Michela Mapelli
INAF - Padova

Dynamics of Stars and Black Holes in Dense Stellar Systems:

Lecture VI:

DYNAMICS AROUND SUPER-MASSIVE BHs

0. nuclear star clusters (NSCs)
1. dynamics around super-massive BHs (SMBHs)
2. formation and dynamics of stars in the central parsec
3. gravitational waves involving NSCs

0. nuclear star clusters (NSCs)

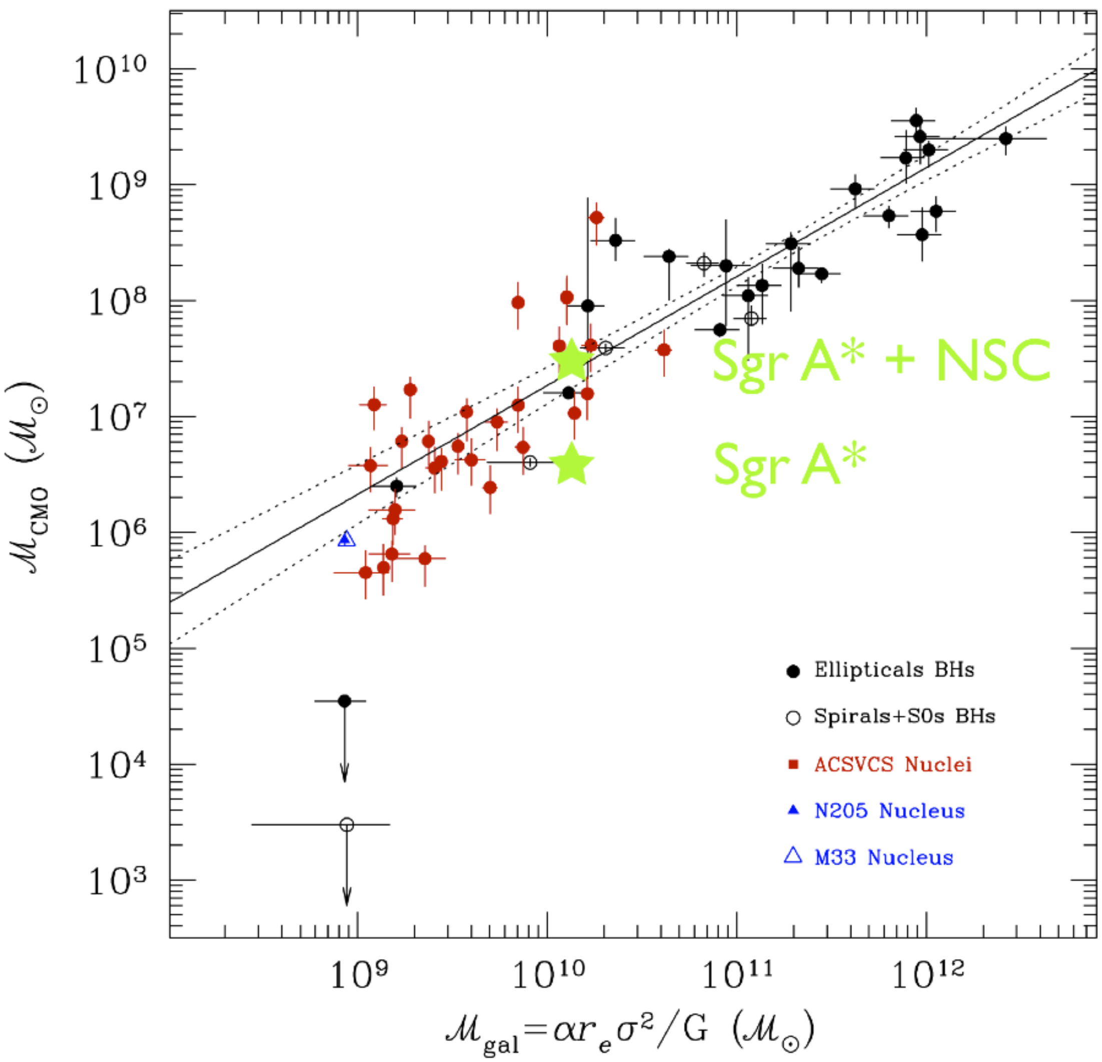
* more massive than globular clusters (M>10^6 Msun)

* **MULTIPLE POPULATION** (<1 Gyr up to 13 Gyr)

* in lower-mass spheroids (<10^10 Msun), NSCs are more common than SMBHs, but sometimes CO-EXISTENT with the SMBH (Milky Way)

* in high-mass spheroids (>10^11 Msun) SMBH alone in most cases

* NSCs obey SCALING RELATIONs as SMBHs (Ferrarese et al. 2006)



