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N-body techniques for astrophysics: Lecture 5 – Sub-grid physics

PhD School in Astrophysics,
University of Padova
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OUTLINE of this lecture:

1 – Radiative transfer

2 – Star formation

3 – Cooling

5 – Supernovae

6 – Non-equilibrium chemistry

WHAT IS SUB-GRID PHYSICS?

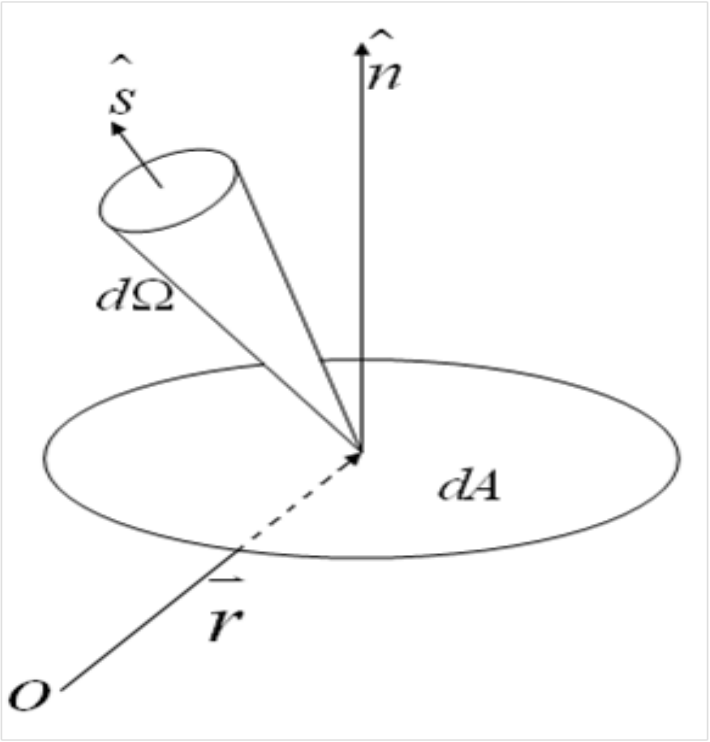
WHAT IS SUB-GRID PHYSICS?

ALL PHYSICS PROCESSES THAT WE DO NOT RESOLVE IN A SIMULATION

BUT WE ADD THEM AS **SIMPLIFIED NON-RESOLVED RECIPES**

1. RADIATIVE TRANSFER: some notes from your previous courses

$$I_\nu \, d\nu \, d\Omega \, dA \, dt$$



energy of photons with frequency over the range ($\nu - d\nu$, $\nu + d\nu$), propagating through the area dA in a solid angle $d\Omega$ around the direction ***n***.

NB: I_ν **= specific intensity**

(erg Hz^-1 sr^-1 cm^-2 s^-1)

→ **Equation of radiative transfer:**

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = -\kappa_\nu I_\nu + \eta_\nu$$

Variation of flux intensity in time

Variation of flux intensity in space

Absorption term

Source term

1. RADIATIVE TRANSFER

How can we implement RT in N-body codes?

i- LINE RT: only 1 emission line has to be integrated
ii- CONTINUUM RT: entire spectrum of photons has to be integrated

TECHNIQUES:

1- RAY BASED SCHEMES: approximate I_ν as a function of optical depth τ along rays cast from the source

PROS: very accurate angular treatment
CONS: too many rays!

2- MOMENTUM BASED SCHEMES: reduce the angular dimensions by taking angular moments of the RT equation

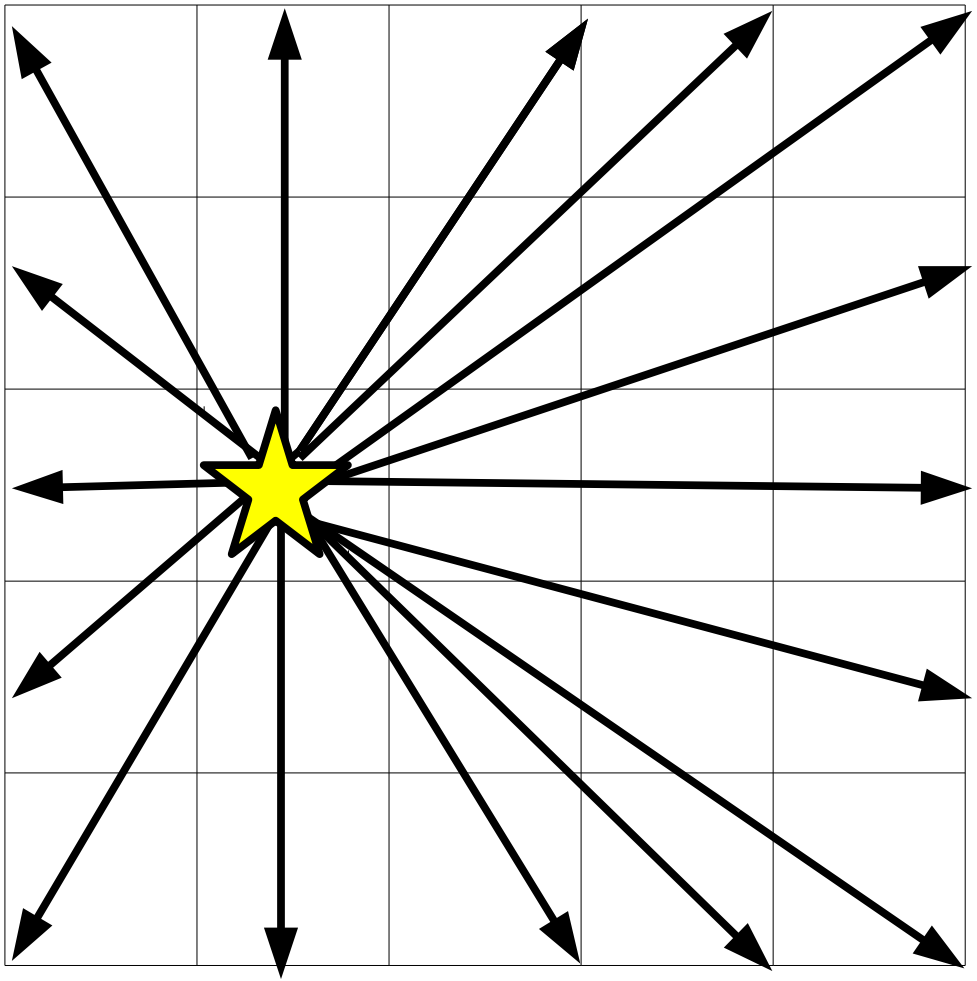
PROS: faster and simpler
CONS: loses angular accuracy (no anisotropy)

1. RT: RAY BASED SCHEMES:

RESOLUTION is given by how many rays cross a cell!

