Michela Mapelli

N-body techniques for astrophysics:
Lecture 1 – General Introduction

PhD School in Astrophysics,
University of Padova
November 3-12, 2015

A few words about the course:

<u>This course is NOT:</u> a programming course,
an advanced algorithm course

<u>The aim of this course:</u> to learn the basic concepts of state-of-the-art techniques of N-body simulations in astrophysics

<u>At the end of the course you should be able to:</u>

- if you are an observer / a technologist, you should be able to critically read and understand a paper about simulations in your field (star clusters, galaxies..) or maybe even run some simple simulation;

- if you are a theorist / you already do simulations, you might acquire experience in different fields (e.g. if you are a cosmologist, you will be able to run simulations of a star cluster)

<u>The lectures:</u>

7 lectures by me, last lecture by my collaborator Mario Spera

A few words about the course:

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Lectures will be published at

http://web.pd.astro.it/mapelli/lectures.html

Note: fill in the course evaluation sheet. It is anonymous, it helps me improving the lectures

Disclaimer: This course is still in the beta version – sorry for the trouble!

ASK MANY QUESTIONS, it is important for you and me!!!!

A few words about the course:

<u>The EXAM:</u>

→ preferred option: choose one code (among those I present or even others), choose a test simulation (e.g. a star cluster, a galaxy, a molecular cloud,...), run the test simulation, write a SHORT report on the simulation and discuss it with me

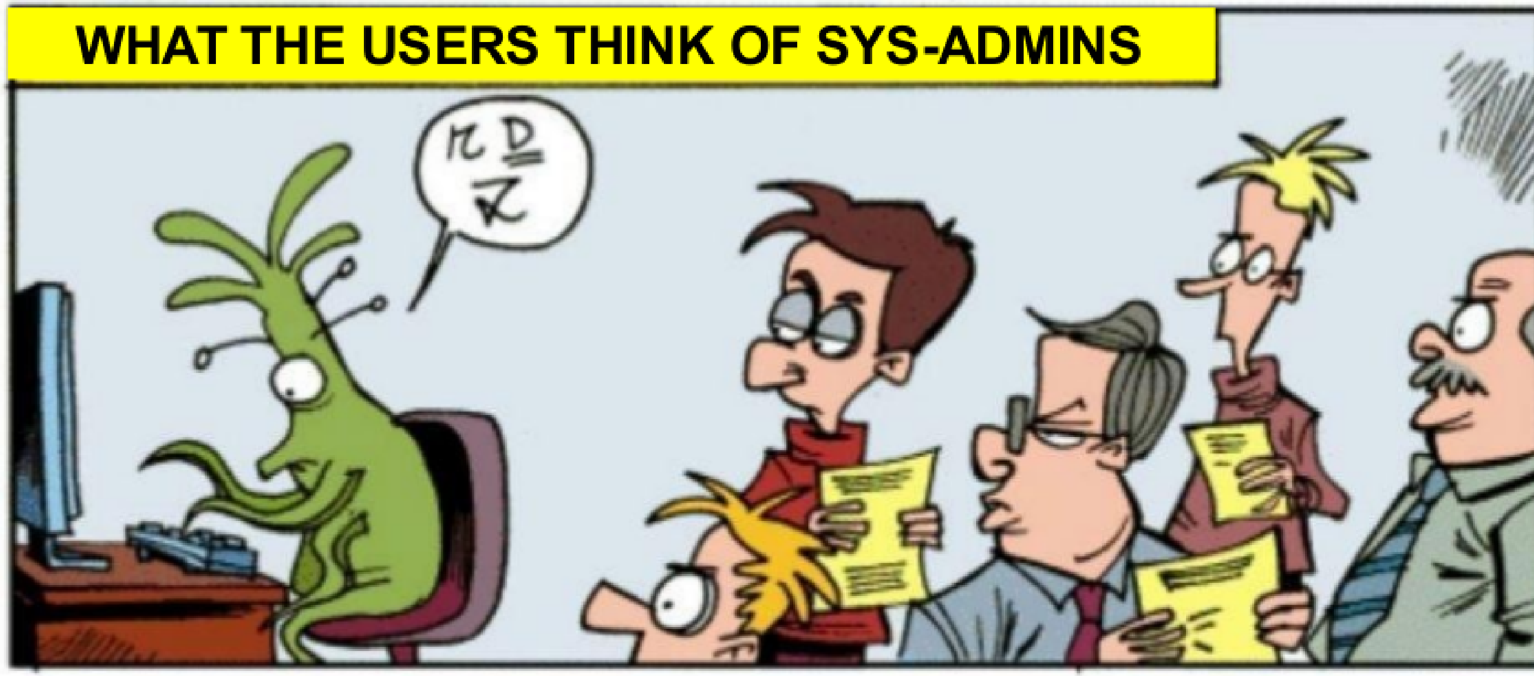
→ disaster option: choose a paper about N-body simulations and discuss it with me (if you really feel you cannot do the preferred option)

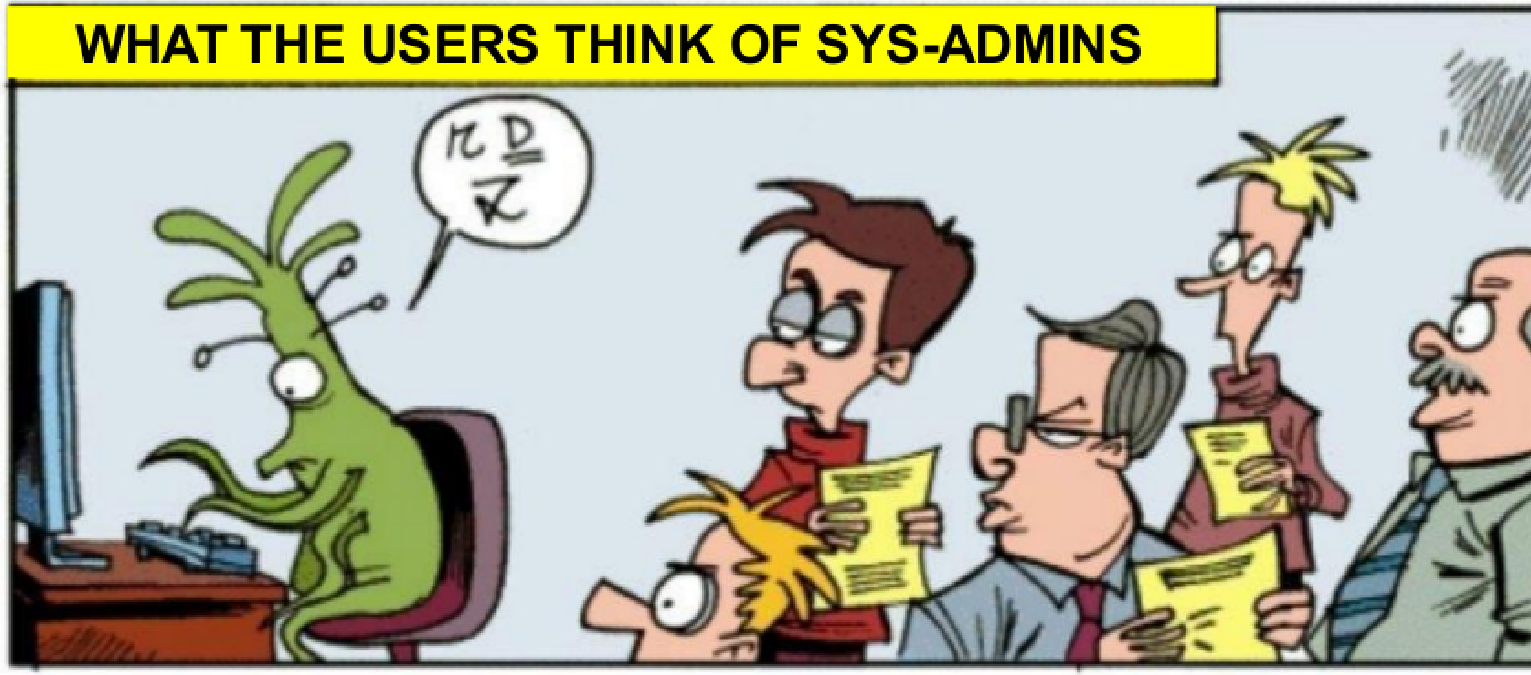
<u>The resources:</u>

you can either use your laptop, or your facilities (if you have computer time), or the computers in aula informatica