Schoedel 2010, arXiv:1001.4238
Ferrarese et al. 2006,ApJ, 644, L21

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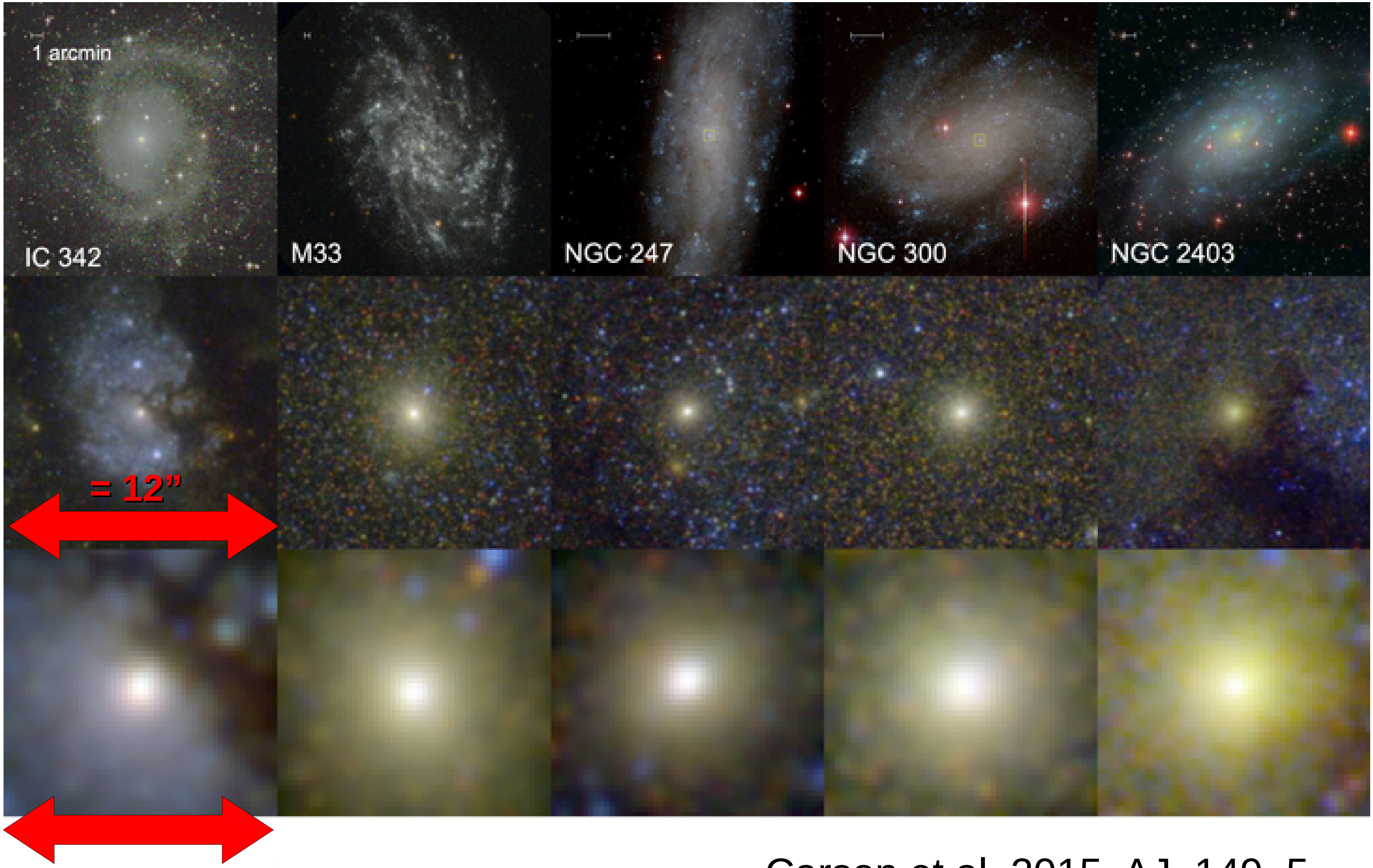

= 2” ~ 20 – 40 pc
for this sample

Carson et al. 2015, AJ, 149, 5

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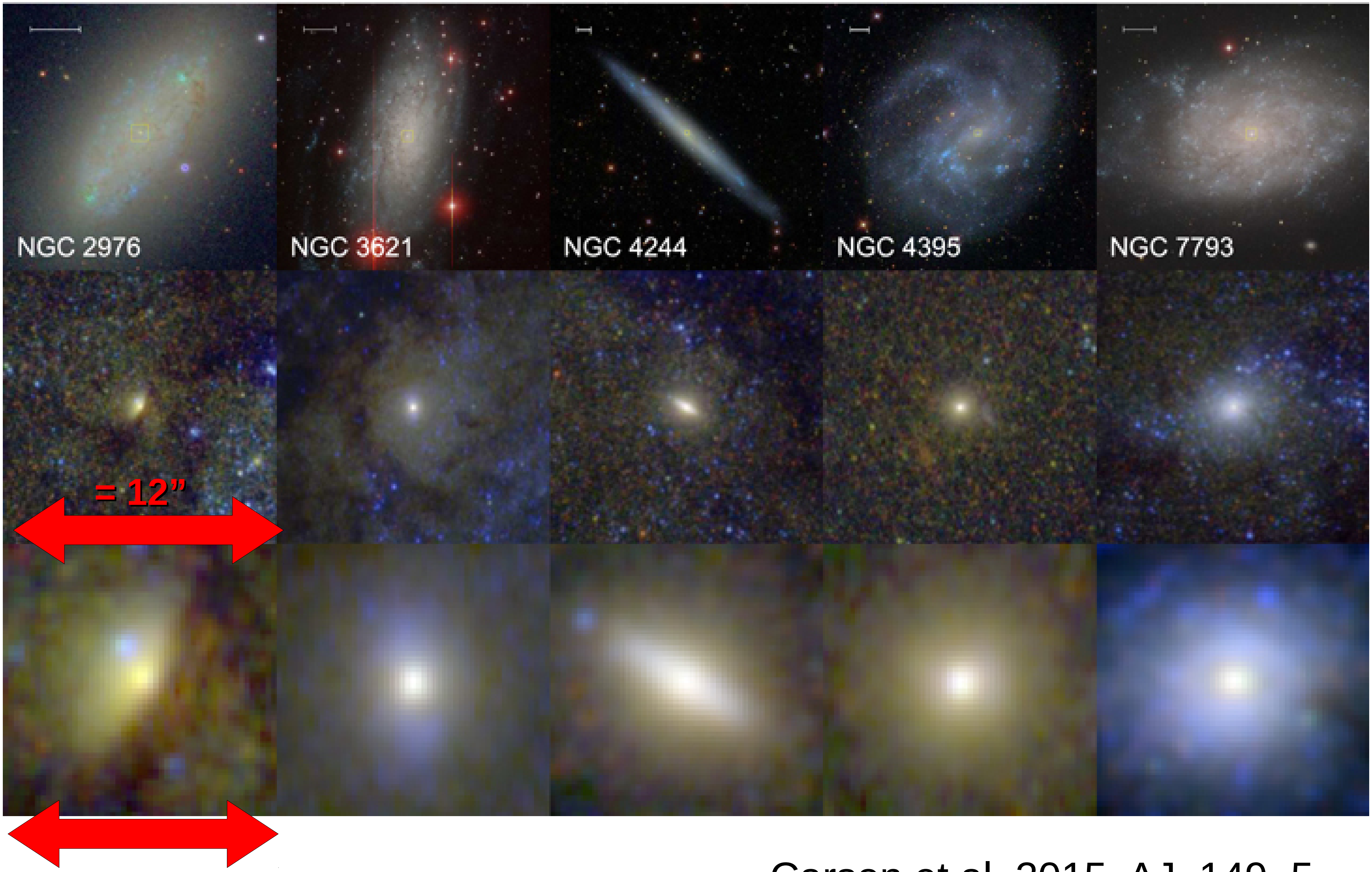