LONG CHARACTERISTICS:

from each source a ray goes directly to each cell



Very accurate but very slow

Too many rays through the cells closer to the source

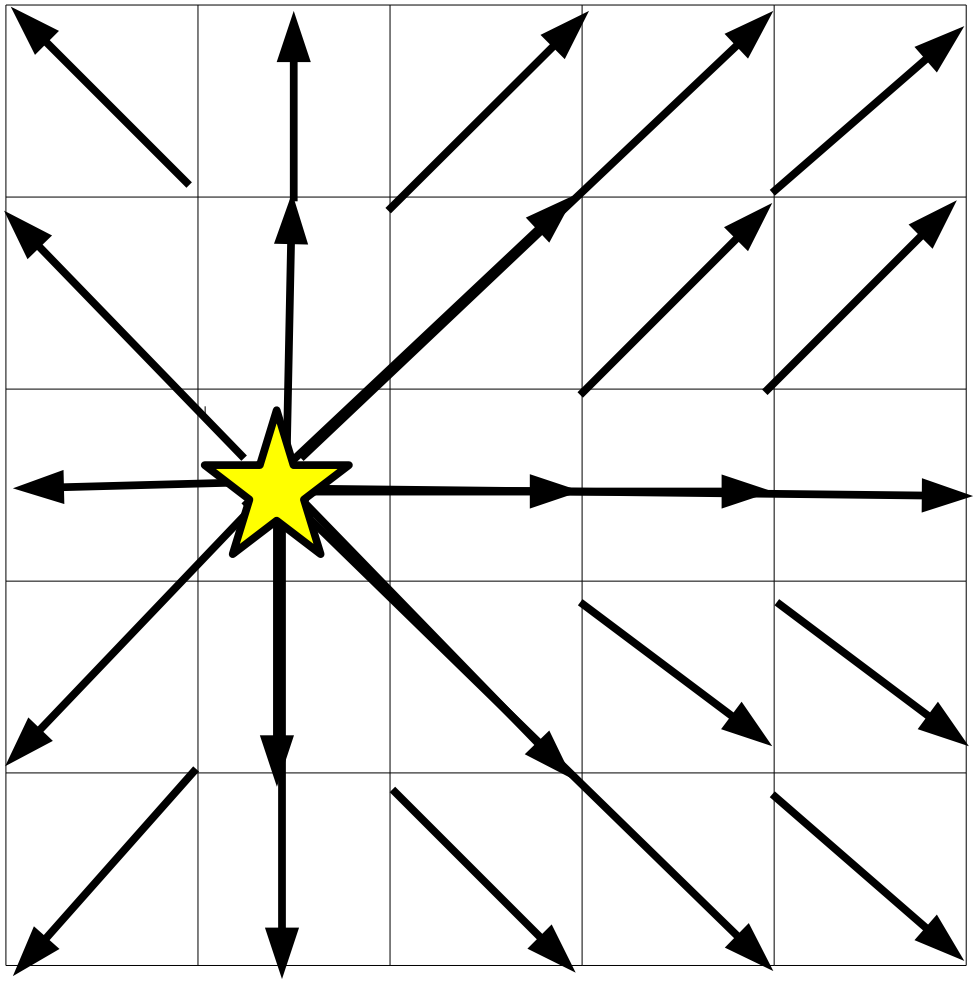
→ needs something to reduce the rays close to the source without reducing rays far from the source

Abel, Norman & Madau (1999), Cen (2002), Susa (2006).

1. RT: RAY BASED SCHEMES:

SHORT CHARACTERISTICS:

each ray goes from one cell to another.
Children rays inherit properties from parent rays



less accurate
still quite slow

PROS:
No redundancy: constant resolution of RT among cells

CONS:
We must interpolate information between cells →
misses some information

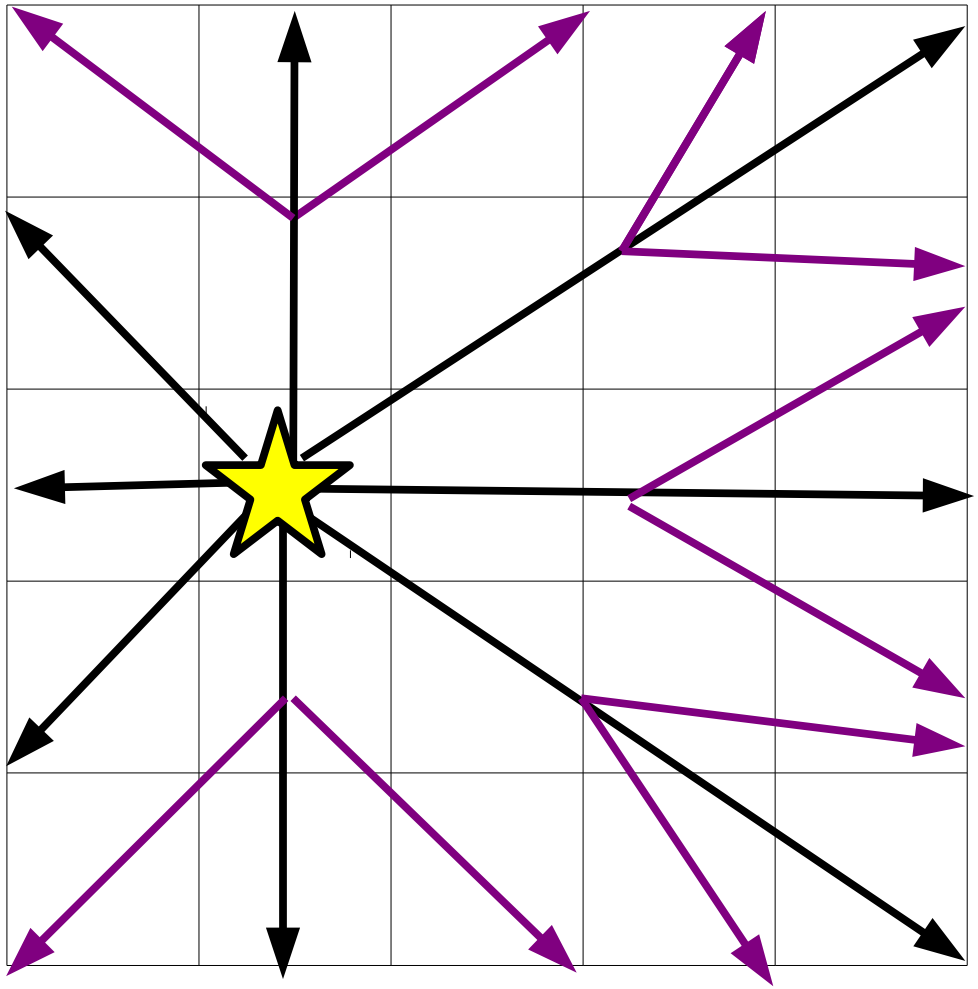
Nakamoto, Umemura & Susa (2001), Mellema et al. (2006), Whalen & Norman (2006), Alvarez, Bromm & Shapiro (2006).

1. RT: RAY BASED SCHEMES:

ADAPTIVE RAY TRACING:

Intermediate between long and short characteristics

Rays are cast from the source but can split into children rays to increase resolution.



→ = parent ray

→ = child ray

PROS:

No redundancy: constant resolution of RT among cells

Rays are split only when needed

CONS:

