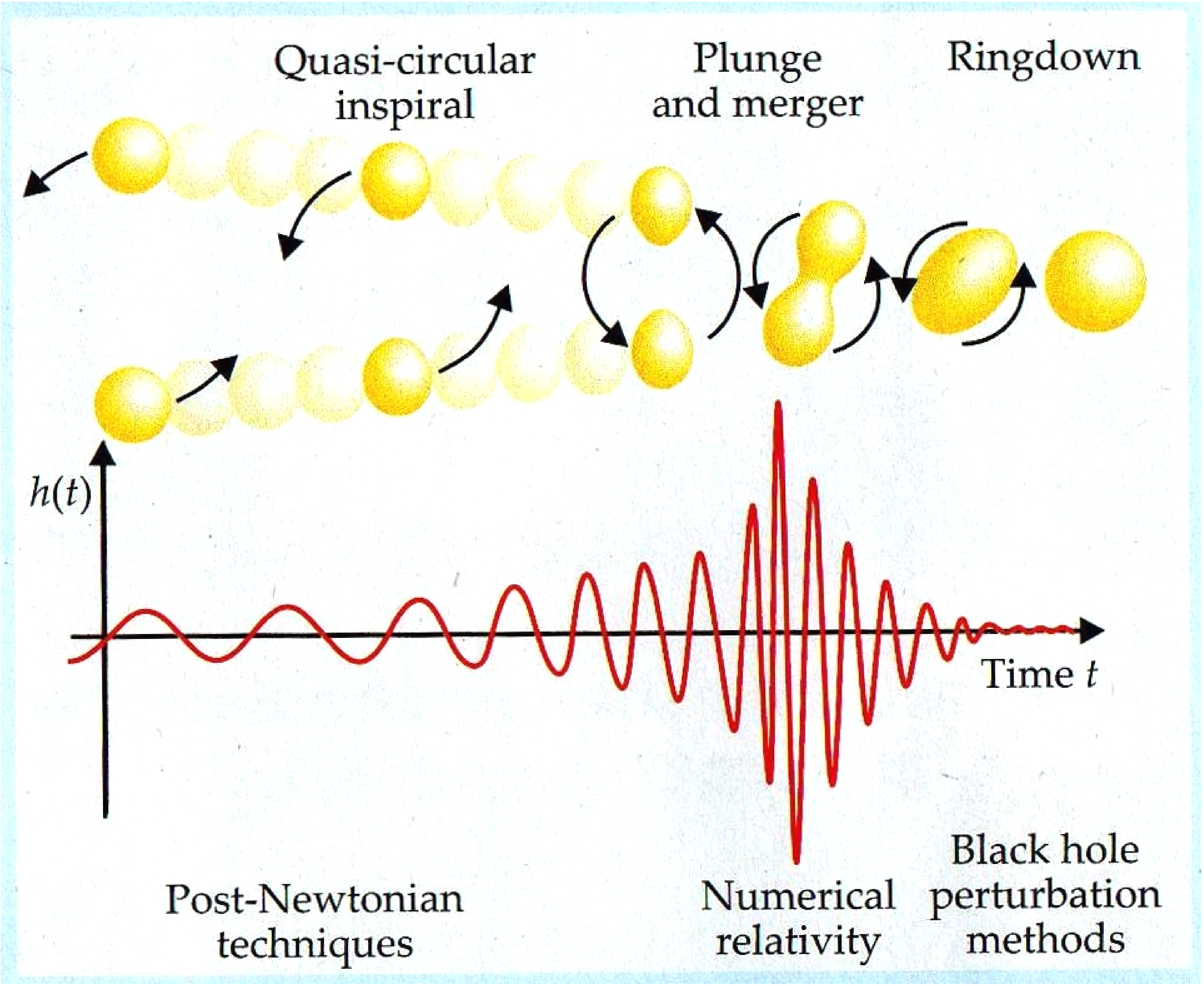


Previous equations are not always true!

Only before merger when binary can be considered Keplerian

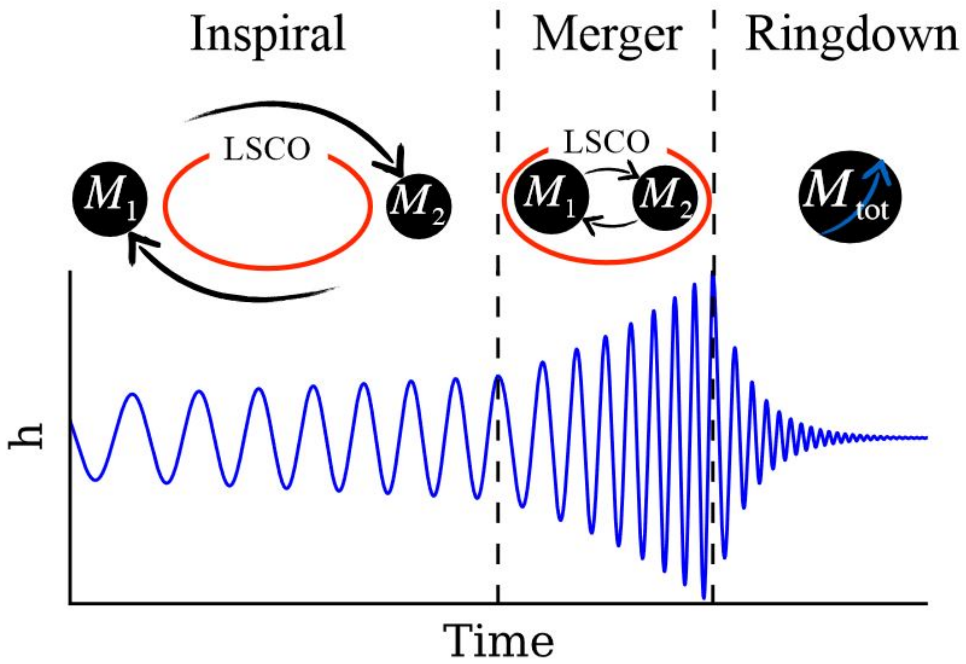
i.e. only during inspiral




Quasi-circular inspiral
Plunge and merger
Ringdown
h(t)
Time t
Post-Newtonian techniques
Numerical relativity
Black hole perturbation methods

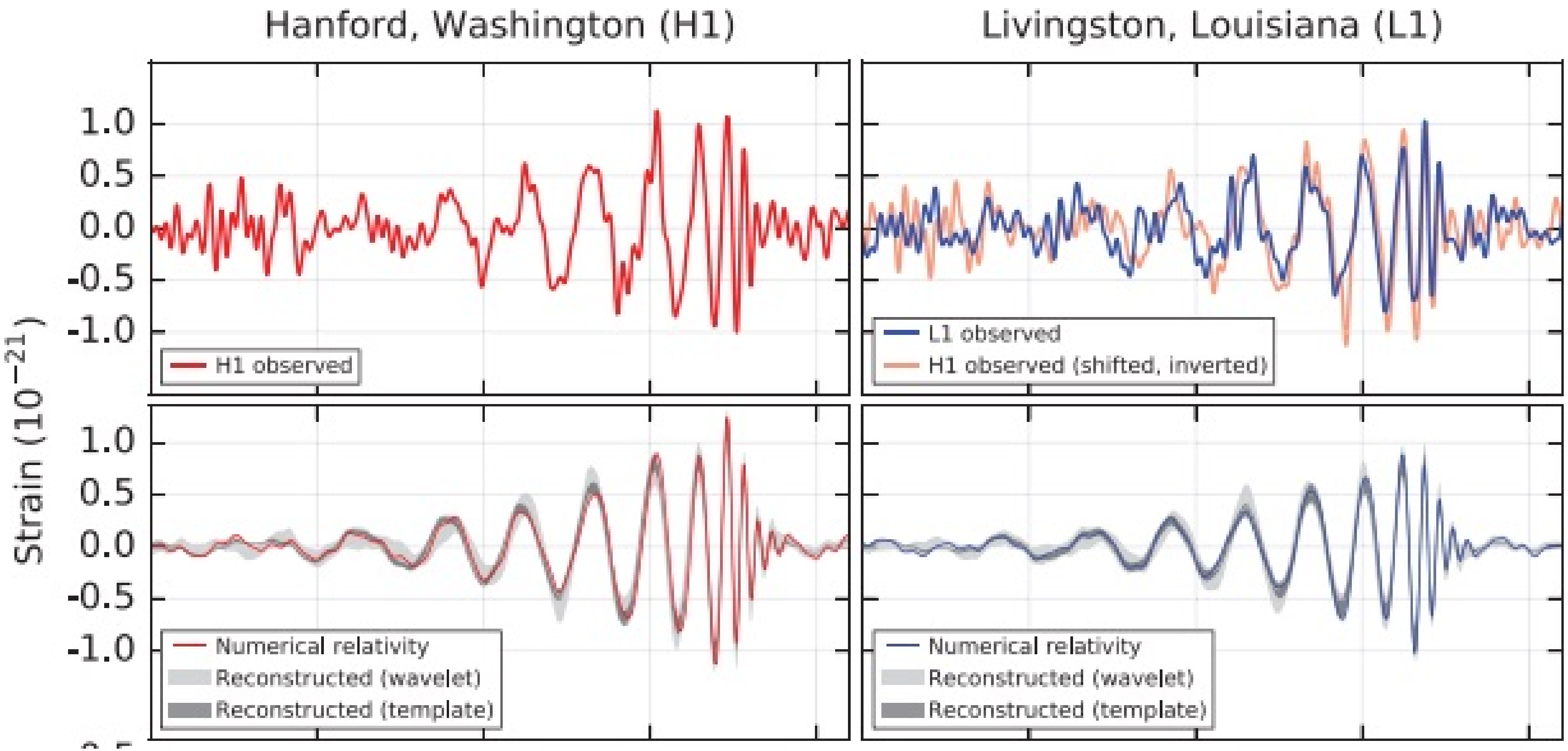
Simple way to estimate frequency at merger:
Last stable circular orbit around a black hole