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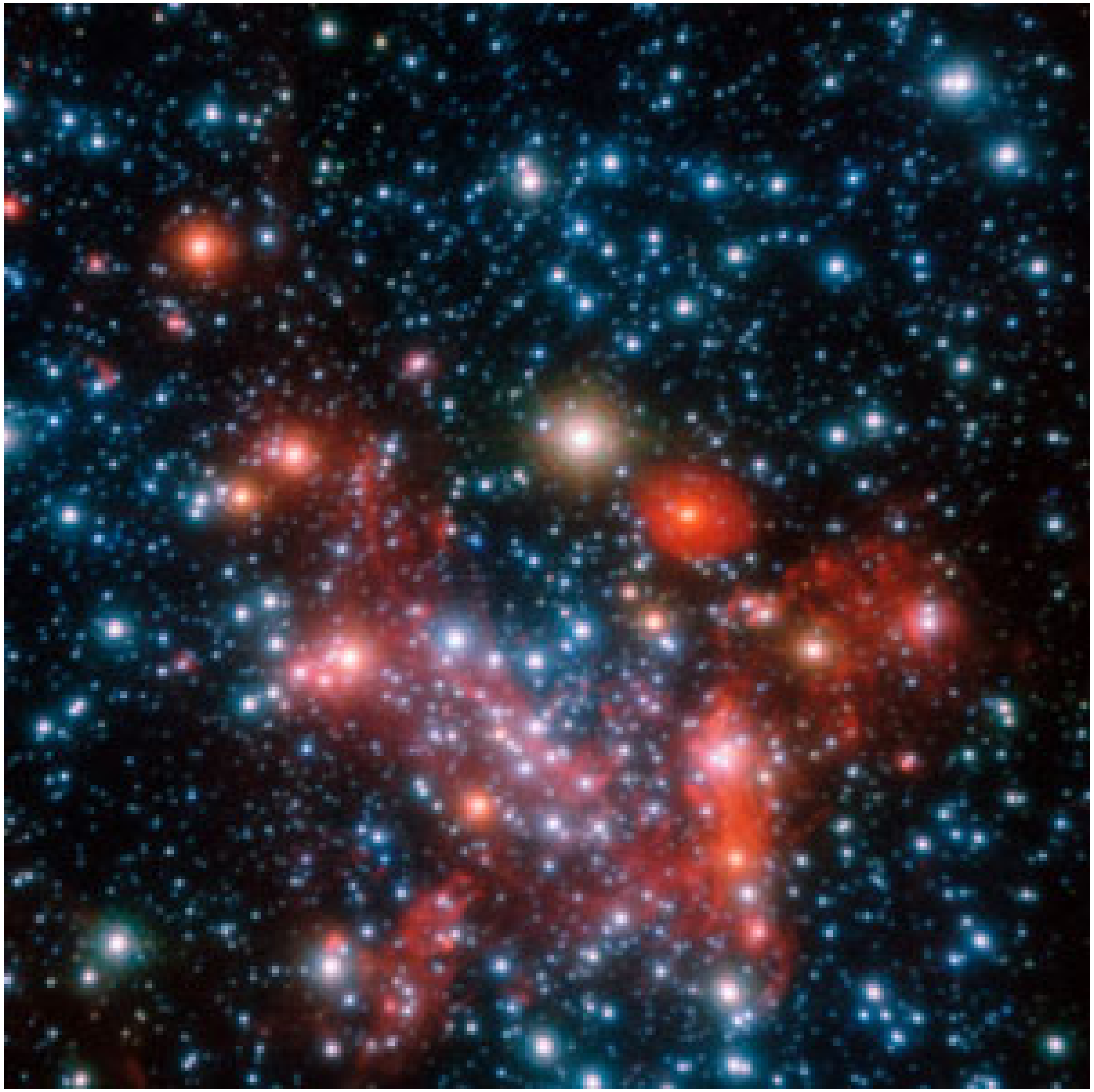
The NSC we know better is the one of the Milky Way:



* ~ 8 kpc away (100 times closer than the 2nd next)