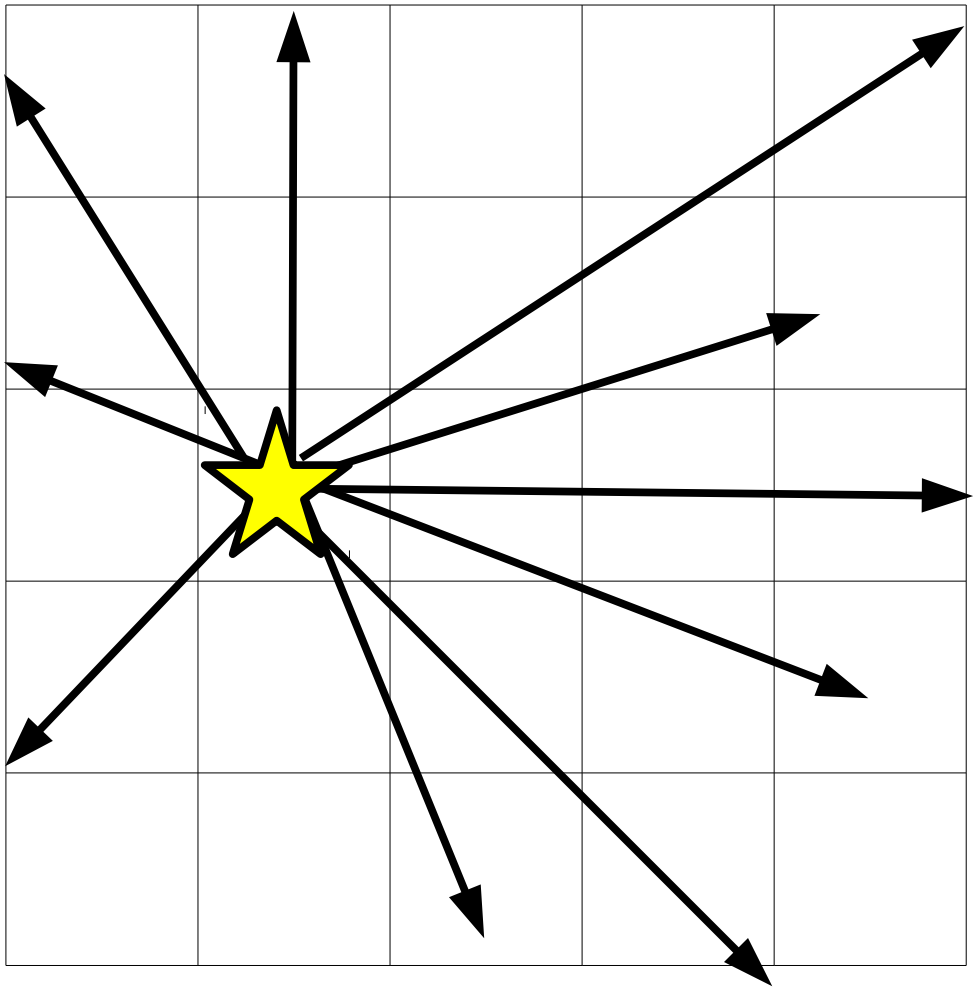
We must interpolate information between cells → misses some information

Abel & Wandelt 2002; Razoumov & Cardall 2005; Wise & Abel 2011)

1. RT: RAY BASED SCHEMES:

MONTE CARLO:

Rays are sampled through Monte Carlo technique



PROS:
Faster

CONS:
Loses information

Ciardi et al. (2001), Maselli, Ferrara & Ciardi (2003), Altay, Croft & Pelupessy (2008), Baek et al. (2009), Cantalupo & Porciani (2011).

1. RT: MOMENTUM-BASED SCHEMES

Energy $E\nu$ = 0-th moment of intensity, Flux $F\nu$ = 1-st moment of intensity, pressure $P\nu$= 2-nd moment of intensity

Rewrite RT equation in terms of 0-th, 1-st, 2-nd moment of intensity

PROS: MUCH MUCH FASTER

CONS: LOSE DIRECTIONALITY OF PHOTONS

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

Energy *Eν* = 0-th moment of intensity, Flux *Fν* = 1-st moment of intensity, pressure *Pν*= 2-nd moment of intensity

Rewrite RT equation in terms of 0-th, 1-st, 2-nd moment of intensity:

$$\frac{\partial E_\nu}{\partial t} + \nabla F_\nu = -\kappa_\nu \, c \, E_\nu + S_\nu$$

$$\frac{\partial F_\nu}{\partial t} + c^2 \nabla P_\nu = -\kappa_\nu \, c \, F_\nu$$

The algorithm is simpler for discretized number of photons

$$N_\nu = E_\nu / (h\nu)$$

$$\frac{\partial N_\nu}{\partial t} + \nabla \tilde{F}_\nu = -\kappa_\nu \, c \, N_\nu + \tilde{S}_\nu$$

$$\frac{\partial \tilde{F}_\nu}{\partial t} + c^2 \nabla \tilde{P}_\nu = -\kappa_\nu \, c \, \tilde{F}_\nu$$

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

For the source term I distinguish: energy emitted from the source, energy that comes from recombinations (scattered material):

$$S_\nu = \dot{N}^*{}_\nu + \dot{N}^{\rm rec}_\nu$$

If I assume that all my gas is Hydrogen (neglect He and metals):

$$\frac{\partial N_{13.6}}{\partial t} + \nabla F_{13.6} = -n_{H0}\, c\, \sigma_{13.6}\, N_{13.6} + \dot{N}^*_{13.6} + \dot{N}^{\rm rec}_{13.6}$$

$$\frac{\partial F_{13.6}}{\partial t} + c^2 \nabla P_{13.6} = -n_{H0}\, c\, \sigma_{13.6}\, F_{13.6}$$

with $$N_{13.6} = \int_{13.6\,{\rm eV}}^{\infty} N_\nu d\nu$$

$$\sigma_{13.6}\, N_{13.6} = \int_{13.6\,{\rm eV}}^{\infty} \sigma_\nu\, N_\nu d\nu$$

NB: if I have other species this becomes a sum!!!

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

FINALLY THE ALGORITHM:

OPERATOR SPLITTING = we decompose the full equation in multiple steps

1. stellar source step (ionizing photons from the star)

2. transport step (solves RT equation in conservative form)

3. thermo-chemical step (solves the right-hand side of RT equation, together with the evolution of neutral hydrogen density and gas temperature)

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

FINALLY THE ALGORITHM:

OPERATOR SPLITTING = we decompose the full equation in multiple steps

1. stellar source step (ionizing photons from the star):

we perform in each cell of the computational grid, indexed i, the following update:

$$N_i^{n+1} = N_i^n + \dot{N}^* \Delta t$$

Number of ionizing (>13.6eV) photons in cell i at time n+1

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

FINALLY THE ALGORITHM:

OPERATOR SPLITTING = we decompose the full equation in multiple steps

2. transport step:

$$\frac{\partial N}{\partial t} + \nabla F = 0 \qquad \frac{\partial F}{\partial t} + c^2 \nabla P = 0$$

$$\frac{N_i^{n+1} - N_i^n}{\Delta t} + \frac{F_{i+1/2}^m - F_{i-1/2}^m}{\Delta x} = 0$$

$$\frac{F_i^{n+1} - F_i^n}{\Delta t} + c^2 \frac{P_{i+1/2}^m - P_{i-1/2}^m}{\Delta x} = 0$$

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

FINALLY THE ALGORITHM:

OPERATOR SPLITTING = we decompose the full equation in multiple steps

3. thermo-chemical step: must solve following equations

$$\frac{\partial N}{\partial t} = -n_{H0}\, c\, \sigma\, N + n_e\, n_{H+} \left(\alpha_A - \alpha_B\right)$$

Recombination coefficients

$$\frac{\partial F}{\partial t} = -n_{H0}\, c\, \sigma\, F$$

Rate of collis. ionizations

$$\frac{\partial n_{H0}}{\partial t} = -n_{H0}\, c\, \sigma\, N + \alpha_A\, n_e\, n_{H+} - \beta\, n_e\, n_{H0}$$

thermal energy **cooling terms**

$$\frac{\partial e}{\partial t} = n_{H0}\, c\, \sigma\, \epsilon\, N - n_{H0}\, n_e\, \Lambda_{e\,H0} - n_{H+}\, n_e\, \Lambda_{eH+}$$

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

1. RT: MOMENTUM-BASED SCHEMES

FINALLY THE ALGORITHM:

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1. stellar source step (ionizing photons from the star)

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I DON'T NEED TO SAY THAT THIS PROCEDURE SLOWS DOWN THE CODE REMARKABLY.....