BUT DON'T MESS WITH THE SYSTEM MANAGERS, PLS

- don't unplug cables
- don't smash computers
- don't use the computers for something not related to the course


WHAT THE USERS THINK OF SYS-ADMINS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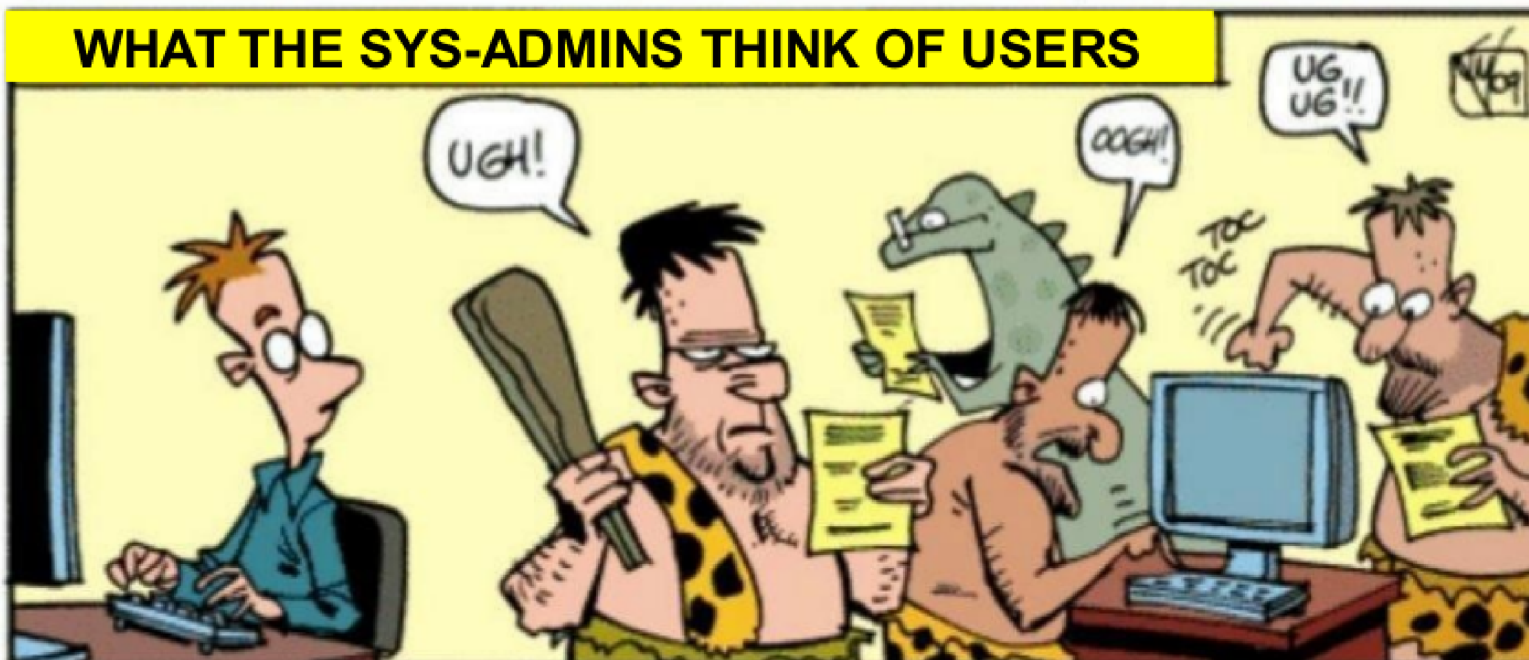

WHAT THE SYS-ADMINS THINK OF USERS
UGH!
OOGH!
TOC TOC
UG UG!!

OUTLINE of the LECTURES:

1. **WHAT IS an N-Body SIMULATION?**
   - **Definitions**
   - **Examples of solvers: Euler, Leapfrog**
   - **Computational complexity**
   - **N-body units!!**

2. **DIRECT N-body for COLLISIONAL SYSTEMS**
   - **DO WE NEED DIRECT N-BODY CODES?**
   - **Hermite integrator scheme**
   - **Time step**
   - **Regularization**
   - **Stellar evolution**
   - **Hardware: Grape – GPUs**
   - **Example codes: STARLAB, HiGPUs, Archain, BSE**

3. **N-body methods for collisionless systems:**

- Softening
- Tree code
- Particle Mesh and Fast Multipole codes
- Clusters of computers
- Example codes: ChanGa no gas

4. **GAS:**

- Equations of gas
- Smoothed-particle hydrodynamics (SPH) codes
- Mesh codes/ Adaptive mesh refinement codes (AMR)
- Example codes: ChanGa with gas, RAMSES

5. **OTHER PHYSICS:**

- Radiative transfer
- SNae
- Star formation

6. **INITIAL CONDITIONS:**

- Random sampling of a distribution function
- Examples

1. WHAT IS an N-Body SIMULATION?

?

1. WHAT IS an N-Body SIMULATION?

numerical integration of the forces acting on *N* particles for a time *t*

- ***astrophysics***
- ***fluid-dynamics***
- ***molecular dynamics***
- ***...***

WHAT IS an N-Body SIMULATION ** IN ASTROPHYSICS **?

numerical integration of the force of GRAVITY acting on *N* particles for a time *t*

1. **WHAT IS an N-Body SIMULATION?**

numerical integration of Newton equation

$$\ddot{\vec{r}}_i = -G \sum_{j \neq i} m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

Or the equivalent system of 2xNxndim 1st ord. differential eqs

$$\begin{cases} \dot{v}_i = -\sum \dfrac{G\, m_j}{r_{ij}^3} x_{ij} \\ x_i = v_i \end{cases}$$

DOES IT HAVE ANALYTIC SOLUTION?

1. WHAT IS an N-Body SIMULATION?

$$\ddot{\vec{r}}_i = -G \sum_{j \neq i} m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

- 1687: Newton finds the equation

- 1710: Bernoulli derives analytic solution for N=2

- 1885: a challenge was proposed to be answered before 21st January 1889, in honor of the 60th birthday of King Oscar II of Sweden and Norway.

'Given a system of arbitrarily many mass points that attract each according to Newton's law, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly.

NOBODY WON THE PRIZE

- 1991: Wang finds a convergent power series solution for a generic number of bodies. Mathematically correct, but too difficult and slow convergence

1. WHAT IS an N-Body SIMULATION?

- Analytic problem only for N=2 (and restricted N=3)

- gravity force does not fade off (even far away particle interact)
→ calculation of system cannot be decomposed in smaller pieces

$$\begin{cases} \dot{v}_i = -\sum \dfrac{G\, m_j}{r_{ij}^3} x_{ij} \\ x_i = v_i \end{cases}$$

A system of 2 x *N* x ndim differential equations with NO known analytic solution

→ FIND A NUMERICAL SOLUTION

WHAT WOULD YOU DO?