* co-existent with SMBH

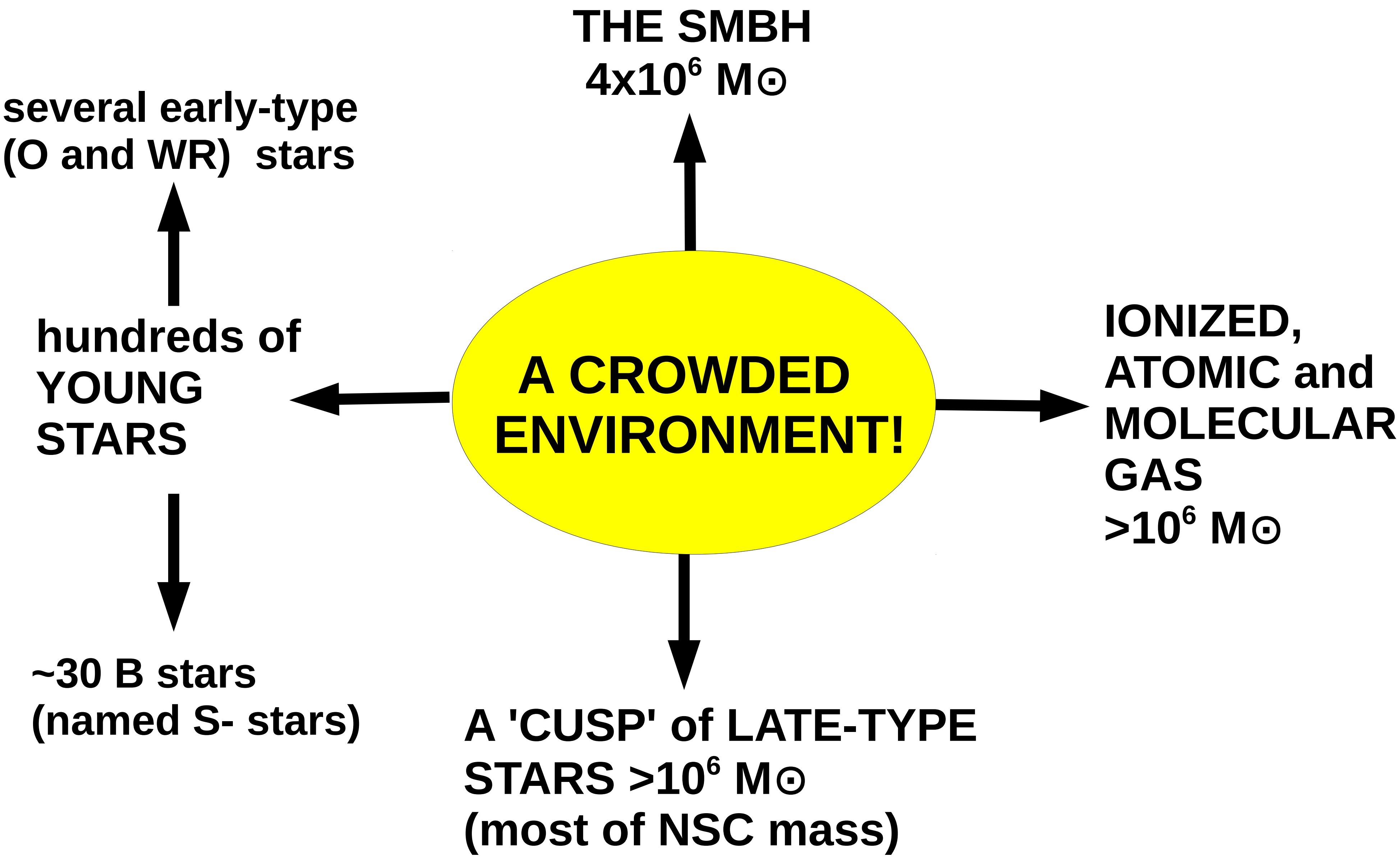
* massive (>10^7 Msun)

* both old (>1 Gyr) and young (~Myr) population

* extremely interesting to understand interplay between stars, gas and SMBH

Review from Genzel+ 2010, Reviews of Modern Physics, arXiv:1006.0064

What do we observe in the Galactic centre?



THE SMBH
$4x10^6$ M⊙

several early-type
(O and WR) stars

hundreds of
YOUNG
STARS

A CROWDED
ENVIRONMENT!