$$r_{\rm LSCO}=6\,\frac{G\,(m_1+m_2)}{c^2}$$



$$\omega_{\rm GW,LSCO}=2\,\sqrt{\frac{G\,(m_1+m_2)}{r^3_{\rm LSCO}}}=\frac{2\,c^3}{6^{3/2}\,G\,(m_1+m_2)}$$

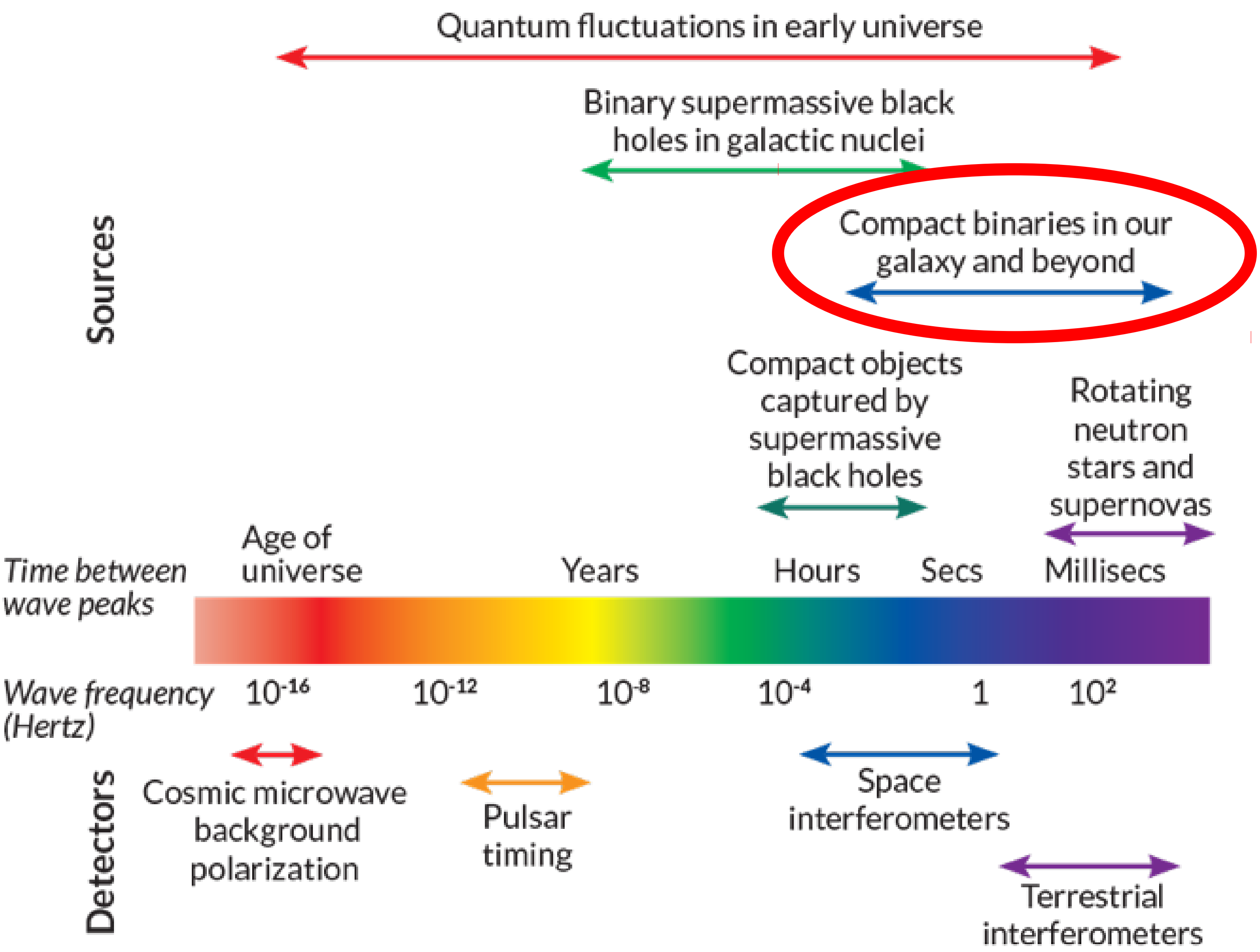
$$\omega_{\rm GW,LSCO}=460\,{\rm Hz}\,\frac{60\,M_{\rm sun}}{(m_1+m_2)}$$



Abbott et al. 2016

Observational facts

Detectors:


Quantum fluctuations in early universe
Binary supermassive black holes in galactic nuclei
Compact binaries in our galaxy and beyond
Sources
Compact objects captured by supermassive black holes
Rotating neutron stars and supernovas
Time between wave peaks
Age of universe
Years
Hours
Secs
Millisecs
Wave frequency (Hertz)
10^{-16}
10^{-12}
10^{-8}
10^{-4}
1
10^{2}
Detectors
Cosmic microwave background polarization
Pulsar timing
Space interferometers
Terrestrial interferometers

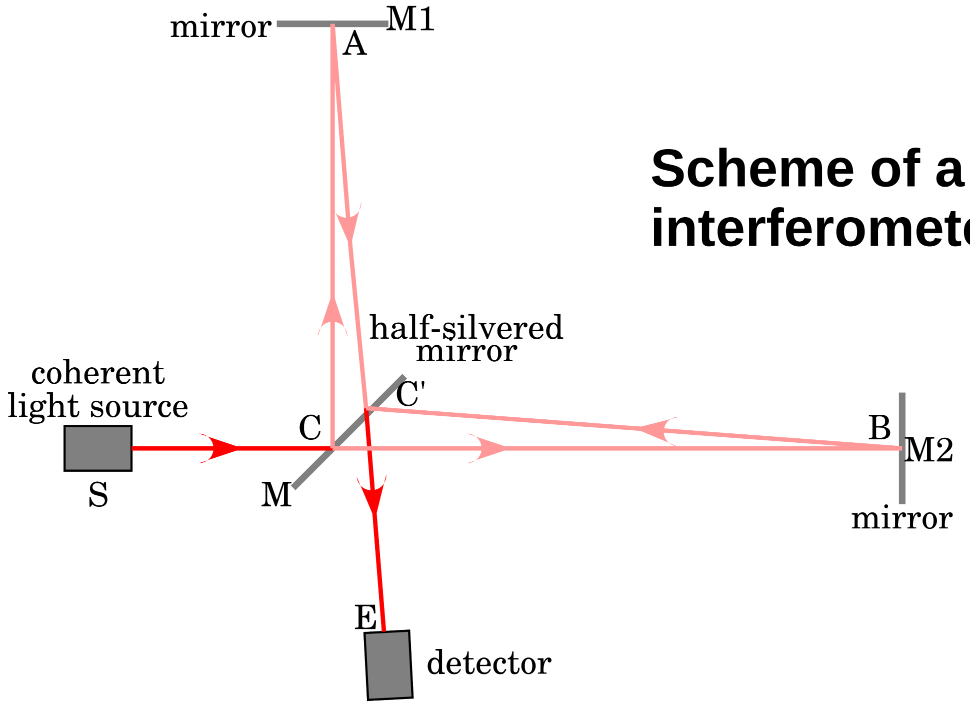
Detectors:

Advanced LIGO (Livingstone + Hanford, US)
Advanced Virgo (Pisa, Italy)

LIGO Lab/Virgo

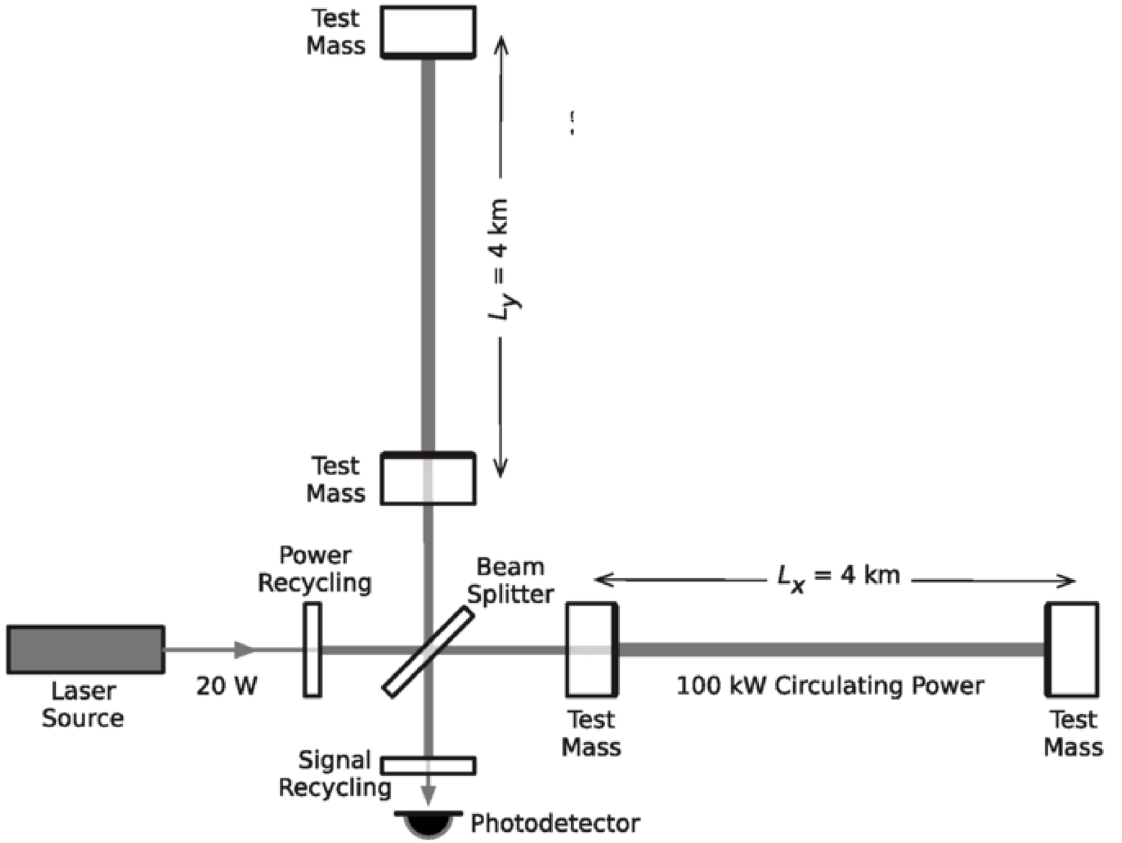
Michelson interferometers

Design started in the '90s
First science runs ~ 2007 (no detection)
Being upgraded in 2007 – 2015
First run advanced detectors 2015