1. **WHAT IS an N-Body SIMULATION?**

TAYLOR EXPANSION

$$x_i(t+\Delta t) = x_i(t) + \frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\,\Delta t + \frac{1}{2}\,\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2}\,\Delta t^2 + O(\Delta t^3)$$

$$v_i(t+\Delta t) = v_i(t) + \frac{\mathrm{d}v_i(t)}{\mathrm{d}t}\,\Delta t + \frac{1}{2}\,\frac{\mathrm{d}^2 v_i(t)}{\mathrm{d}t^2}\,\Delta t^2 + O(\Delta t^3)$$

Predicting time t +Δt with info at time t

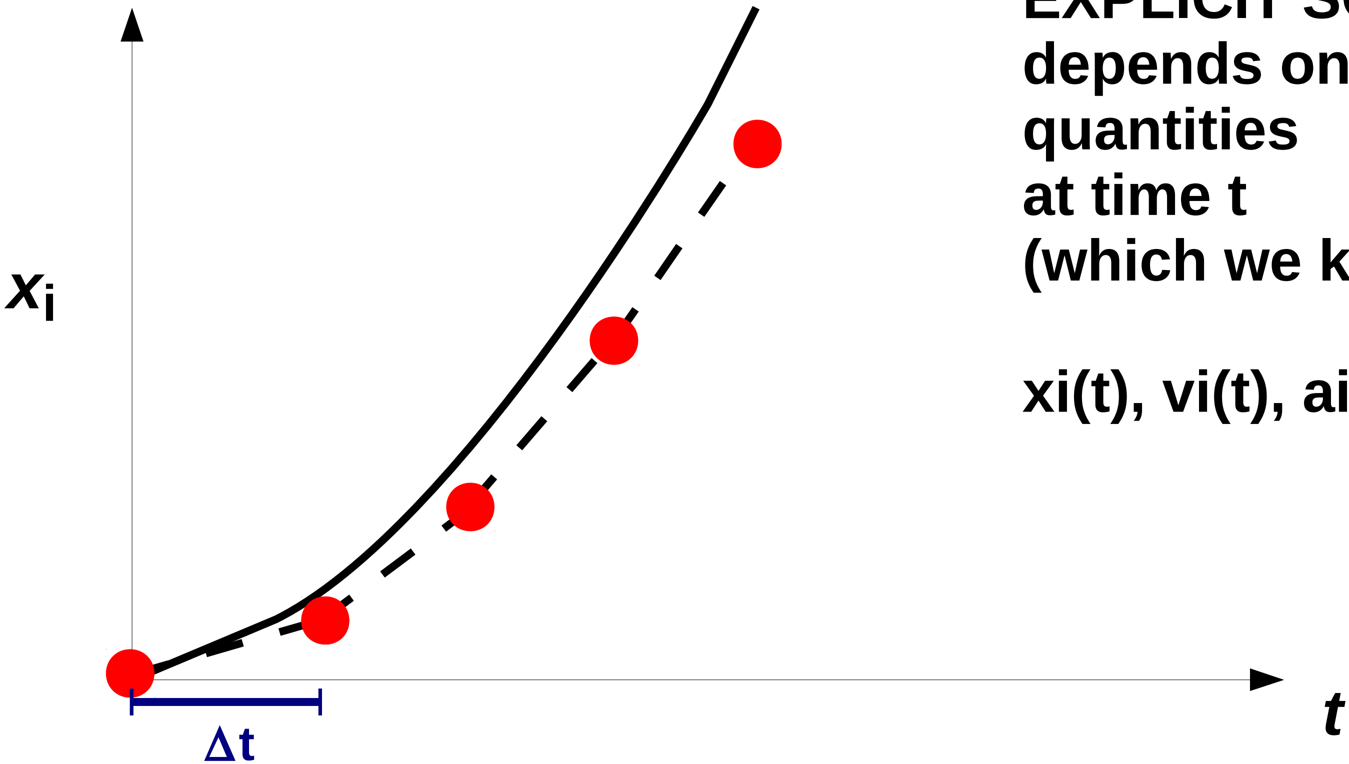
Truncation order gives error order

A 1st order method has Δt order errors
A 2nd order method has Δt^2 order errors
A 3rd order method has Δt^3 order errors
...

EULER METHOD: Taylor expansion at 1st order

$$x_i(t+\Delta t) = x_i(t) + \frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\Delta t$$

$$v_i(t+\Delta t) = v_i(t) + \frac{\mathrm{d}v_i(t)}{\mathrm{d}t}\Delta t$$



EXPLICIT SCHEME = depends only on quantities at time t (which we know)

xi(t), vi(t), ai(t)

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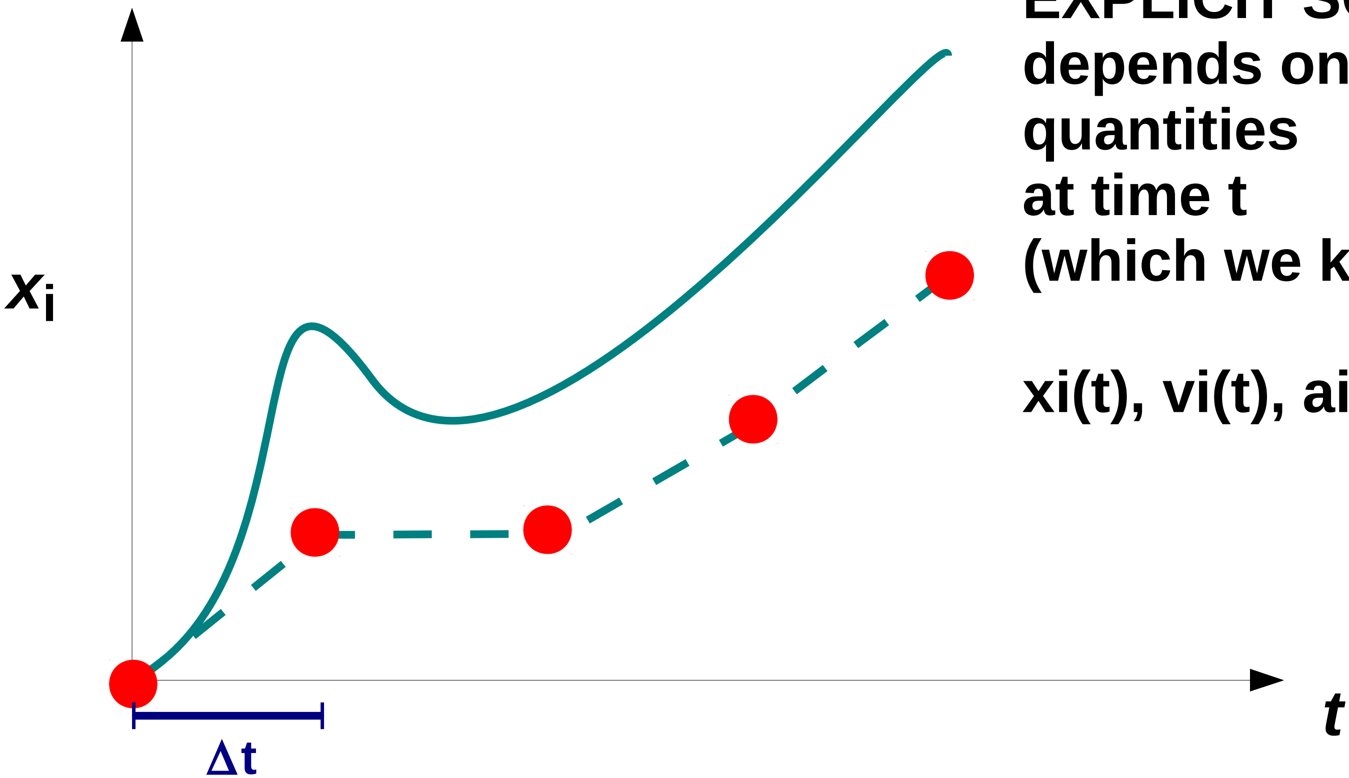