IONIZED,
ATOMIC and
MOLECULAR
GAS
$>10^6$ M⊙

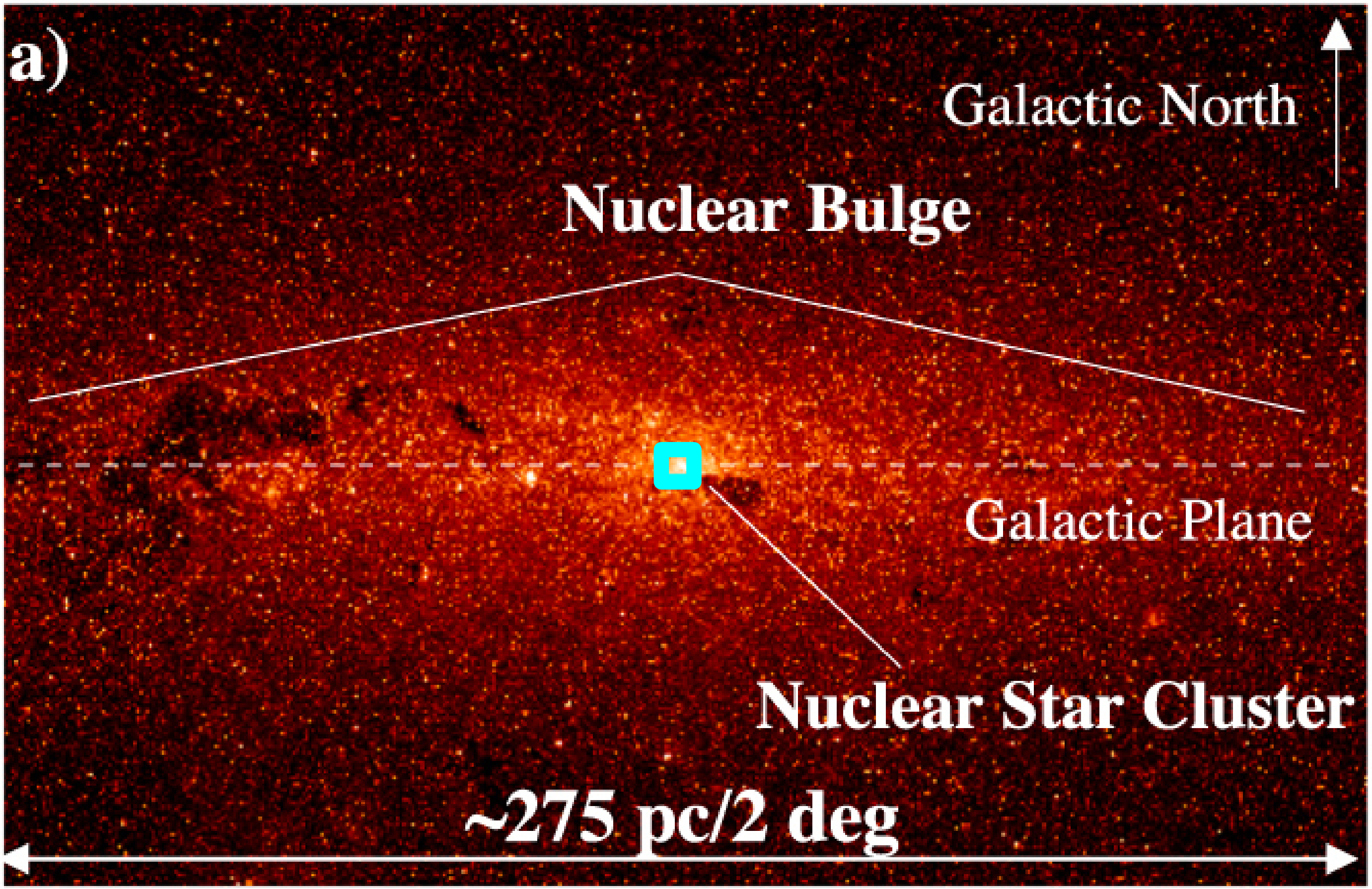
~30 B stars
(named S- stars)

A 'CUSP' of LATE-TYPE
STARS $>10^6$ M⊙
(most of NSC mass)

0. nuclear star clusters (NSCs)

STARS:

270 pc < 1/10 scale length of our Galaxy's disc



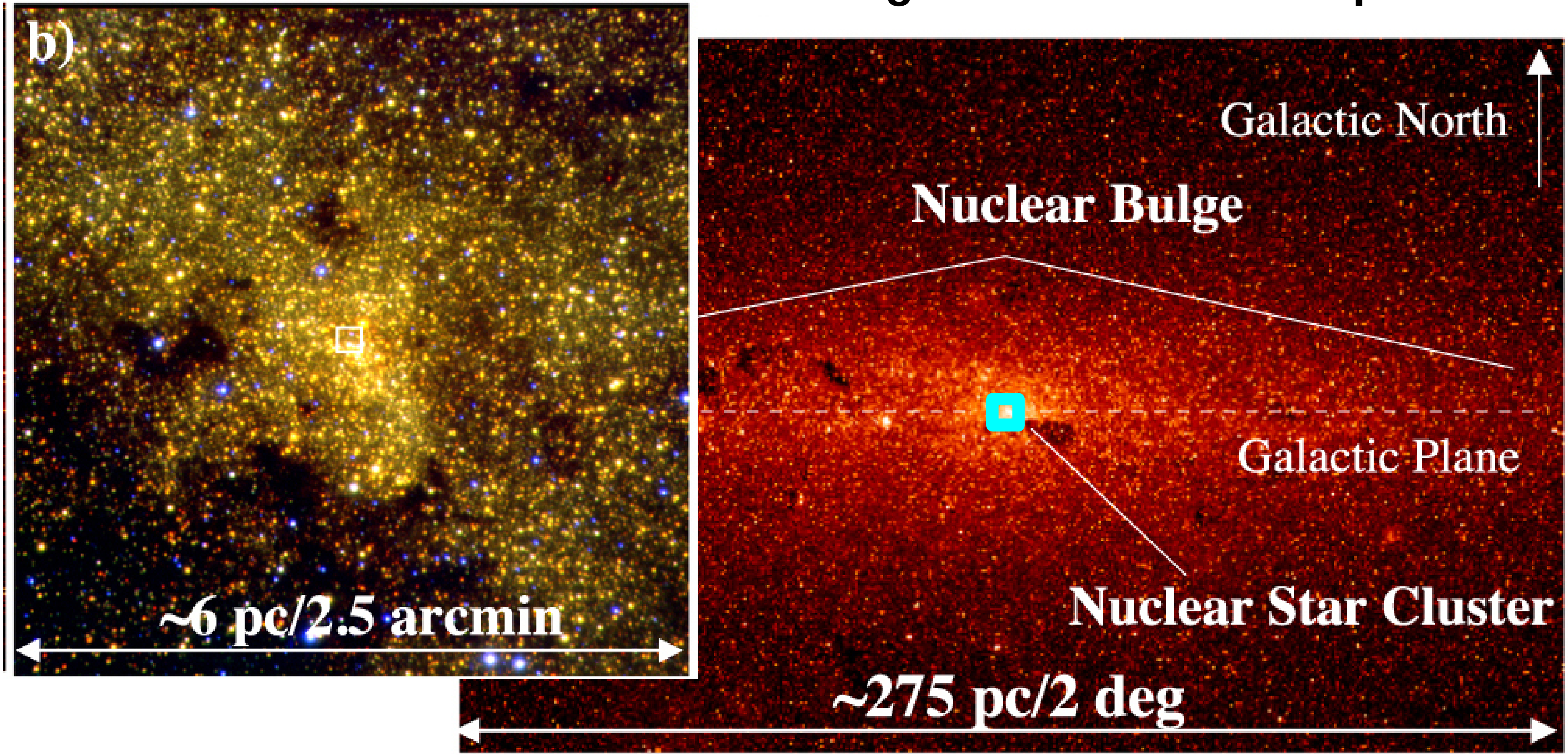
Schoedel et al. 2007, A&A, 469, 125
Schoedel et al. 2014 arXiv:1411.4504