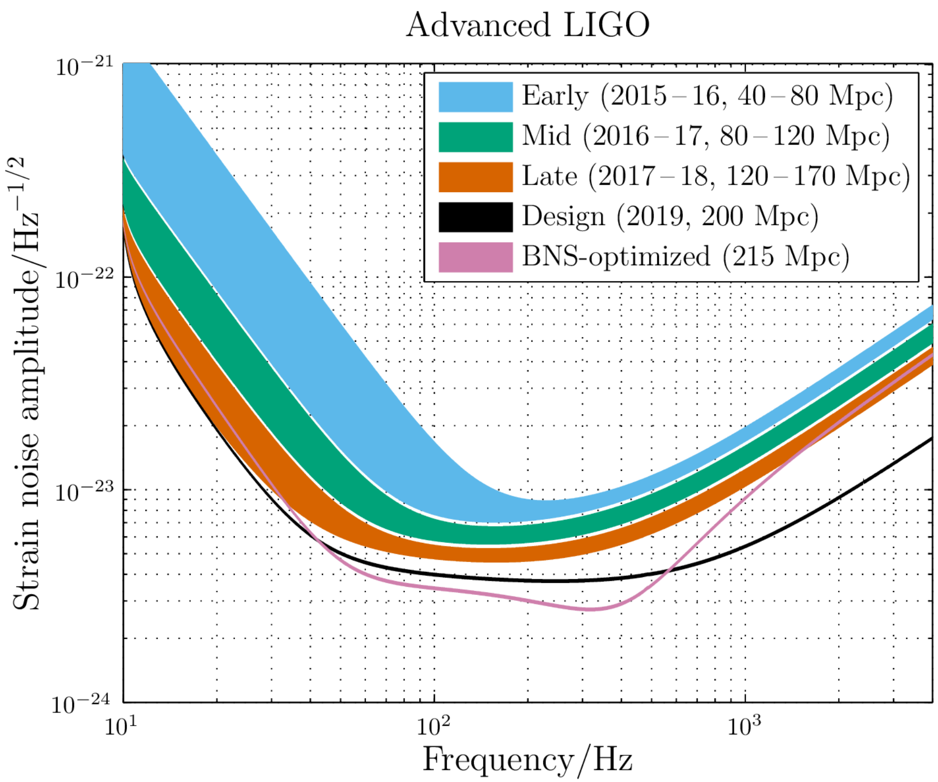
Scheme of a Michelson interferometer

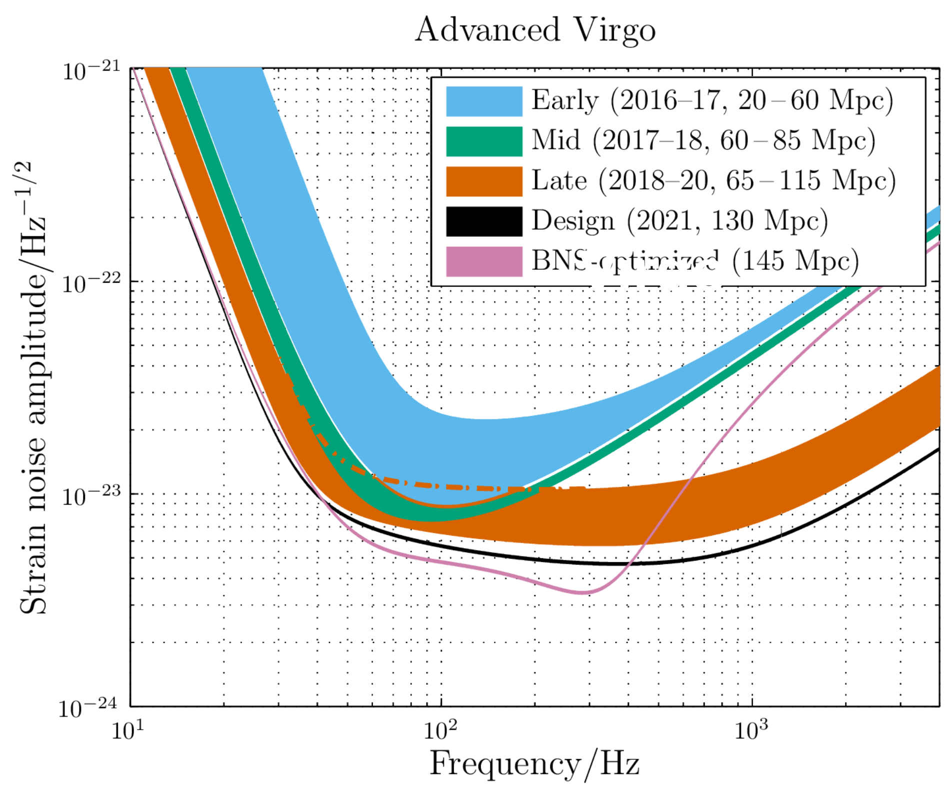
LIGO-Virgo scheme



Frequency range: ~ 10 – 10'000 Hz

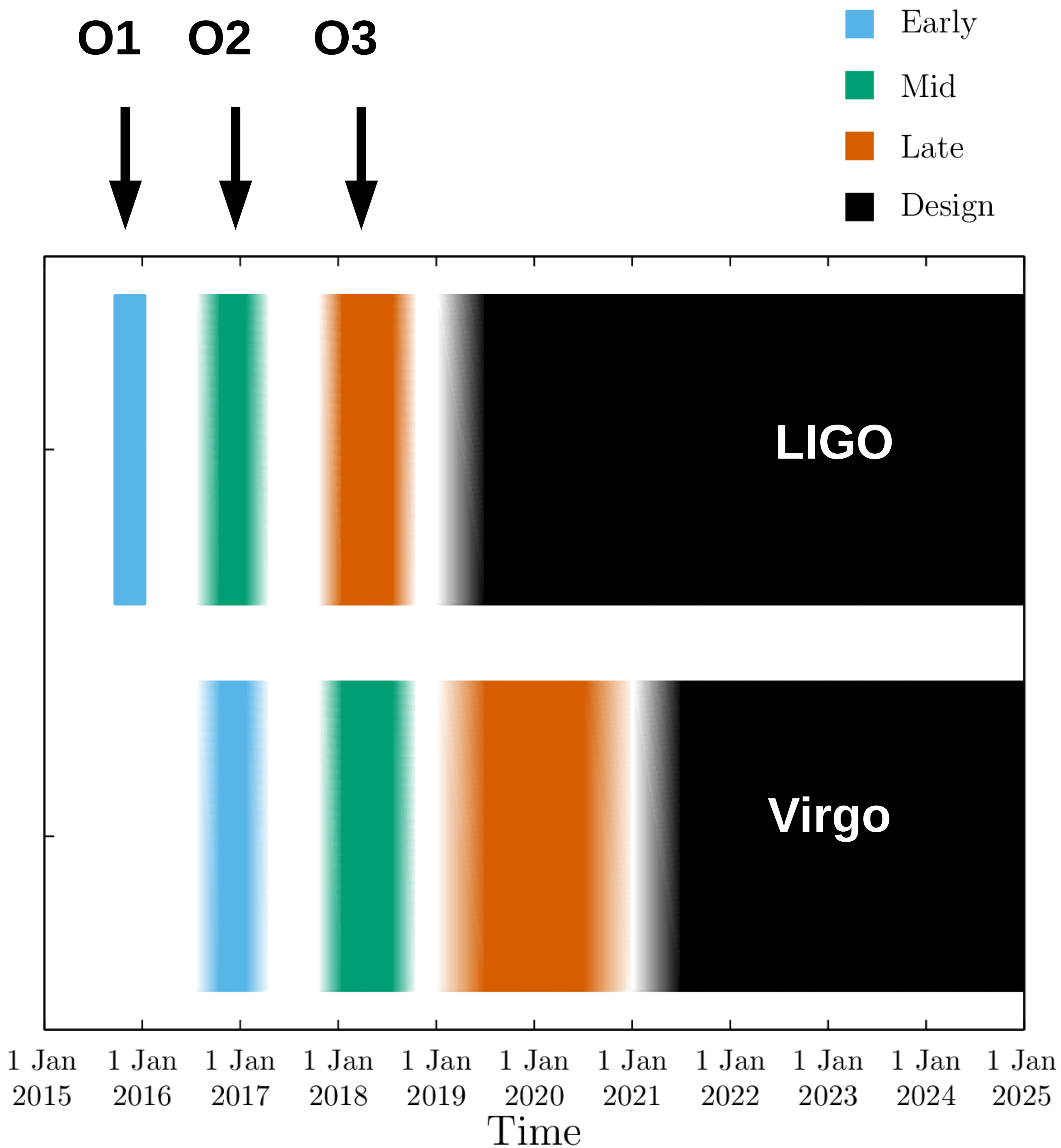
Suitable for mergers of compact binaries





Noise level of LIGO & Virgo (Noise power spectral density)

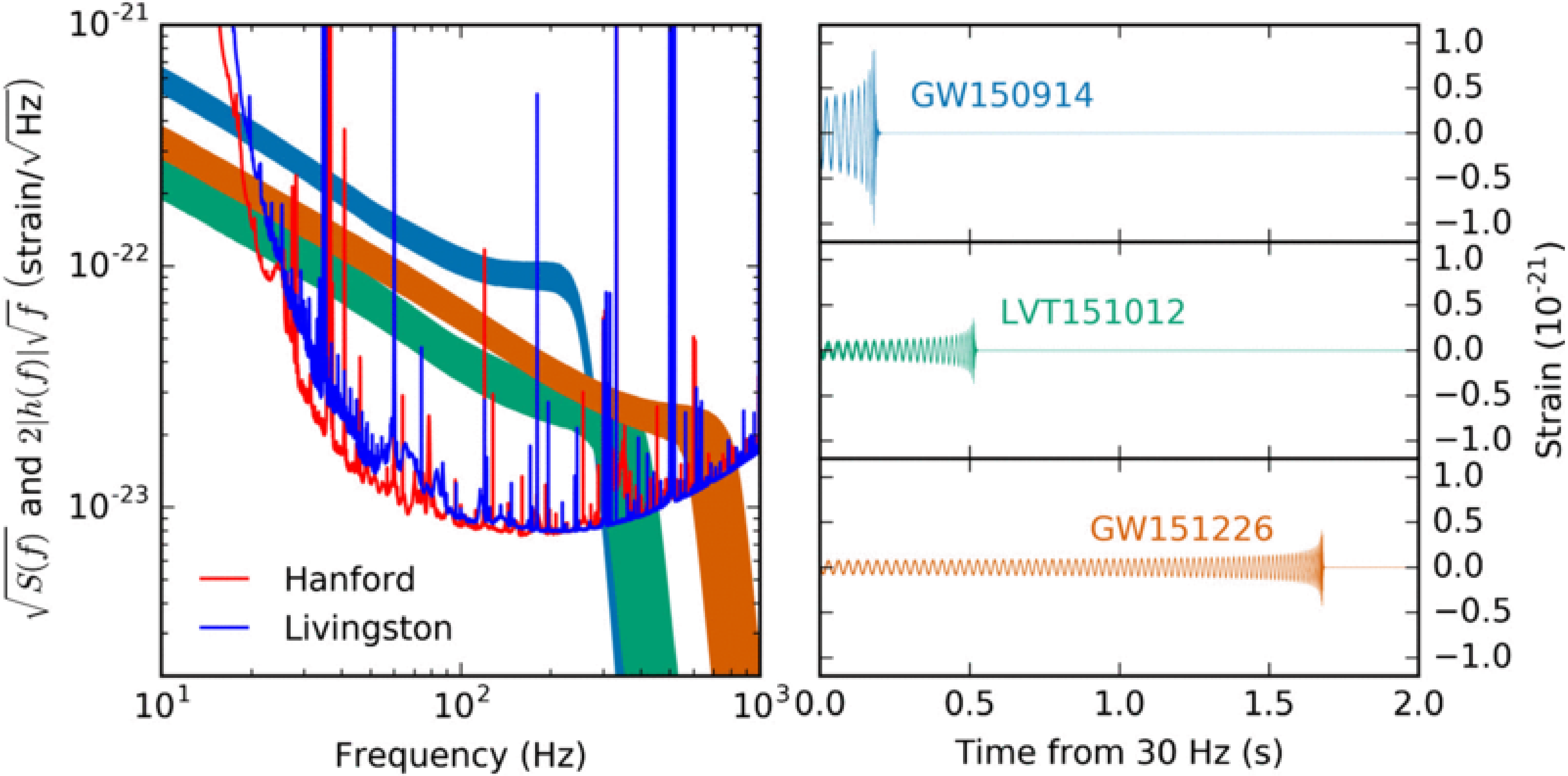
Time schedule



Summary of detections:

2015/09/12	first LIGO run	
2015/09/14	GW150914	black holes (BHs)
2015/10/12	LVT151012	maybe BHs
2015/12/26	GW151226	BHs
2015/01 – 2016/11	detectors switched off	
2017/01/04	GW170104	BHs
2017/08/01	Virgo joins LIGO	
2017/08/14	GW170814	BHs
2017/08/17	GW170817	neutron stars (NSs)
2017/08/25 – now	detectors switched off	

Properties of the first detections:



From Abbott et al. (2016)
https://journals.aps.org/prx/abstract/10.1103/PhysRevX.6.041015#fulltext

15 Observables (+ eccentricity):

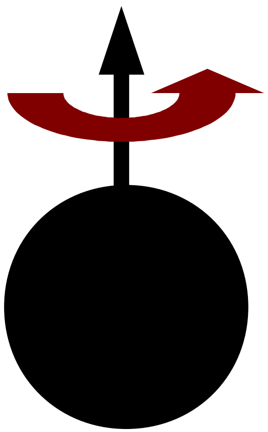
- 2 masses
- 6 spin components
- polarization
- inclination of binary wrt interferometers
- 2 sky positions (RA, DEC)
- redshift of merger
- reference time
- phase at a reference time

Mass and Spins

black holes (BHs) are uniquely defined by mass & spin (electric charge deemed to be negligible)

Spins:

$$\vec{S} = \frac{\vec{J}\,c}{G\,m^2}$$

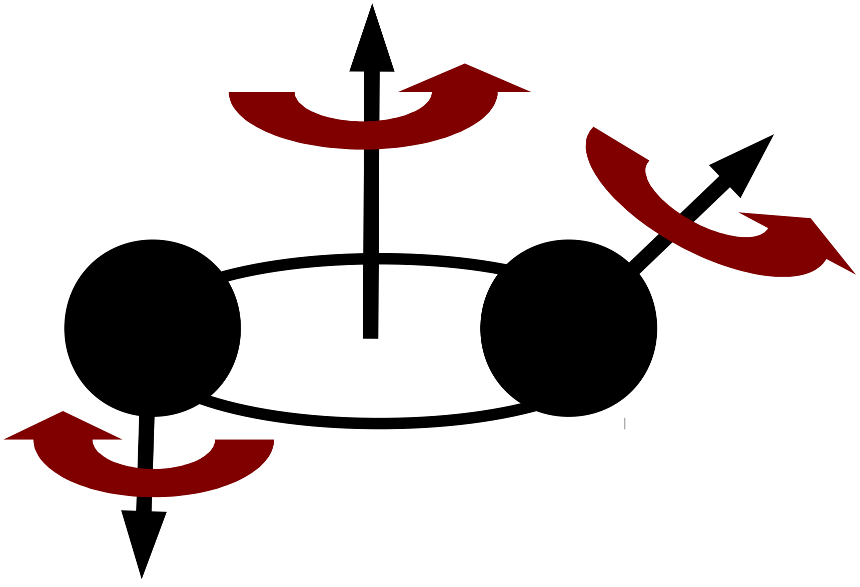


$$S = 0 \;\rightarrow \mathrm{Schwarzschild\,BH}$$

$$S = 0.998 \;\rightarrow \mathrm{Maximally\,Rotating\,BH}$$

If black hole is in binary we have 6 spin components:

3 per each black hole



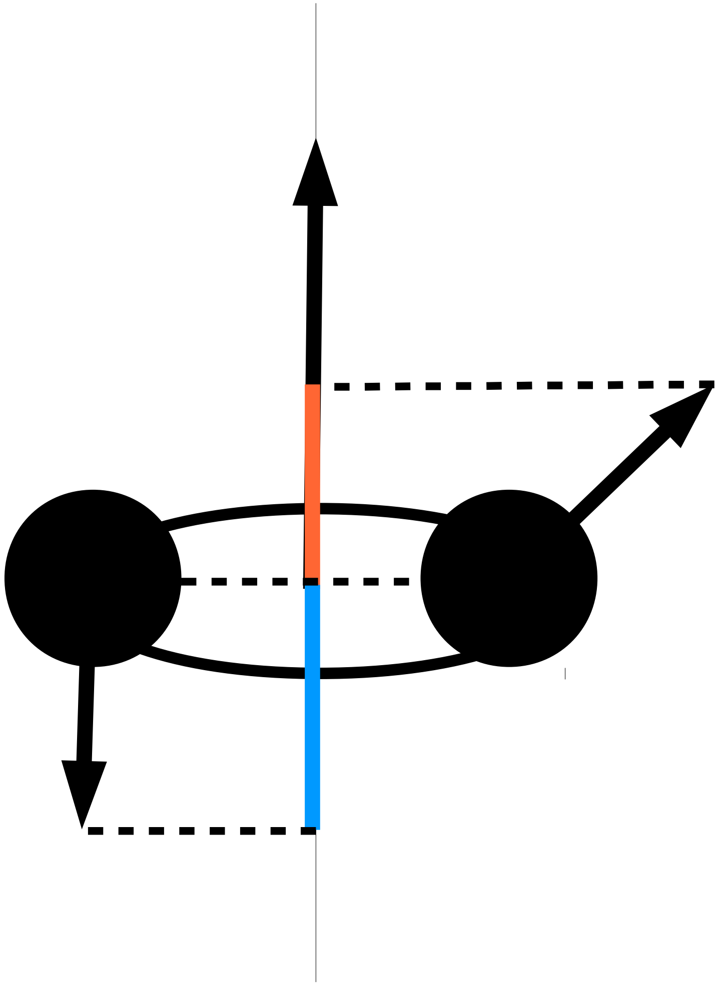
Spins:

LIGO – Virgo not enough to measure 6 spins

Only measured parameter: EFFECTIVE SPIN

$$\chi_{\rm eff} = \frac{(m_1\,\vec{S}_1 + m_2\,\vec{S}_2)}{m_1 + m_2}\cdot\hat{L}$$

$$-1 \leq \chi_{\rm eff} \leq 1$$



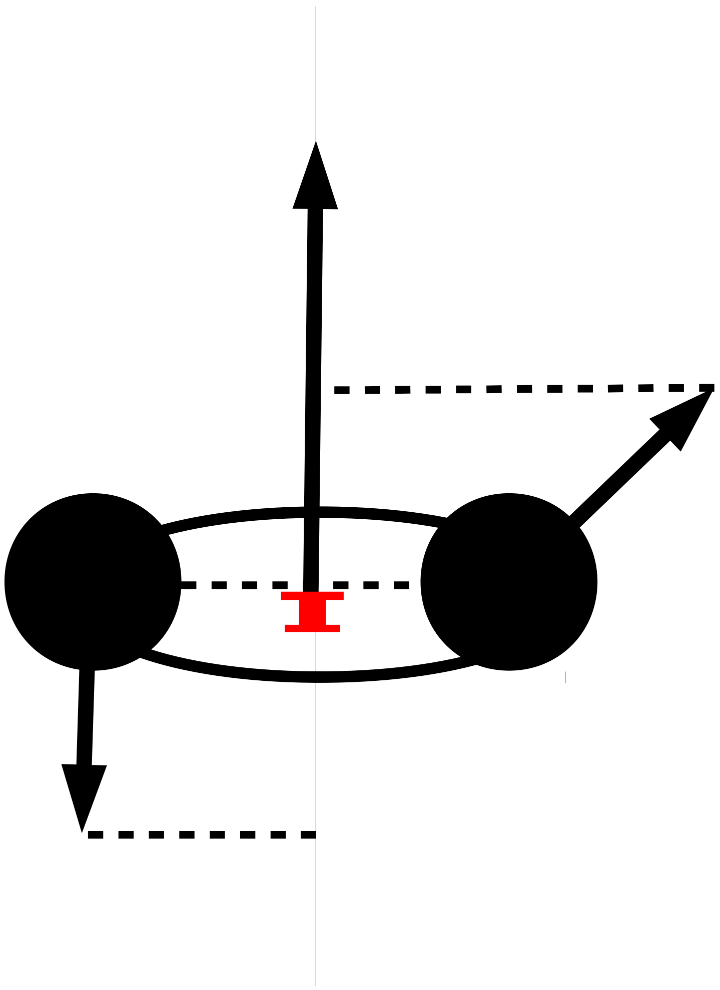
Spins:

LIGO – Virgo not enough to measure 6 spins

Only measured parameter: EFFECTIVE SPIN

$$\chi_{\rm eff} = \frac{(m_1\,\vec{S}_1 + m_2\,\vec{S}_2)}{m_1 + m_2} \cdot \hat{L}$$

$$-1 \leq \chi_{\rm eff} \leq 1$$



In our example $\chi_{\rm eff} = -0.2$

Measured because affects phase of GWs while orthogonal spin to binary ang. mom. measures precession

Masses

If black hole (BH) is in binary 2 masses:

m_1, m_2 = Mass of first BH, Mass of second BH

but LIGO-Virgo measured two combinations of m_1, m_2 :

Chirp mass:

$$m_{\rm chirp} = \frac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

change of frequency during inspiral scales with it

Total mass:

$$M = (m_1 + m_2)$$

frequency at merger scales with it

Other relevant mass (for phase of GWs): mass ratio

$$q = \frac{m_2}{m_1}$$

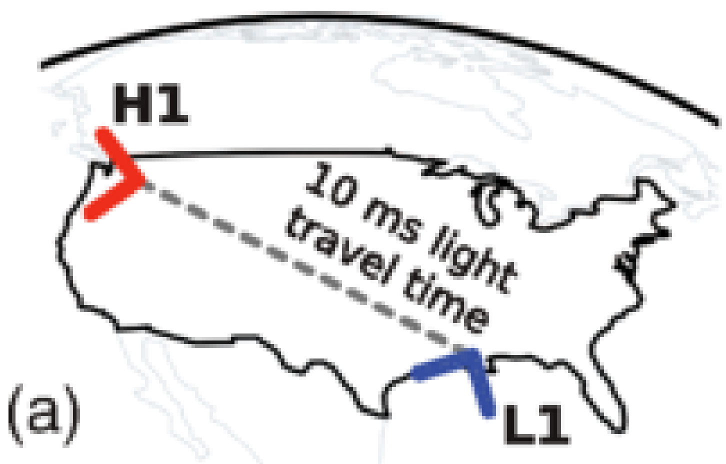
Redshift (or Luminosity distance)

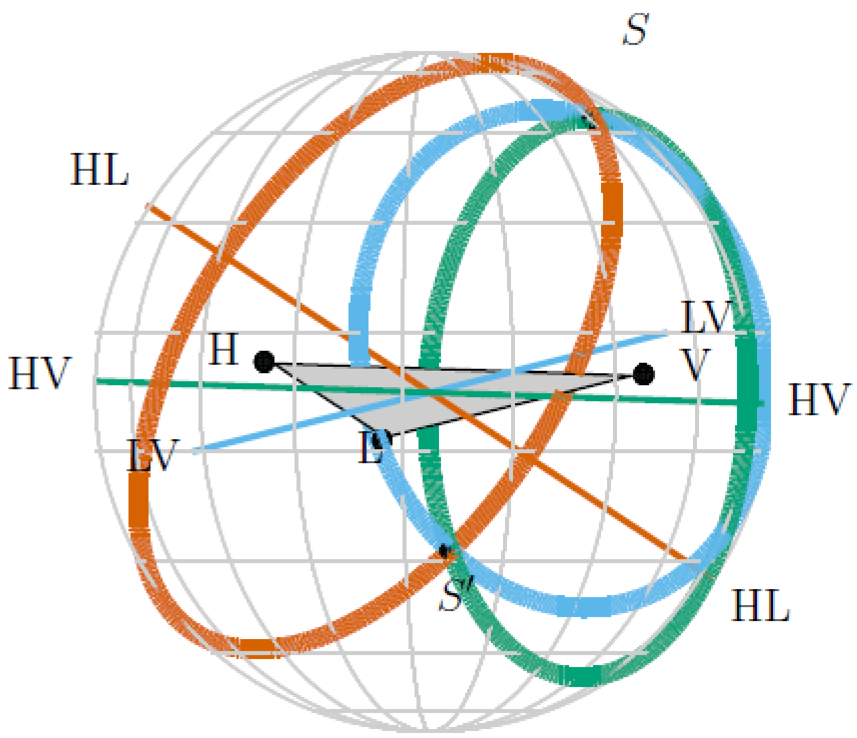
Measured from the strain (GW amplitude) after removing mass contribution

$$h \propto \frac{1}{r}$$

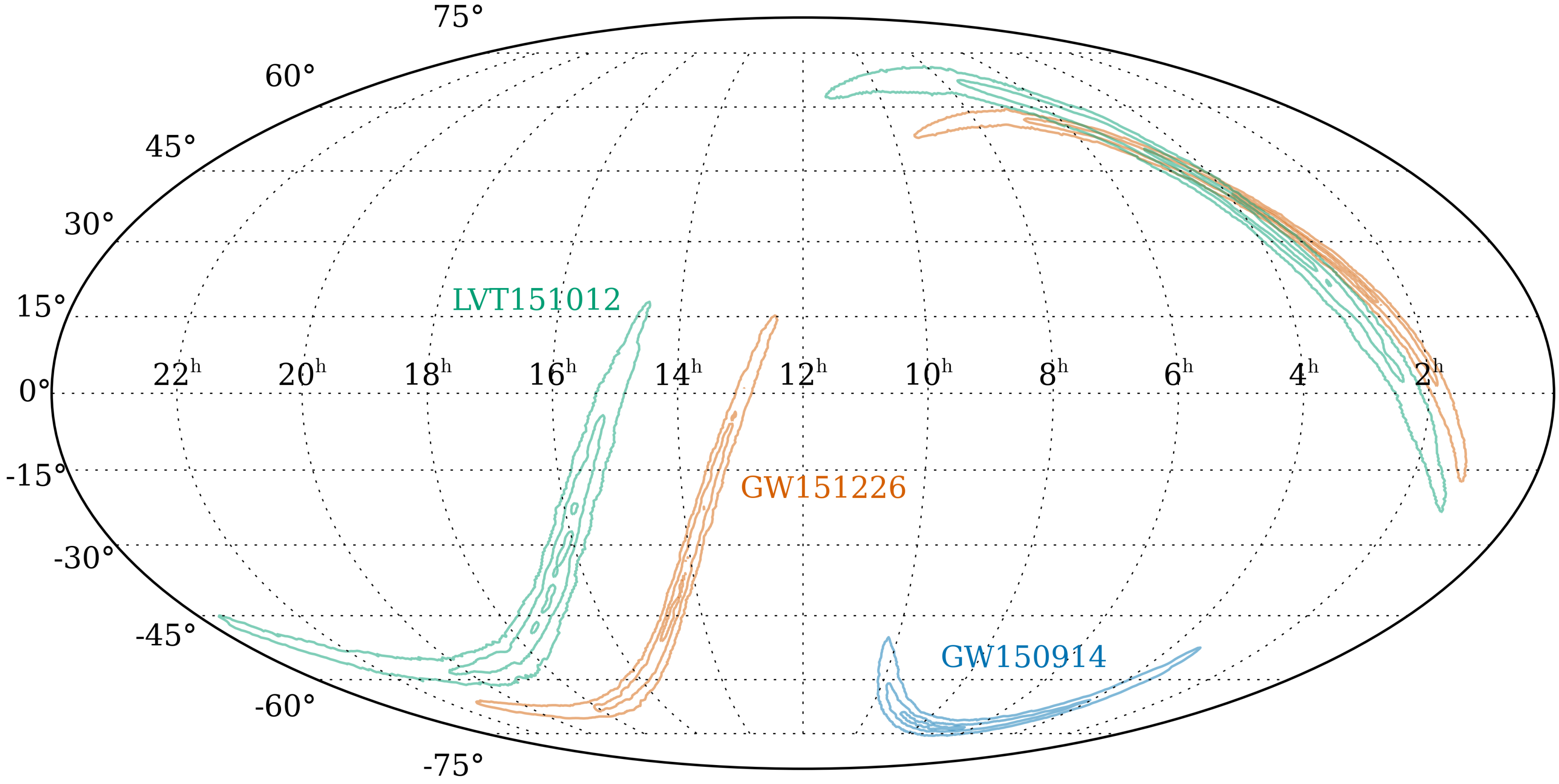
Sky localization (RA, DEC)