(eg RAMSES, Aubert & Teyssier 2008; Rosdahl & Teyssier 2013)

2. Star formation

Most methods based on Schmidt law:

$$\rho_{\rm SFR} = C\,\rho_{\rm gas}^{1.5}$$

Example (ChaNGa):

A gas particle is eligible for SF if

(i) the particle is denser than nmin = threshold (~0.1 cm^-3),

(ii) the particle is in an overdense region,

(iii) the particle is part of a converging flow,

(iv) the particle is Jeans unstable – $(h_i / c_i) > [1/(4\pi G \rho_i)]^{1/2}$

where *hi* and *ci* are the smoothing length and sound speed of i- particle

But fraction of gas particle mass that must be converted to stars is too small with respect to particle mass

→ only some of the gas particles that are eligible to star formation are converted to star according to a STOCHASTIC PROCESS

Stinson et al. (2006)

2. Star formation

STOCHASTIC PROCESS:
For a gas particle that satisfies i-iv
probability *p* is estimated

$$p = \frac{m_{\rm gas}}{m_{\rm star}} \left[1 - \exp\left(-\epsilon \, \Delta t / t_{\rm dyn}\right)\right]$$

Mass of gas particle over mass of future star particle

Efficiency dimen-sionless coefficient

timestep

dynamical timescale

If *p* > random number (0,1)
the gas particle is converted into star

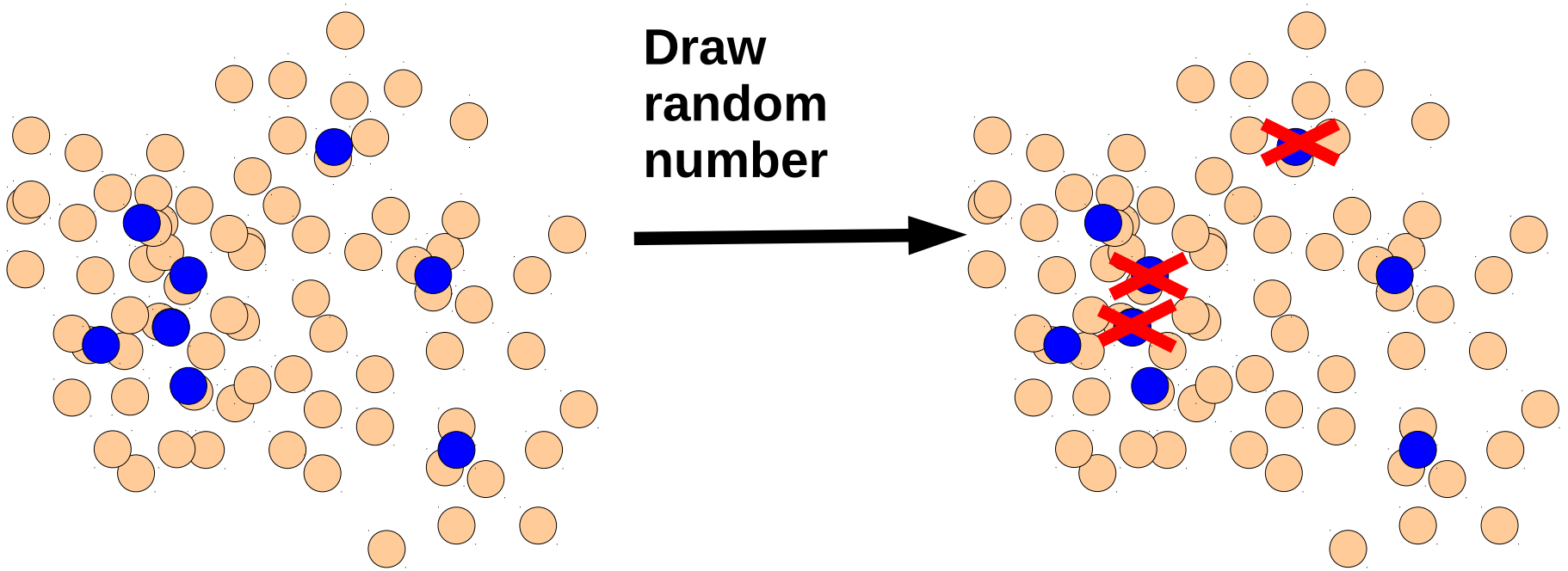
This method enforces Schmidt law

Stinson et al. (2006)

2. Star formation

Estimate criteria i – iv

Transform to star particles those particles with random number < probability



- **Orange: all gas particles in the simulation**
- **Blue marks gas particles that are eligible to become stars according to criteria i-iv**
- **Red cross marks gas particles for which random number < probability**

Stinson et al. (2006)

2. Star formation for small-scale simulations: SINK PARTICLES

Recipes with Schmidt law won't wok
If gas particle mass <~ 1 Msun & simulation box < kpc
i.e. for MOLECULAR CLOUD, STAR CLUSTER or SMALL GALAXY SIMULATIONS!

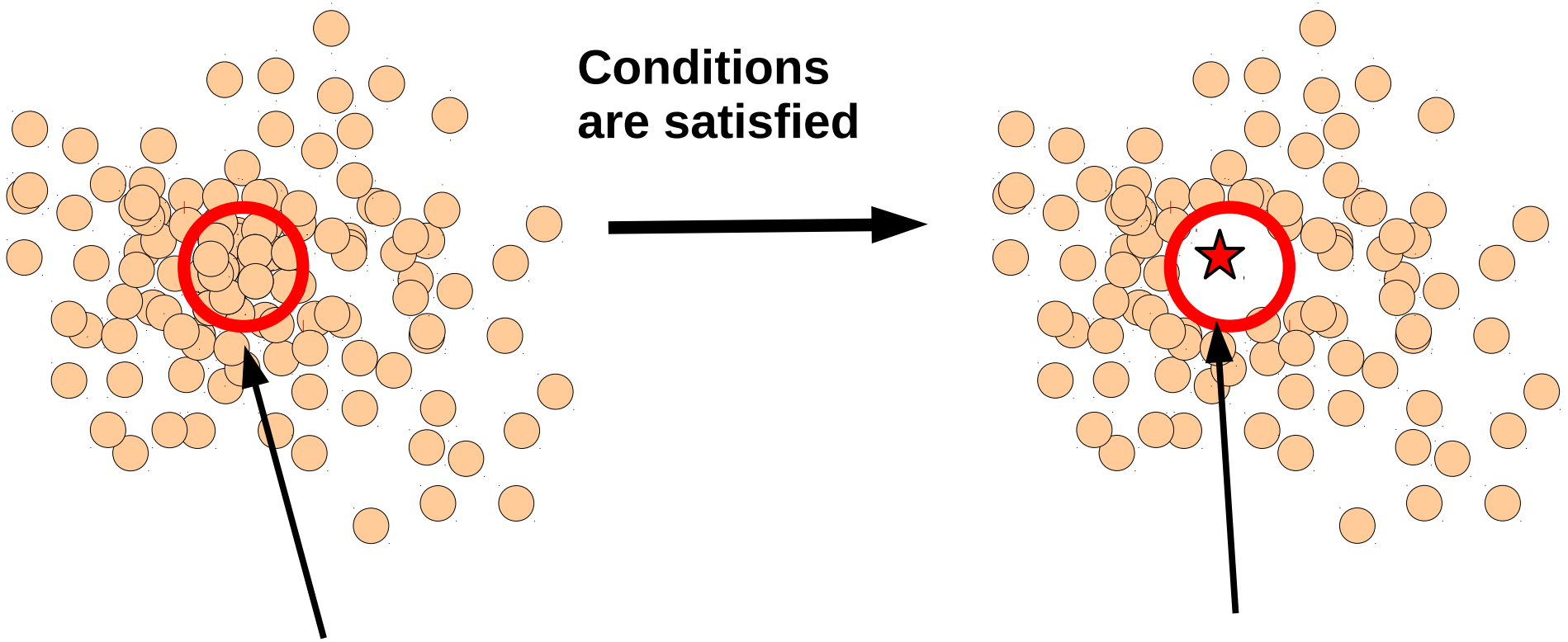
→ SINK PARTICLE TECHNIQUE (e.g. Bate, Bonnell & Price 1995; Bleuler & Teyssier 2014):