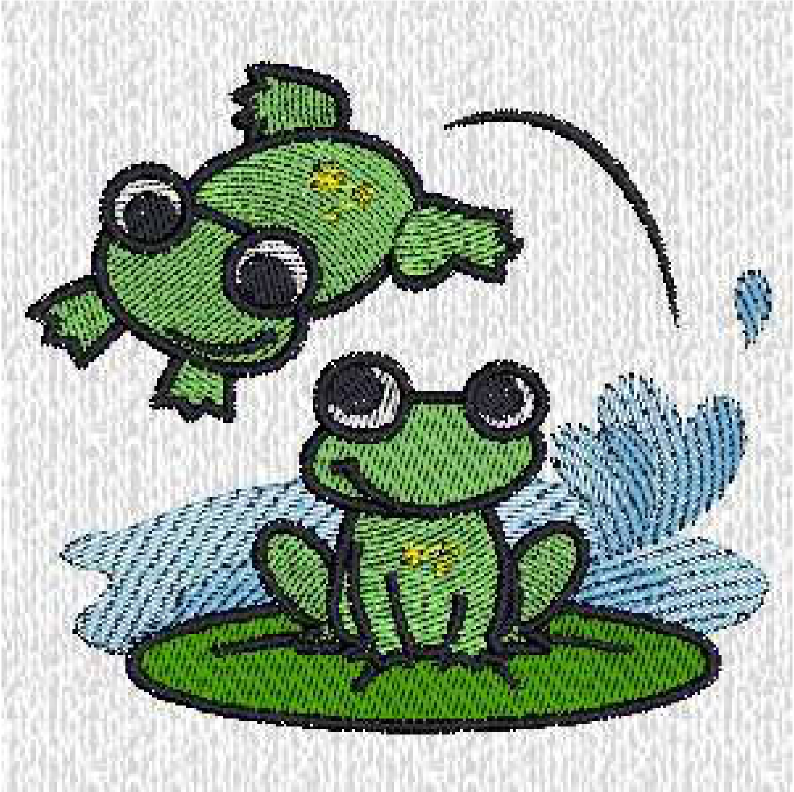
EXPLICIT SCHEME = depends only on quantities at time t (which we know)

xi(t), vi(t), ai(t)

1. **WHAT IS an N-Body SIMULATION?**

LEAPFROG METHOD:

The name comes from the leapfrog game

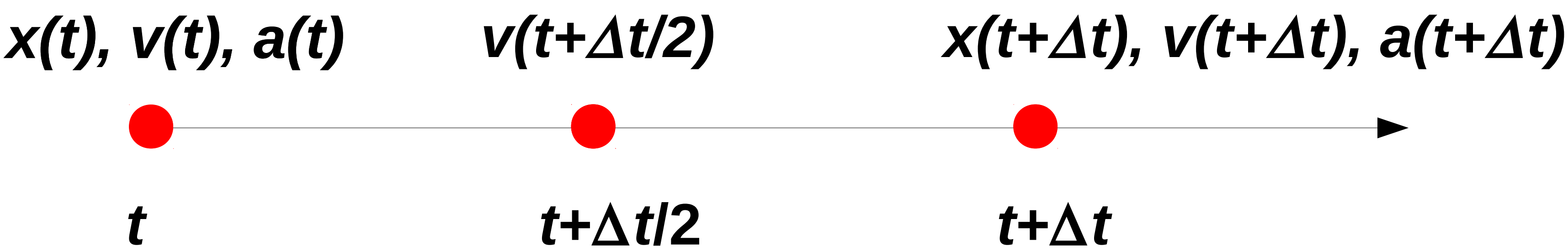


IDEA: evaluation of velocity and position jumps like little happy frogs within a timestep..

1. **WHAT IS an N-Body SIMULATION?**

LEAPFROG METHOD:

same as Euler but evaluated in between a timestep



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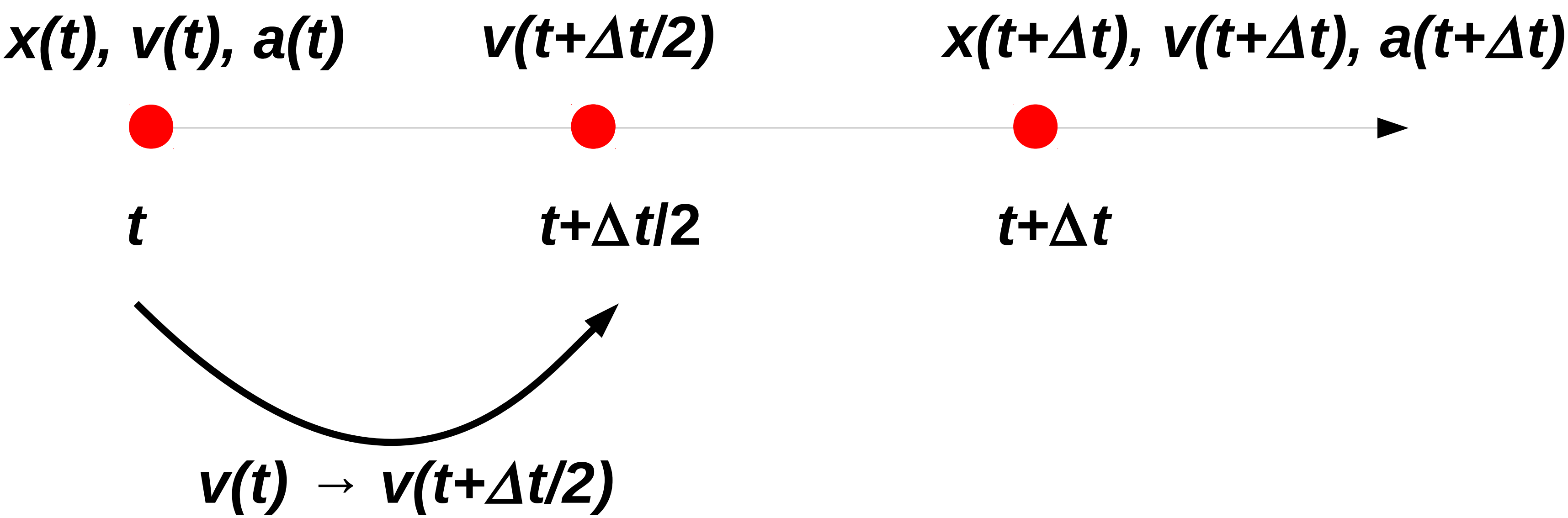

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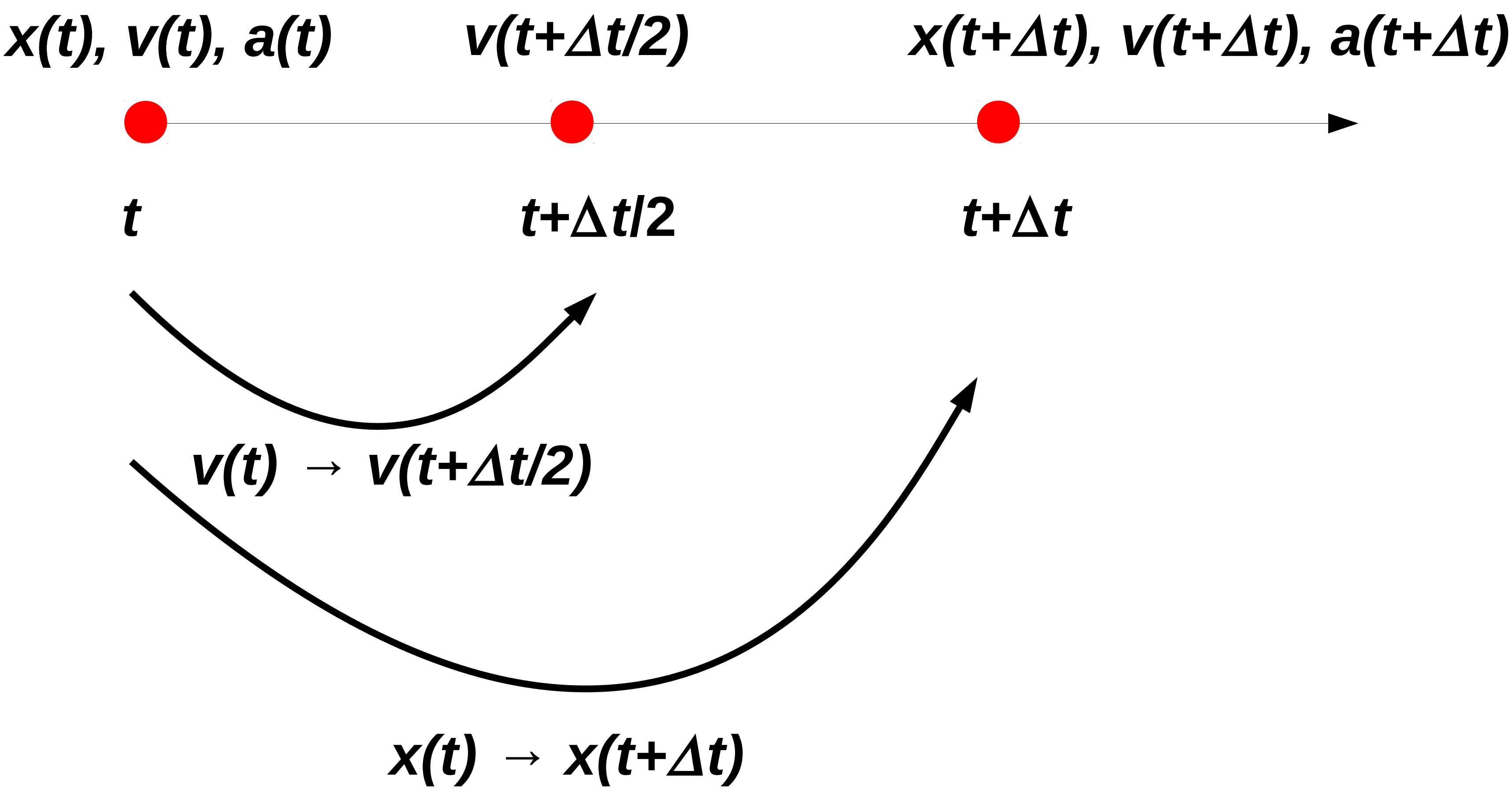