0. nuclear star clusters (NSCs)

OLD STARS: spherical cusp, very high density

~10^6 M⊙ pc^{-3}

solar neighborhood <~ 1 M⊙ pc^{-3}



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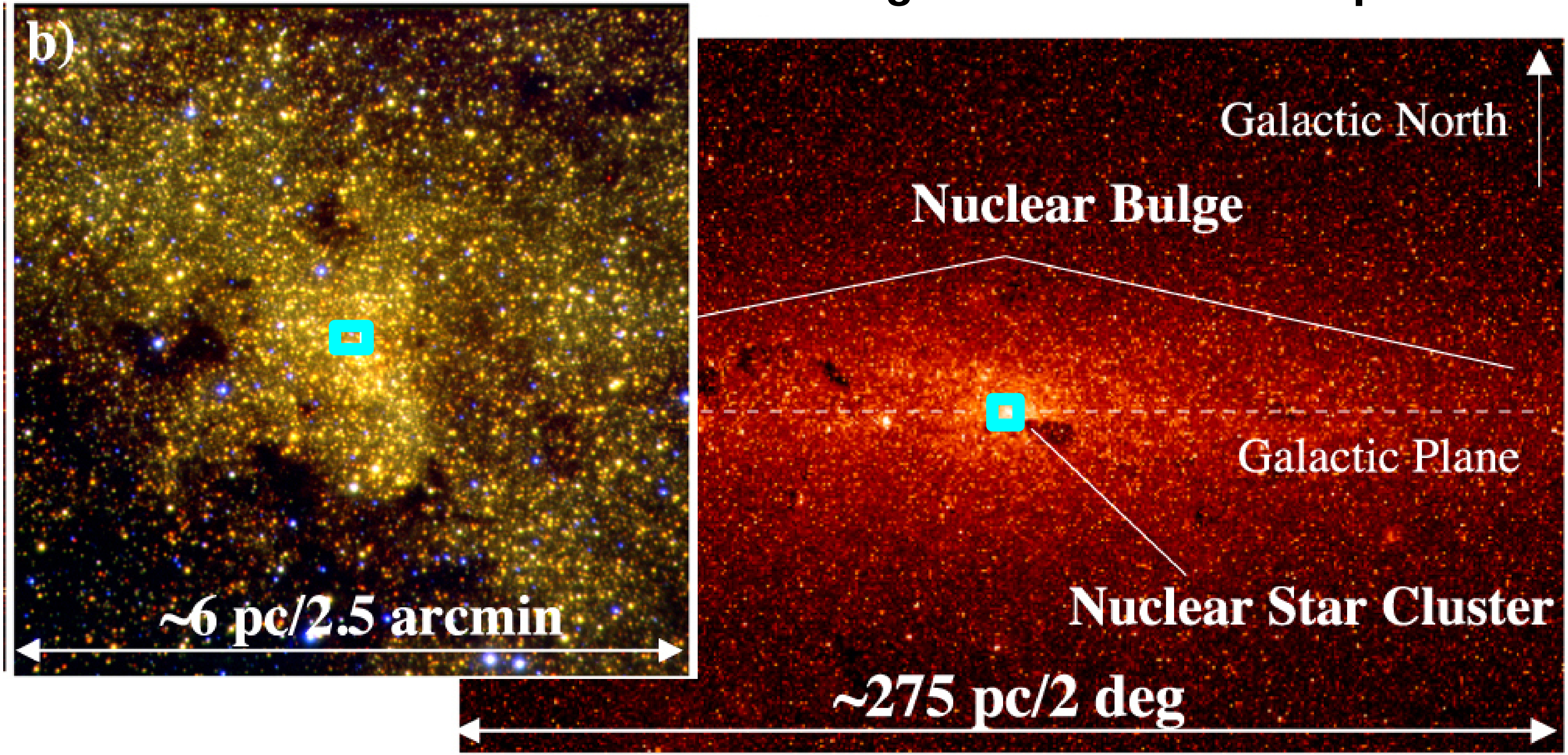