A clump of gas particles (or cells) is converted to a STAR PARTICLE if

1. **density is above user-provided threshold,**
2. **the clump will continue to collapse if not replaced by sink:**
   - **U / W < 0.5**
     (where U=thermal energy, W= gravitational energy)
   - **TOTAL ENERGY < 0**
   - **$\nabla \cdot a < 0$ negative divergence of accelerations**
     (clump of particles is not being disrupted)
3. **other requirements**

2. Star formation for small-scale simulations: SINK PARTICLES

SINK PARTICLE TECHNIQUE (e.g. Bate, Bonnell & Price 1995; Bleuler & Teyssier 2014):



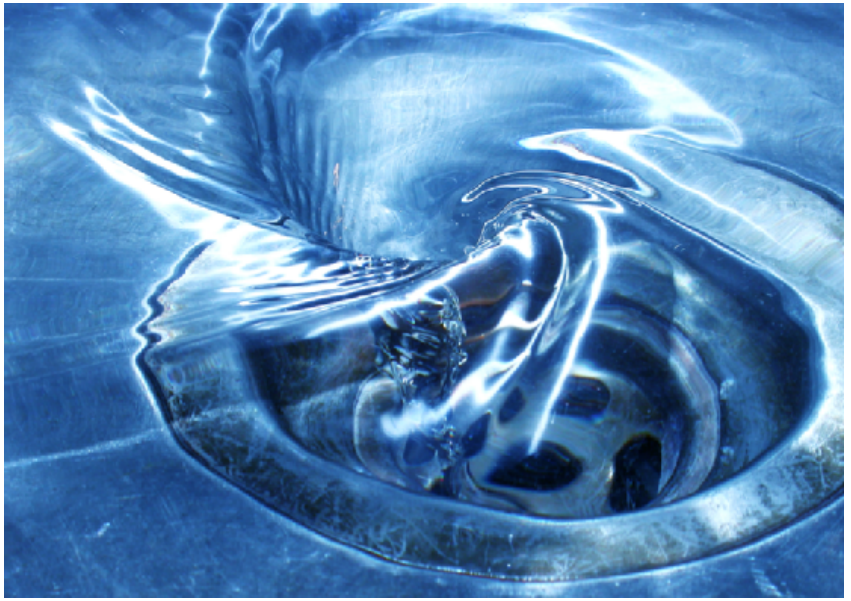
Circle within which I check if conditions are satisfied (~ smoothing length)

Sink particle created in the centre of mass with mass= total mass of clump

2. Star formation for small-scale simulations: SINK PARTICLES

SINK PARTICLE TECHNIQUE (e.g. Bate, Bonnell & Price 1995; Bleuler & Teyssier 2014):

A sink is equal to a STAR:
affected only by GRAVITY
not by Euler equations!



Called SINKS
because they can eat further gas (used both for accreting proto-stars and for black holes)

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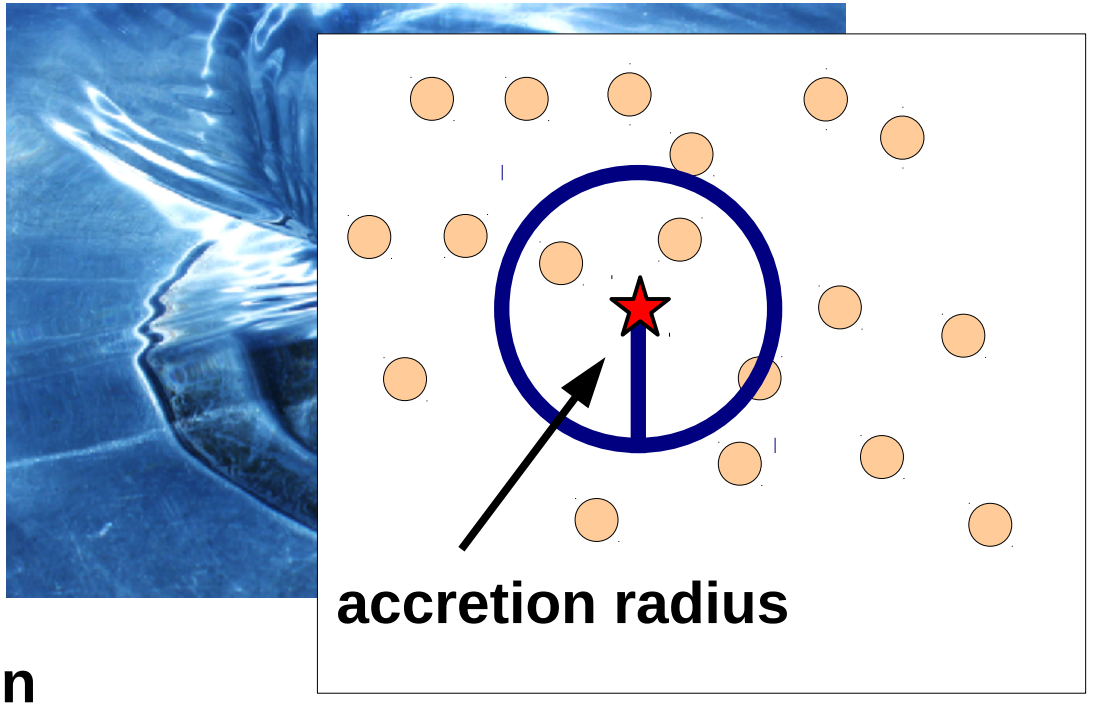


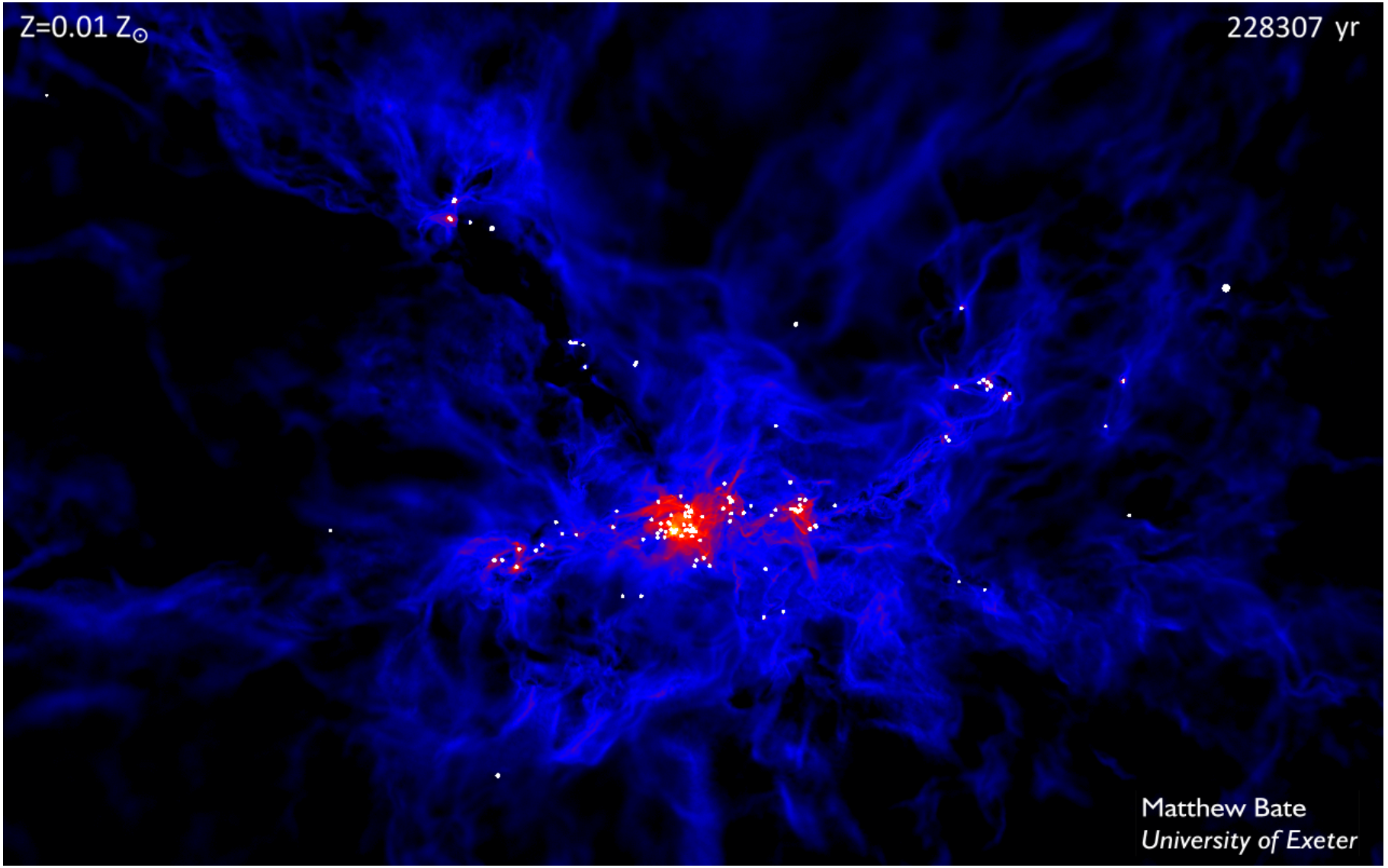
Called SINKS because they can eat further gas (used both for accreting proto-stars and for black holes)