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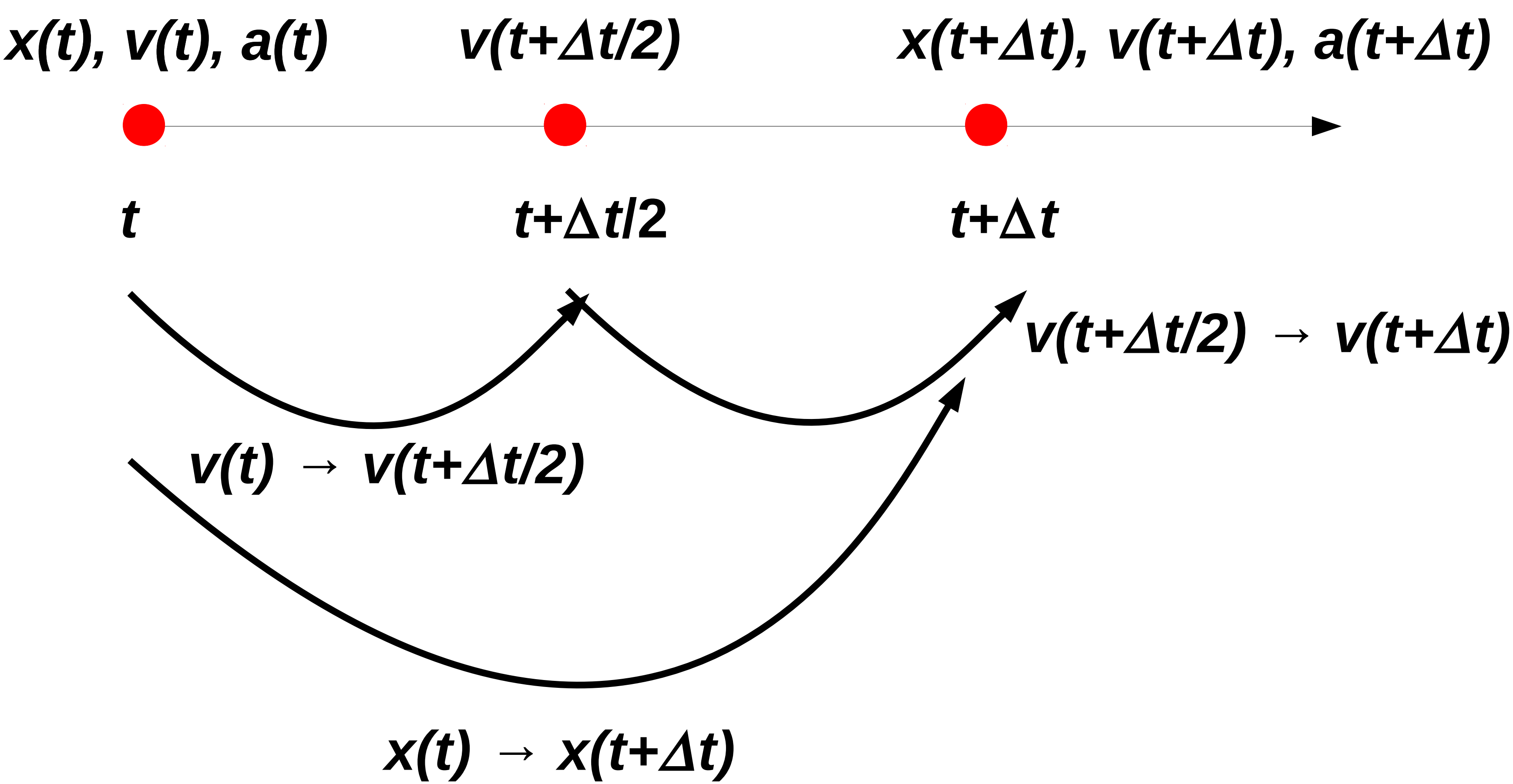

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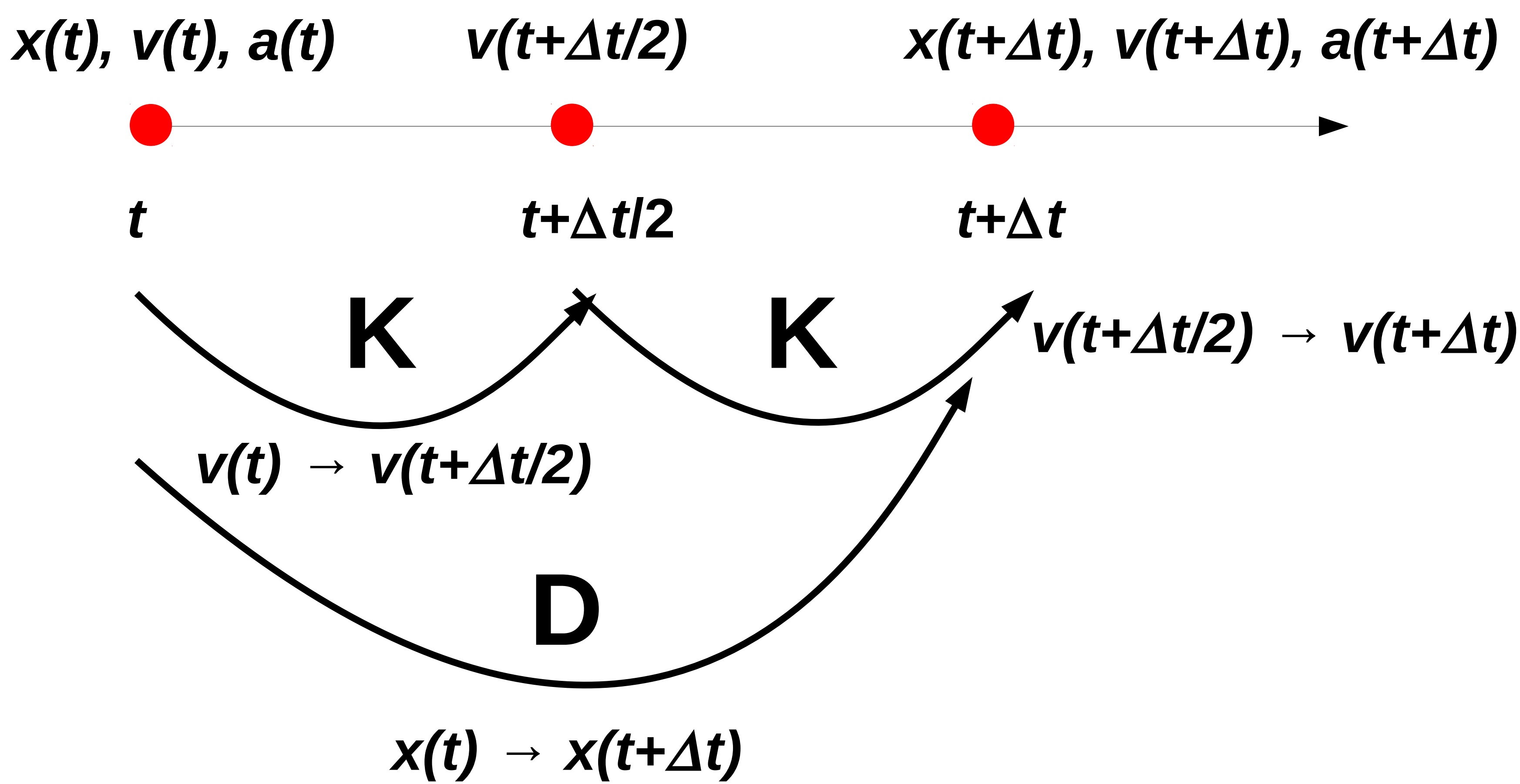