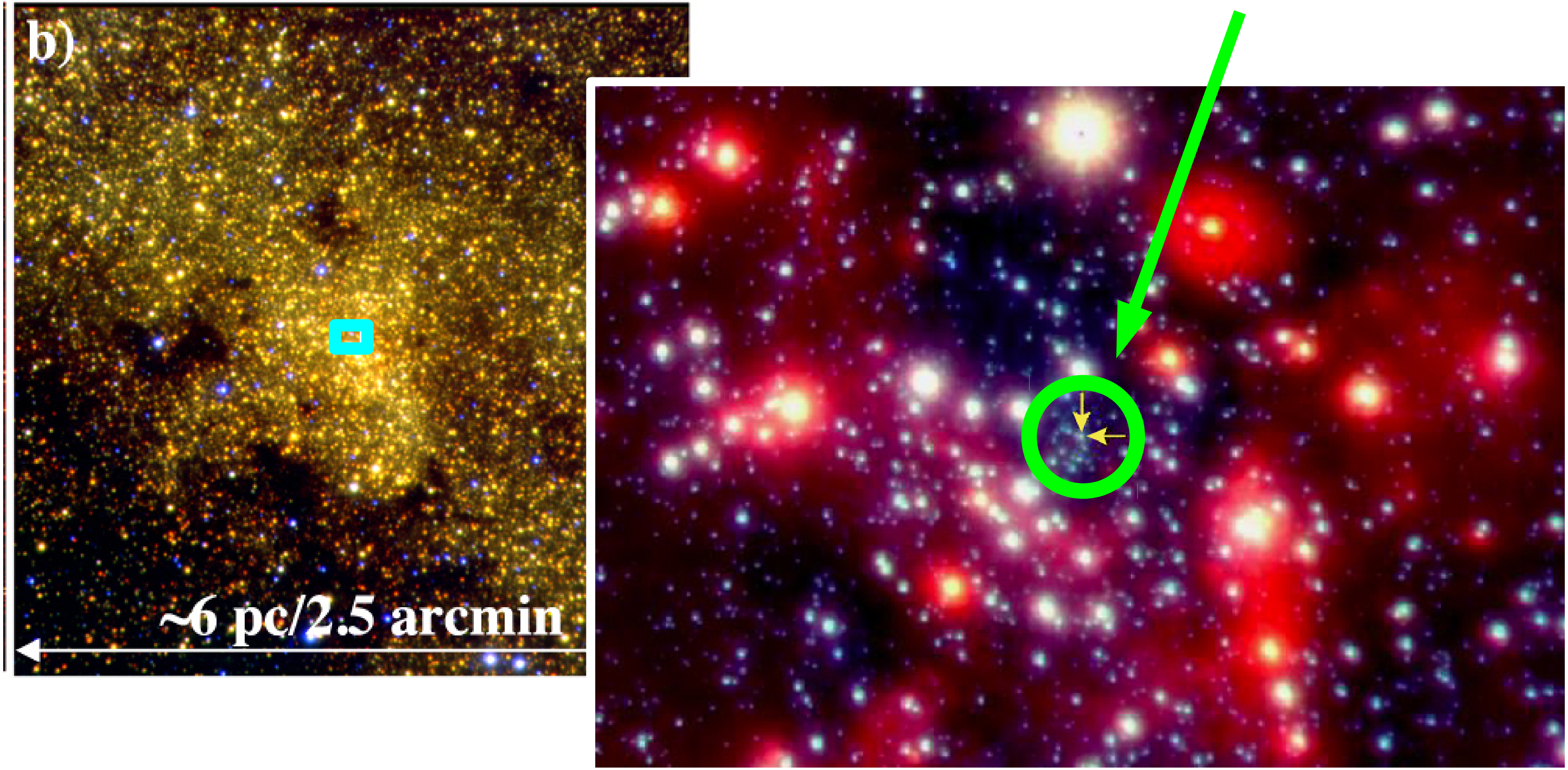
Schoedel et al. 2007, A&A, 469, 125

Schoedel et al. 2014 arXiv:1411.4504

0. nuclear star clusters (NSCs)