A GAS PARTICLE (or cell) is eaten (= deleted from the simulation and its mass transferred to the sink) IF

1. **enters the (user-provided) accretion radius of the sink**
2. **is bound to the sink**
3. **its specific angular momentum is less than required for a circular orbit around the sink**
4. **is more bound to that sink than to any other sink**
5. **any further criterion depending on the code.**

2. Star formation for small-scale simulations: SINK PARTICLES

SINK PARTICLE TECHNIQUE (e.g. Bate, Bonnell & Price 1995; Bleuler & Teyssier 2014):



3. COOLING

Gas can be treated as

- ADIABATIC: no exchange of heat

- THERMAL: temperature of gas is constant

- with COOLING (heating) function:
A function is added to GAS EQUATIONS that accounts for cooling or heating processes

e.g.

$$\frac{de}{dt} = -\frac{P}{\rho}\nabla \cdot \mathbf{v} - \frac{\Lambda(e,\,\rho)}{\rho}$$

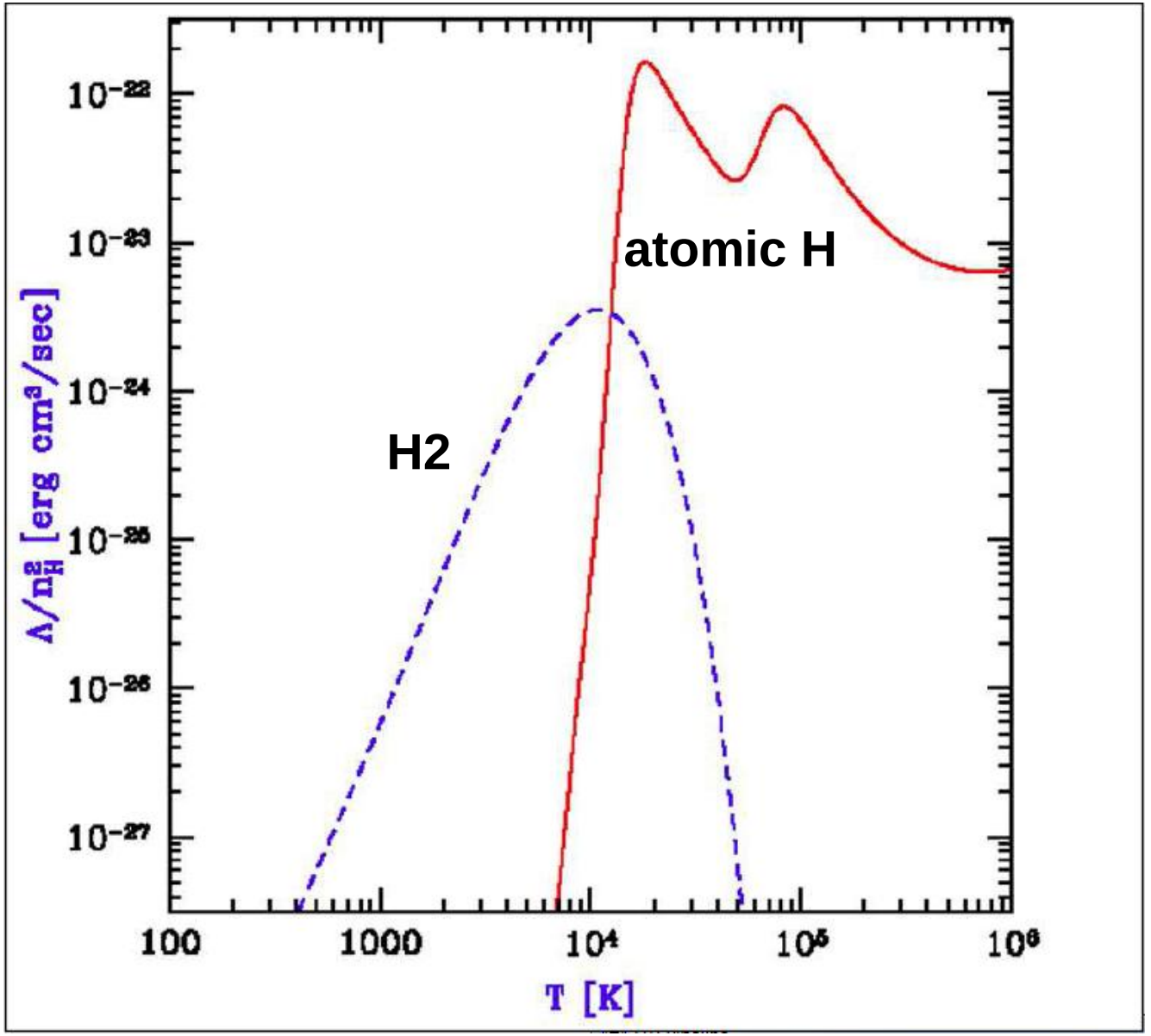
specific thermal energy

cooling function

3. COOLING

What is a cooling function?

- If NO METALS then H2, atomic H:

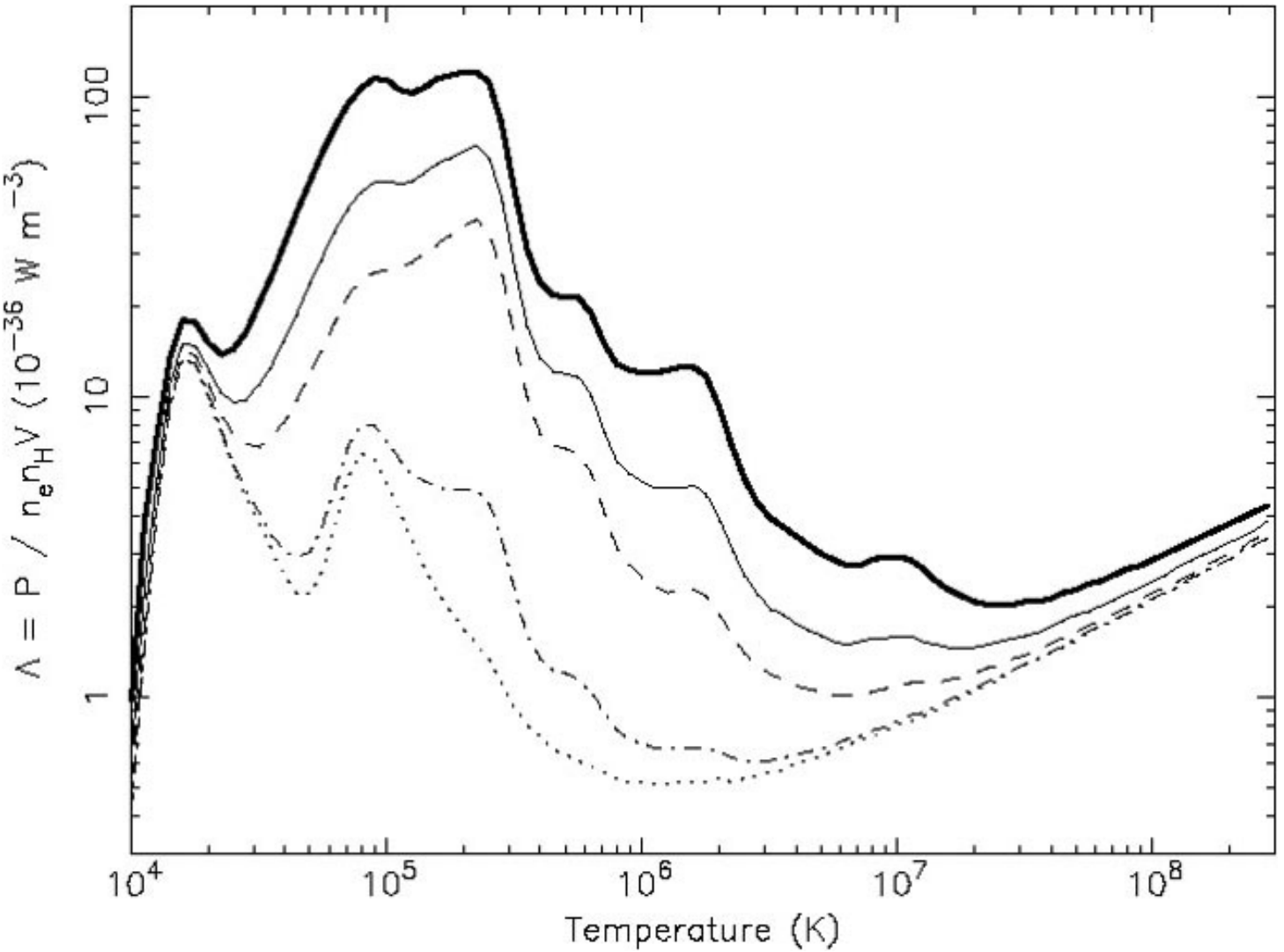


3. COOLING

What is a cooling function?

- If METALS they dominate high Temperature cooling (>10^5 K)

Different curves for different metallicity



Sutherland & Dopita 1993

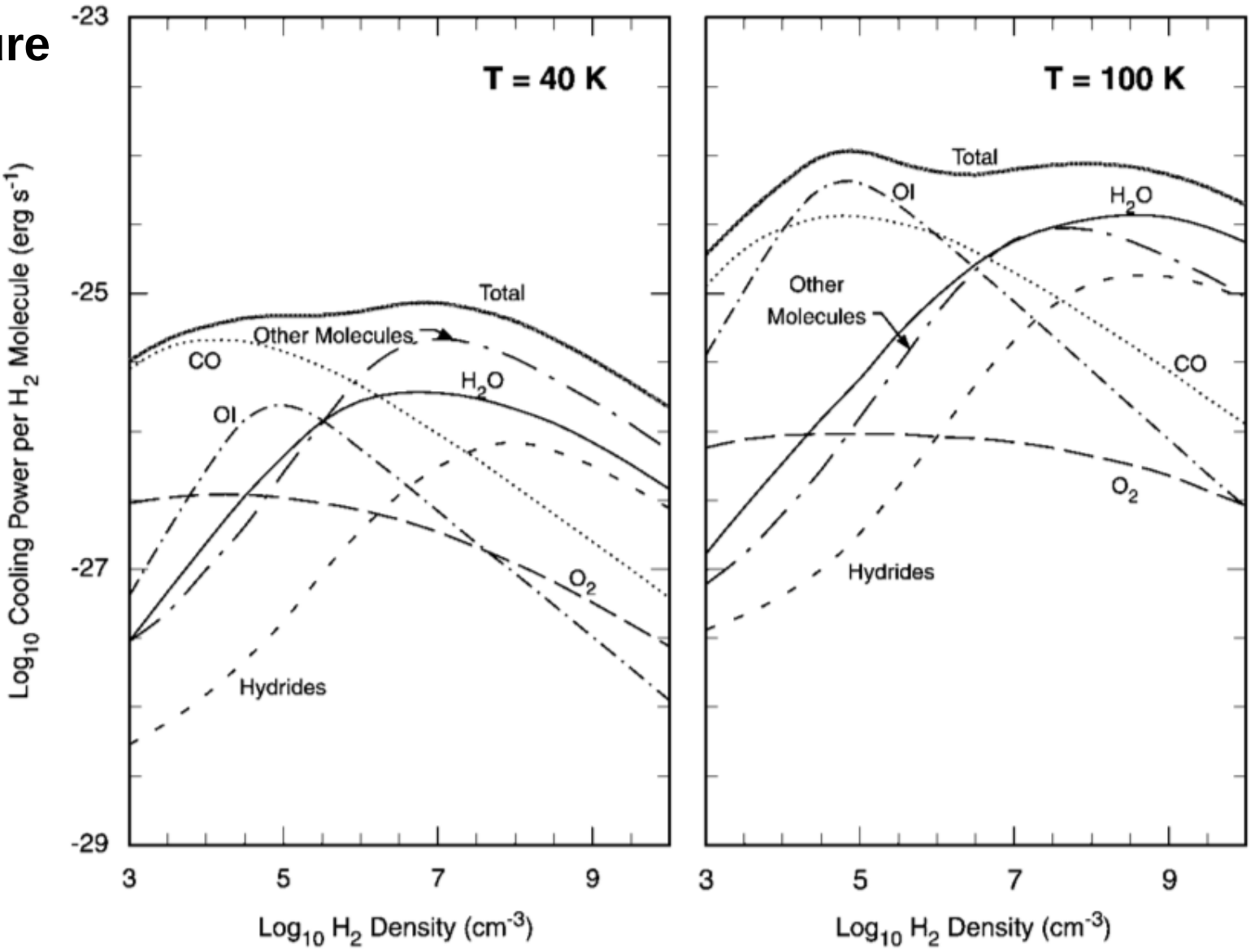
3. COOLING

What is a cooling function?

- Low temperature (<few x 10^3 K) non-zero metallicity

DUST and MOLECULE COOLING is DOMINANT

Depends on opacity!!!