Kick + Drift + Kick (KDK) scheme

1. **WHAT IS an N-Body SIMULATION?**

LEAPFROG METHOD:

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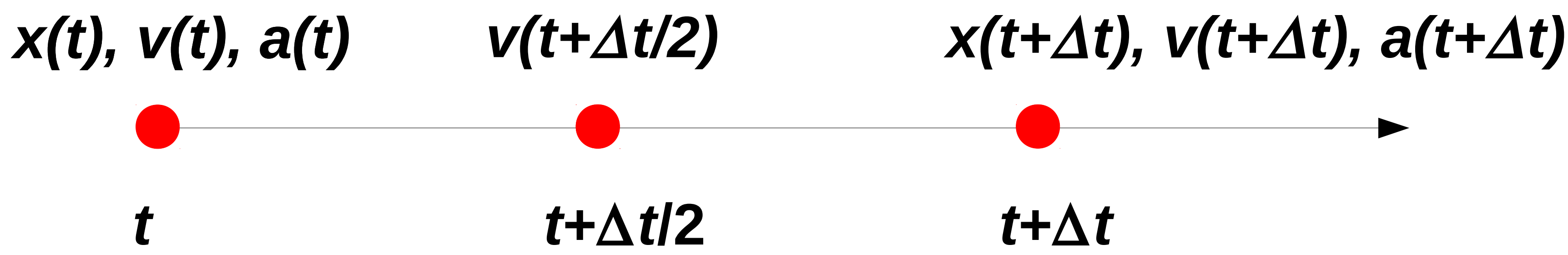


$$v\left(t+\frac{\Delta t}{2}\right) = v(t) + \frac{\Delta t}{2}\,a(t)$$

$$x(t+\Delta t) = x(t) + v\left(t+\frac{\Delta t}{2}\right)\,\Delta t$$

$$v(t+\Delta t) = v\left(t+\frac{\Delta t}{2}\right) + \frac{1}{2}\Delta t\,a(t+\Delta t)$$

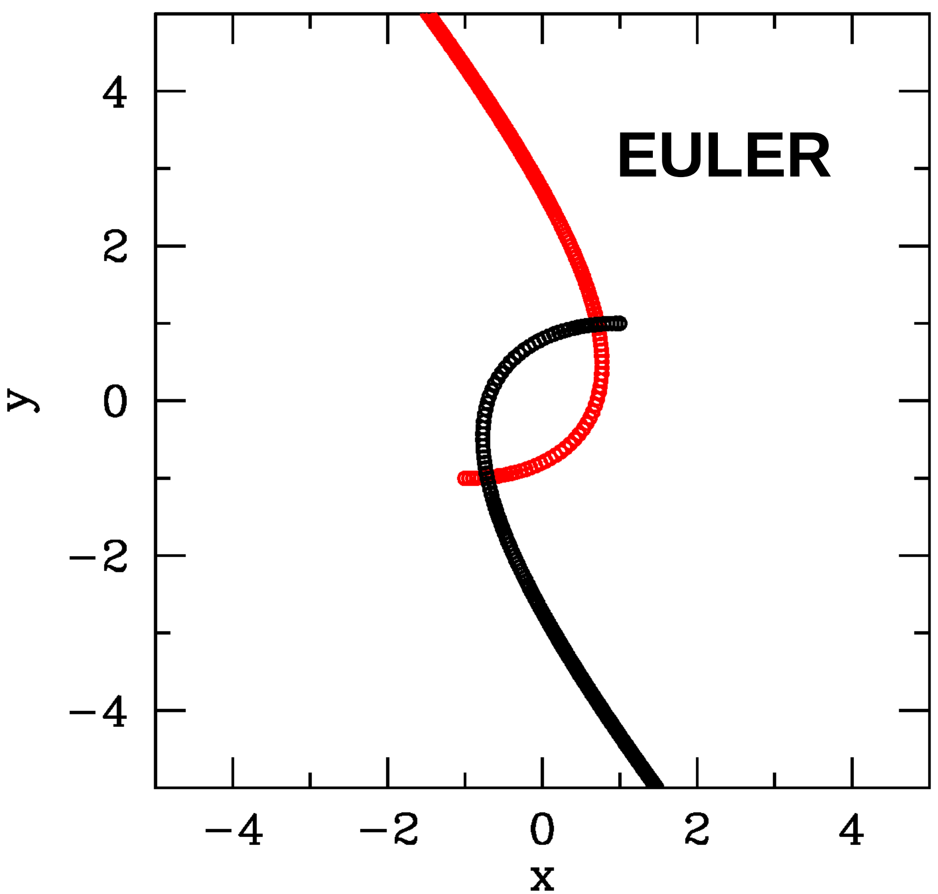
$$\longrightarrow \quad x(t+\Delta t) = x(t) + v(t)\,\Delta t + \frac{1}{2}a(t)\Delta t^2$$

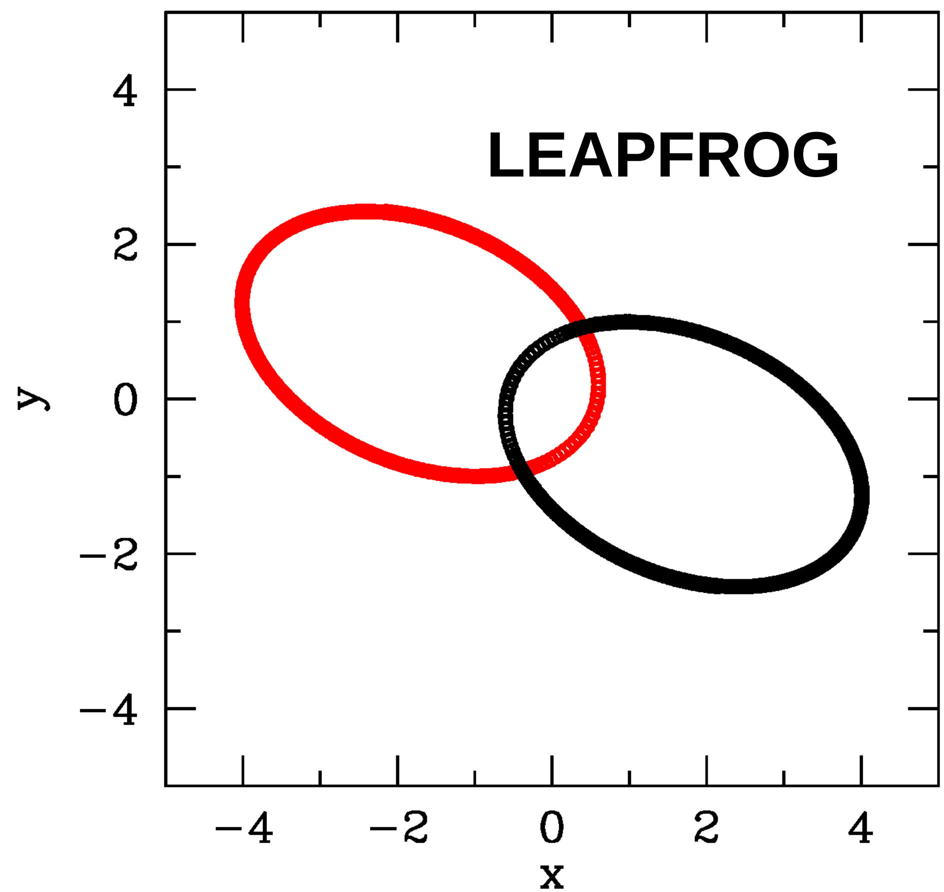
$$v(t+\Delta t) = v(t) + \frac{1}{2}a(t)\,\Delta t + \frac{1}{2}a(t+\Delta t)\,\Delta t$$

1. **WHAT IS an N-Body SIMULATION?**

EULER VS LEAPFROG METHOD a simple test:

Both Euler and Leapfrog are 2d order methods, but..