Measured by time delay between two detectors (+ phase, + amplitude)



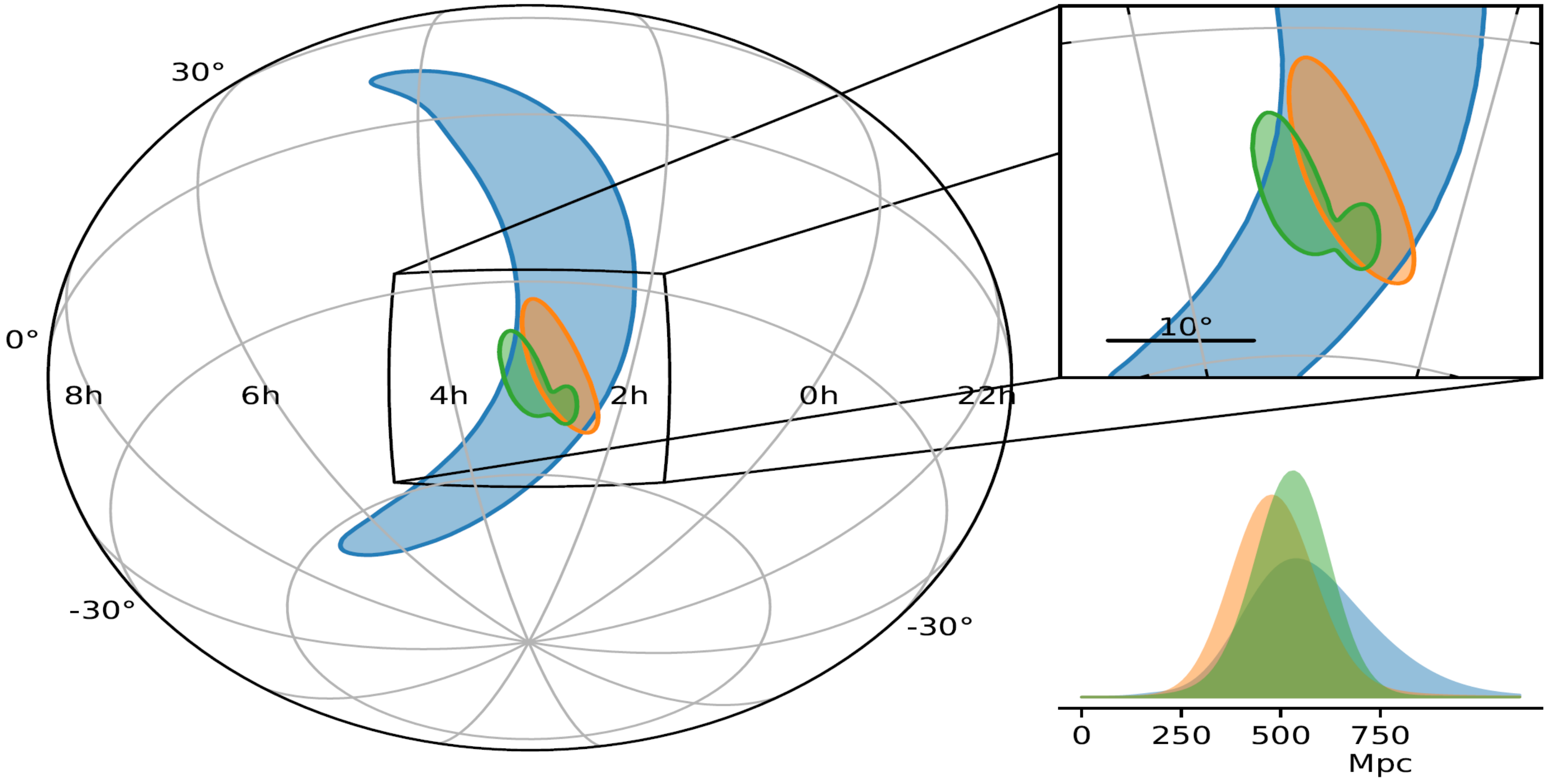


Sky map with only 2 detectors (GW150914, GW151226, LVT151012) (hundreds of square degrees 90% credible area)

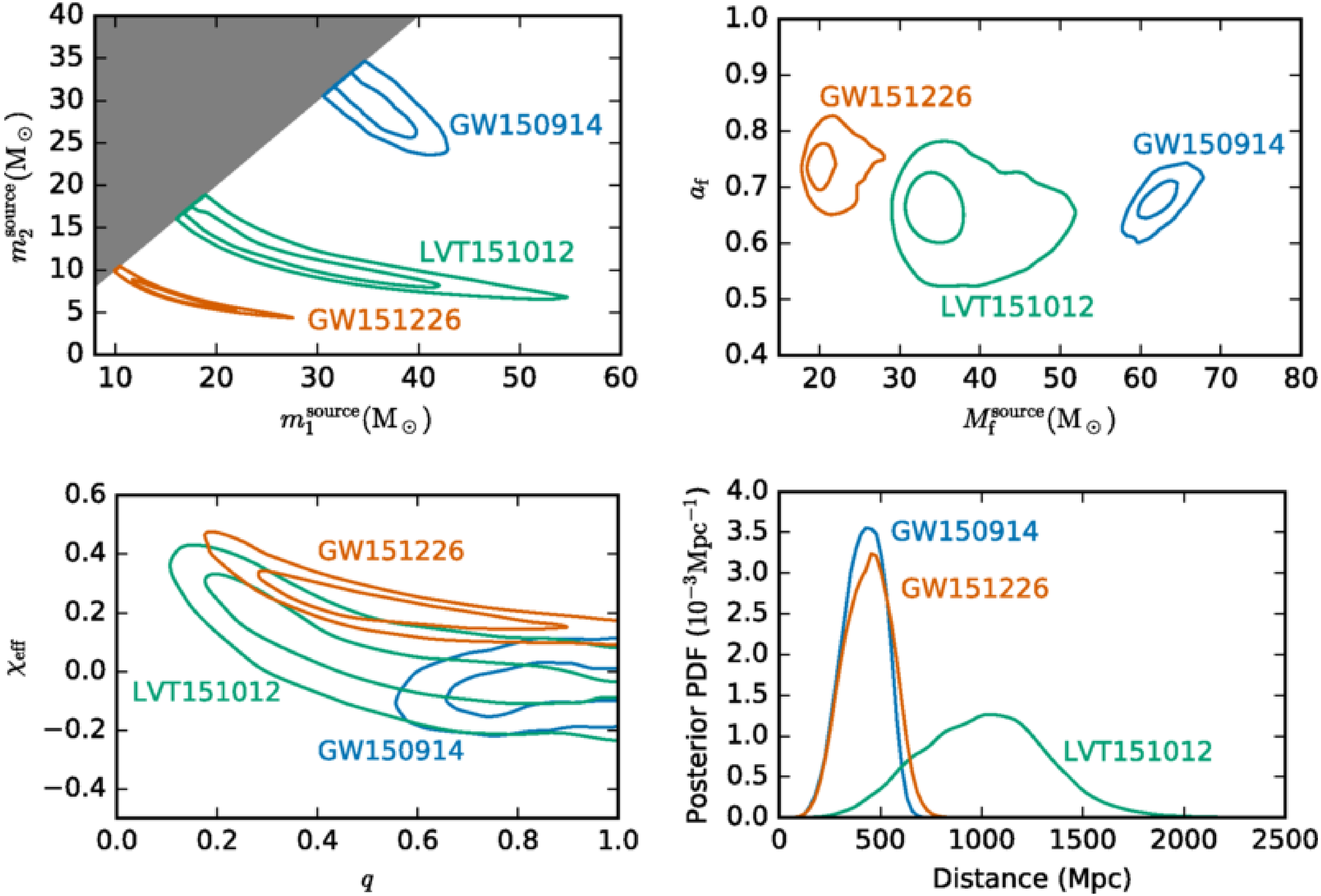


Sky map with 3 detectors (GW170814) (TENS of square degrees 90% credible area)

Rapid LIGO only localization
Rapid LIGO+ Virgo localization
Refined LIGO+ Virgo localization