4. Supernovae:

Only SNII because they are the most energetic
Energy ejected is (i) kinetic, (ii) thermal

Some recipes include both, but for galactic scale or cosmological simulations (NO smaller scales) kinetic feedback converts to thermal in << 1 timestep

→ ONLY THERMAL FEEDBACK

Procedure:

1. Associate single stellar population (SSP) recipes to each star particle: a star particle is decomposed into stars with a IMF and LIFETIMES are calculated

→ we know the number of SNe that explode at time t N_{SN}

2. Energy N_{SN} E_{SN}= f 10^{51} erg with f<1
is distributed to neighbor gas particles
E.g. for SPH the criterion is:

$$\Delta E_{SN,i} = \frac{m_i \, W(|\mathbf{r}_i - \mathbf{r}_{SN}|, h_{SN}) \, \Delta E_{SN}}{\sum_{j=1}^{N} m_j \, W(|\mathbf{r}_j - \mathbf{r}_{SN}|, h_{SN})}$$

4. Supernovae:

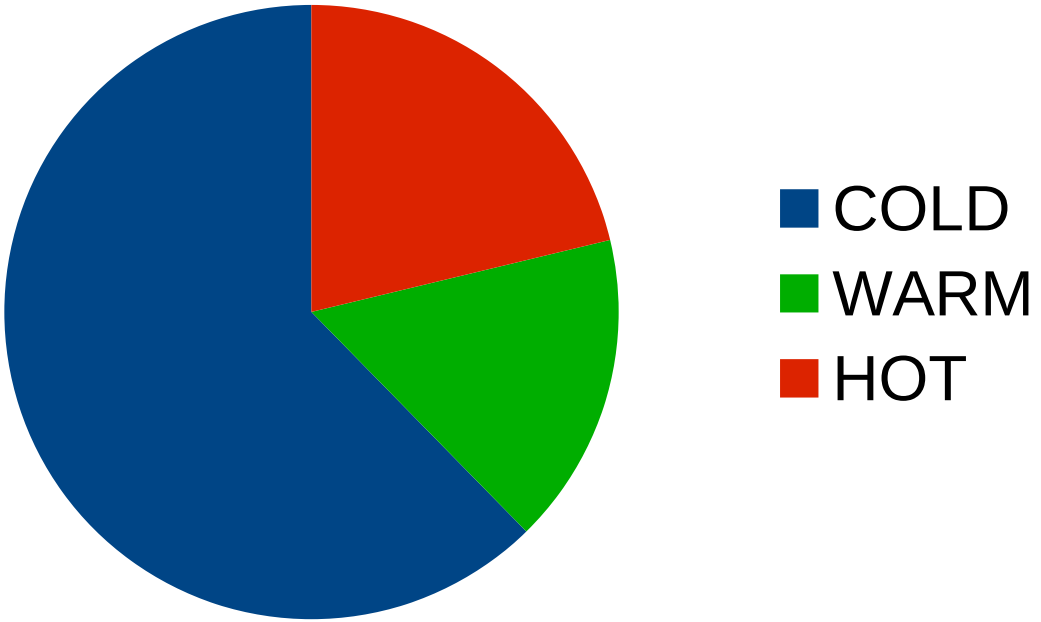
Overcooling:

In most codes this causes OVERCOOLING
Energy is radiated away too fast and SN does not have effects

Possible solutions:

- COOLING SWITCH OFF (aka blastwave mechanism): cooling is switched off for a time t (depending on the density and pressure), Stinson et al. 2006

- IF MULTIPHASE MEDIUM (hot, warm, cold, gas), SN energy is injected only in the HOT medium (e.g. Murante et al. 2015)



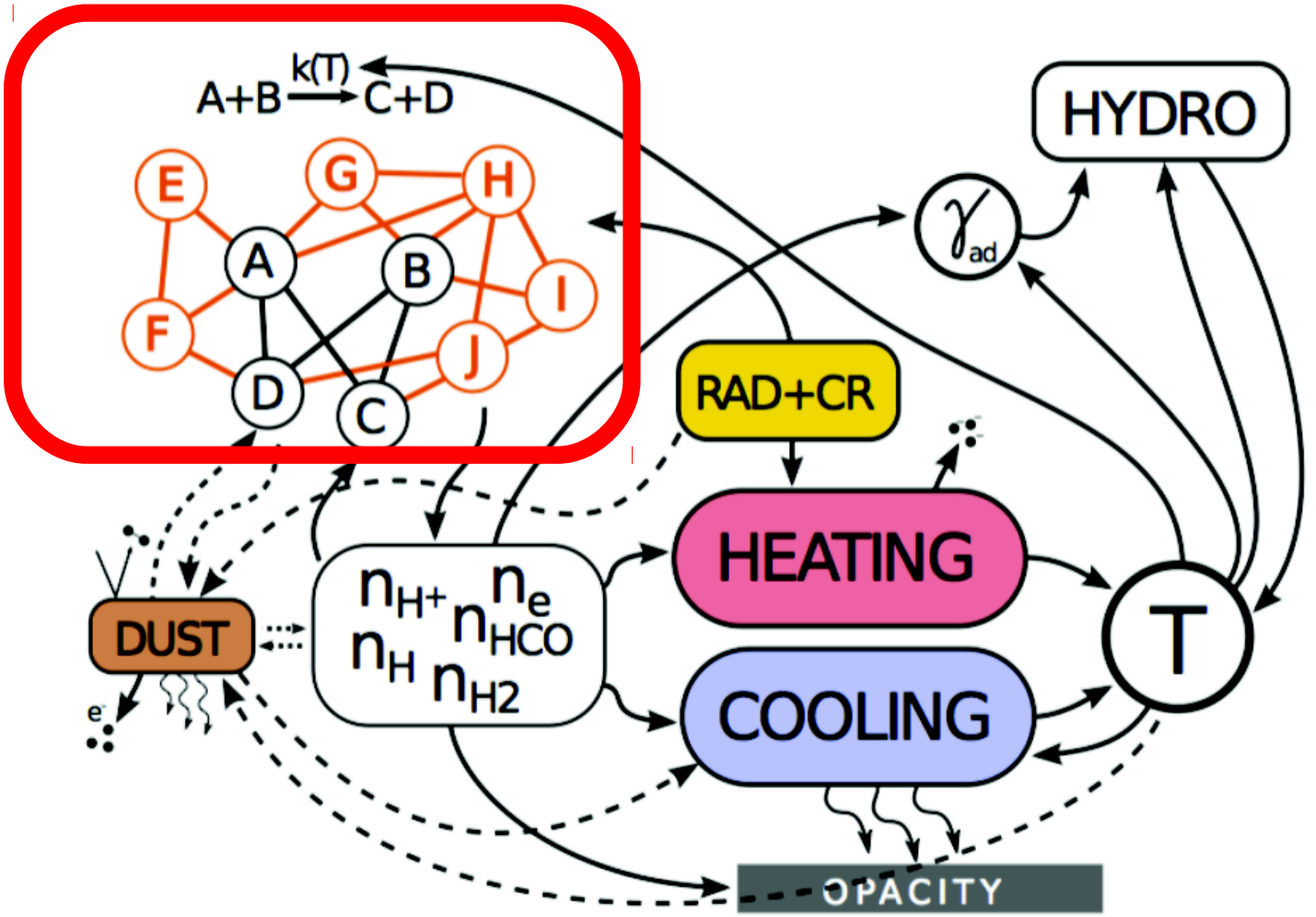
Solving 3 different Euler equations per particle

5. Non-equilibrium chemistry:

Cooling described so far assumes equilibrium (ionizations=recombinations), Saha equation

What if NO EQUILIBRIUM?
Chemical networks must be solved together with Euler!!!

Chemical reactions are a live calculation in non-equilib. chemistry (substituted by average cooling/ heating function Λ in equilibrium)



Grassi, Bovino, Schleicher et al. (2014)

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What if NO EQUILIBRIUM?
Chemical networks must be solved together with Euler!!!

EXAMPLE of the simpler reaction: H2 formation from H

H_2^+ route:

$$H + H^+ \rightarrow H_2^+ + \gamma,$$
$$H_2^+ + H \rightarrow H_2 + H^+$$

H^- route:

$$H + e \rightarrow H^- + \gamma,$$
$$H^- + H \rightarrow H_2 + e.$$

Three-body:

$$H + H + H \rightarrow H_2 + H,$$
$$H + H + He \rightarrow H_2 + He,$$
$$H + H + H_2 \rightarrow H_2 + H_2.$$

5. Non-equilibrium chemistry:

Cooling described so far assumes equilibrium (ionizations=recombinations), Saha equation

What if NO EQUILIBRIUM?
Chemical networks must be solved together with Euler!!!

What is going on in a MOLECULAR CLOUD: Carbon + Oxygen!

From Szucs et al. 2014

Things you can do with

KROME package

Grassi et al. 2014

http://kromepackage.org/