Same initial conditions: integration of a Keplerian binary

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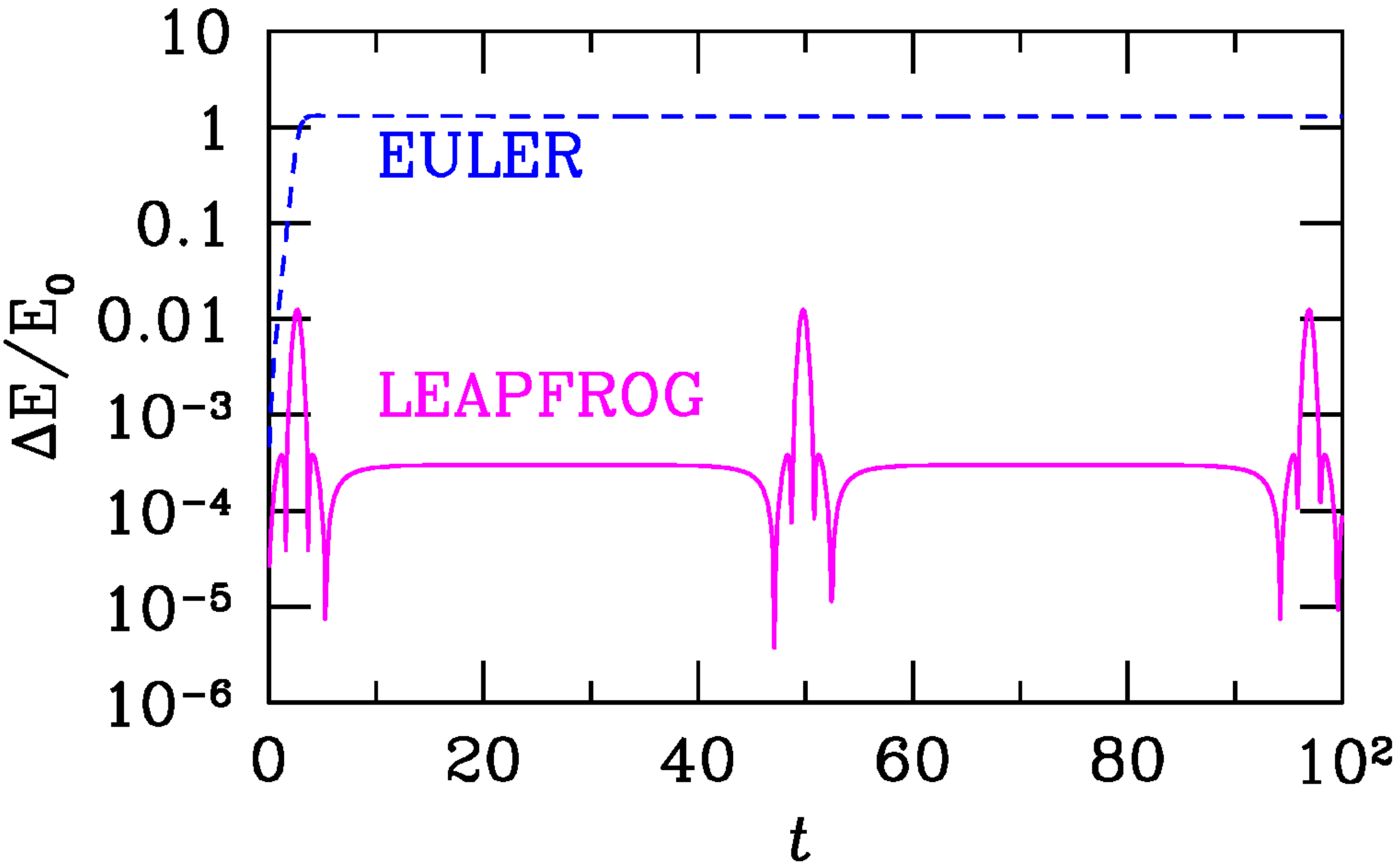

Same initial conditions: integration of a Keplerian binary

EULER VS LEAPFROG METHOD a simple test:

Both Euler and Leapfrog are 2d order methods, but..

A possible hint for the final exam: if you like programming, you can write a simple LEAPFROG code!!!!

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numerical integration of Newton equation for

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SCALING of a NUMERICAL PROBLEM:

Numerical complexity:
How many calculations I have to do for N-particles?

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SCALING of a NUMERICAL PROBLEM:

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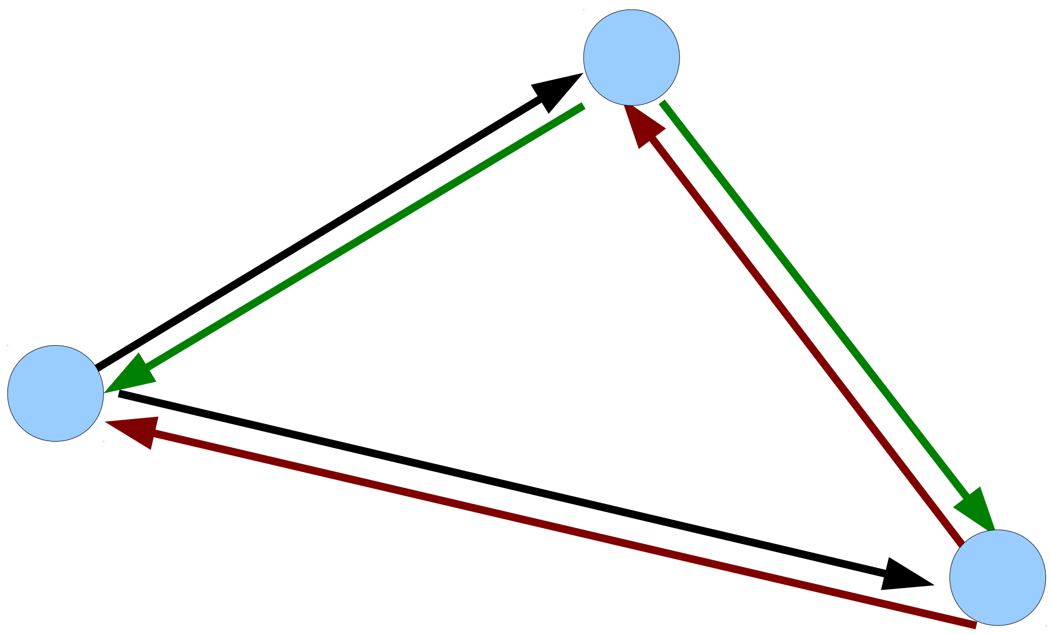
How many calculations I have to do for N-particles



3 particles
6 forces
N (N - 1)

1. **WHAT IS an N-Body SIMULATION?**

SCALING of a NUMERICAL PROBLEM:

Numerical complexity:

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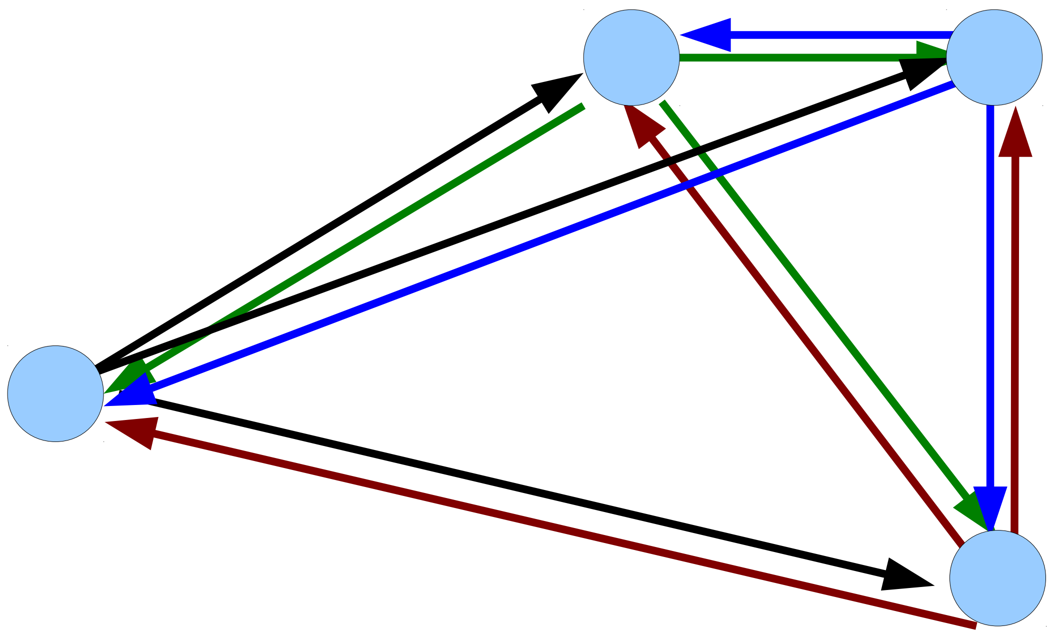


4 particles
12 forces
N (N - 1)

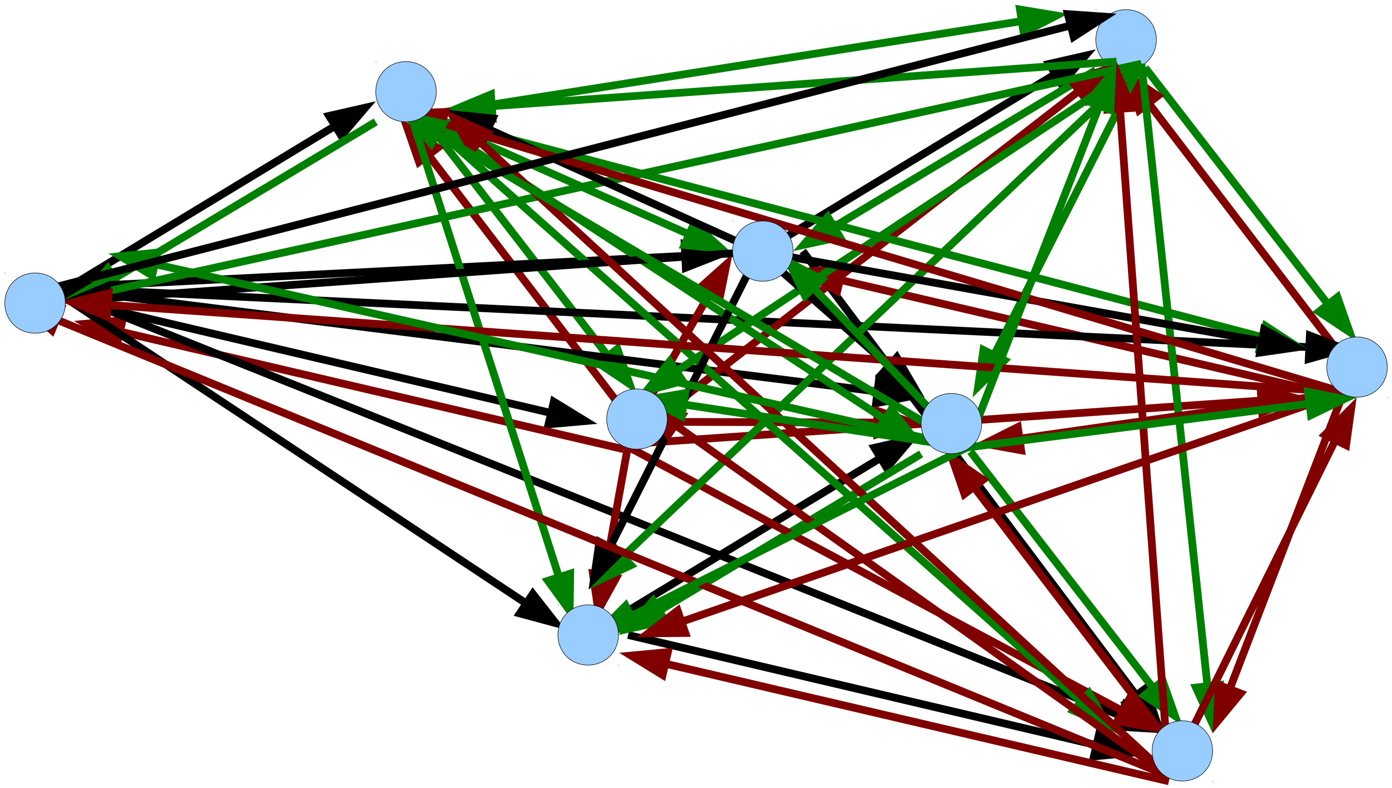
1. WHAT IS an N-Body SIMULATION?

SCALING of a NUMERICAL PROBLEM:

Numerical complexity:

$$\ddot{\vec{r}}_i = -G \sum_{j \neq i} m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

How many calculations I have to do for N-particles



9 particles
72 forces
N (N – 1)

COMPLEXITY GROWS as N^2 - VERY FAST !!!!

1. WHAT IS an N-Body SIMULATION?

SCALING of a NUMERICAL PROBLEM:

COMPLEXITY GROWS as N^2 - VERY FAST !!!!

- CAN I REDUCE COMPLEXITY?

YES, BUT IT IS NOT ALWAYS THE RIGHT CHOICE
→ See direct vs indirect N-body LECTURE 2

- HOW CAN I REDUCE COMPLEXITY?
E.G. with BARNES-HUT TREE METHOD
and/or with MULTIPOLE EXPANSION
→ See LECTURE 3

1. WHAT IS an N-Body SIMULATION?

N-body UNITS:

MOST N-body codes work in N-body units

since Gravity is the main force, choose units so that G=1

DEFINITION OF CODE UNITS (SO THAT G = 1)

$$6.67 \times 10^{-8} \frac{cm^3}{s^2 g} \simeq G = \frac{U_l^3}{U_t^2 \, U_m}$$

$$\Rightarrow U_t^2 = \frac{U_l^3}{G \, U_m}$$

1. WHAT IS an N-Body SIMULATION?

N-body UNITS:

DEFINITION of M_{SCALE}, T_{SCALE}, L_{SCALE}

$$M_{scale} := \frac{1}{U_m}$$

$$T_{scale} := \frac{1}{U_t}$$

$$L_{scale} := \frac{1}{U_L}$$

N-body UNITS:

FROM CODE UNITS TO PHYSICAL UNITS and VICEVERSA:

$$M_{code} = M_{phys}\; M_{scale} = \frac{M_{phys}}{M_{TOT}}$$

M_{code}: mass of particles in the code (mass in code units)

M_{phys}: mass of particles in physical units

$M_{scale} := \frac{1}{m_{TOT}}$

$$T_{code} = T_{phys}\; T_{scale} = \frac{T_{phys}}{0.25\ Myr}$$

T_{code}: time in code units

$T_{scale} := \frac{1}{U_t} = 1/0.25\,Myr$ (in our example)

$$L_{code} = L_{phys}\; L_{scale} = \frac{L_{phys}}{1\,pc}$$

L_{code}: length in code units

$L_{scale} := \frac{1}{U_L} = \frac{1}{1\,pc}$ (in our example $\frac{1}{1\,pc}$)

$$V_{code} = V_{phys}\; \frac{L_{scale}}{T_{scale}} = V_{phys}\; \frac{U_T}{U_L} = V_{phys}\; \frac{0.25\,Myr}{1\,pc}$$

V_{code}: Velocity in code units