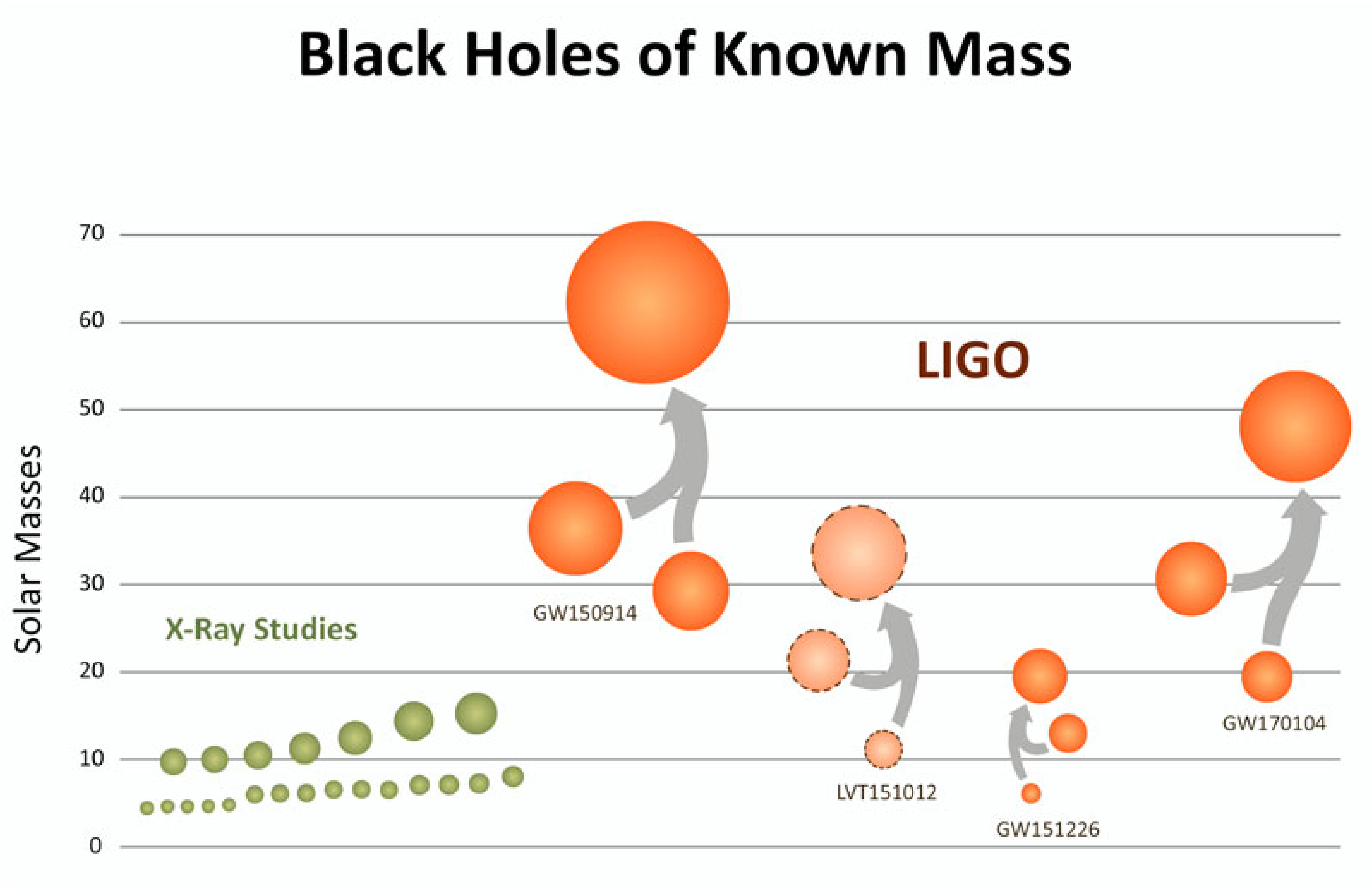
Properties of the first detections:



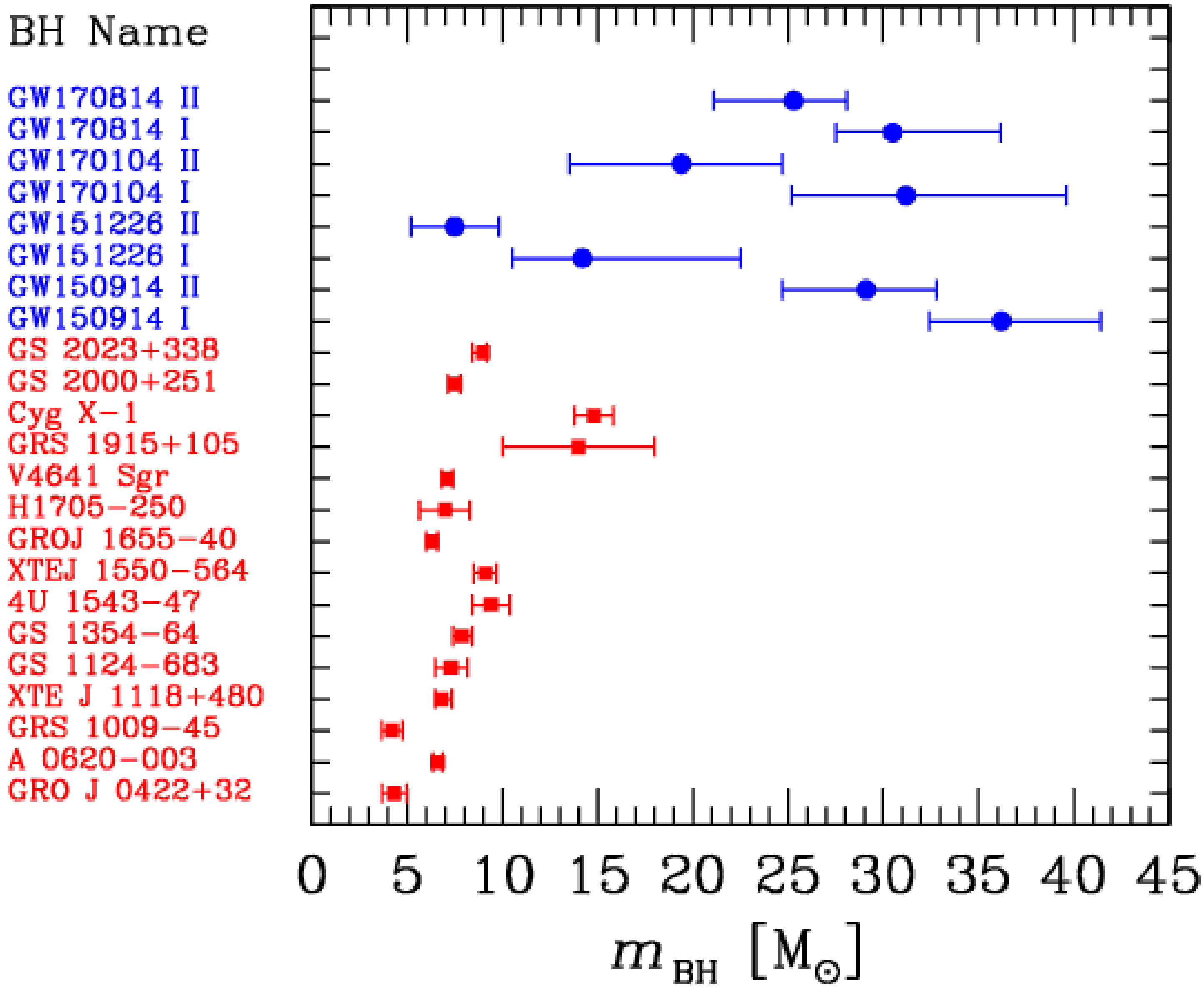
From Abbott et al. (2016)

https://journals.aps.org/prx/abstract/10.1103/PhysRevX.6.041015#fulltext

Focus on black hole masses



Focus on black hole masses



Two different classes of black holes?

Fundamental questions this course will address:

* How do black holes (BHs) / neutron stars (NSs) form?

* What determines their mass?

* How do BH / NS binaries form?

* What determins BH / NS spins?

* BH binaries from GWs and from X-ray:
Two different classes of black holes?

* Can we predict / study redshift distribution of mergers?

THANK YOU