YOUNG STARS: <100 Myr

S-cluster r ~ 0.04 pc

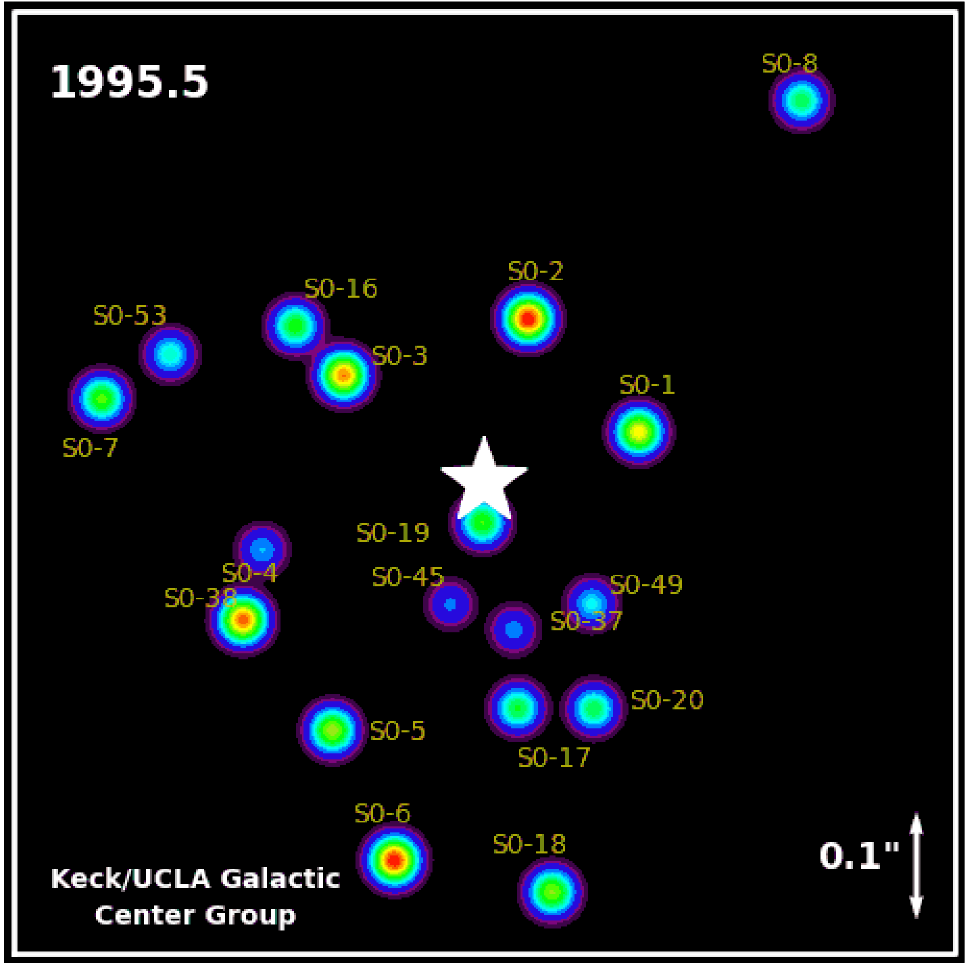


0.5 pc ~ 12 arcsec

Schoedel et al. 2002, Nature, 419, 694

S-cluster

0.2”~ 0.008 pc



- “cluster” radius ~ 0.04 pc

~ 30 stars

- age ~ 20 – 100 Myr

- eccentricity ~ 0.7-0.9

-allow dynamical measurement of SMBH mass (S02, yellow line)

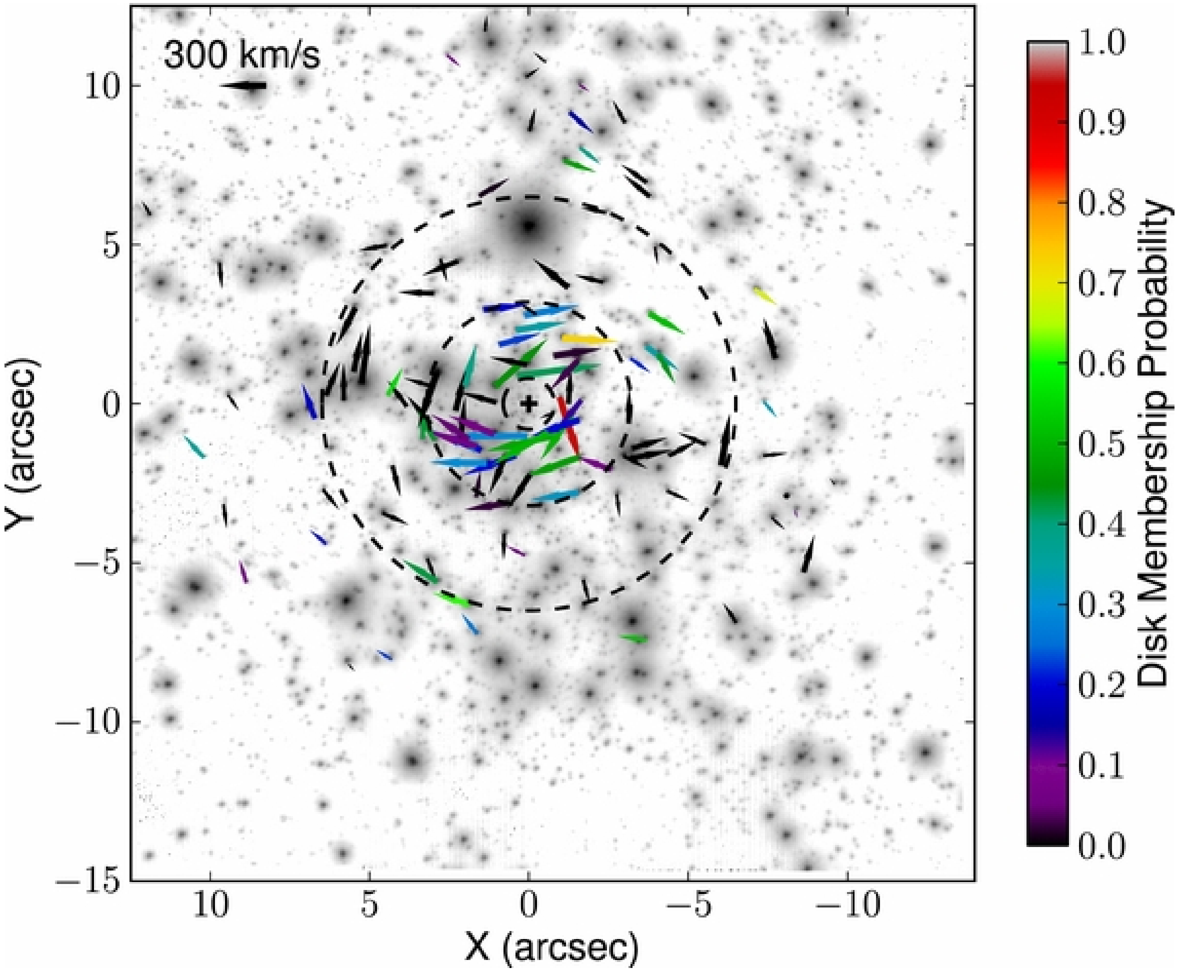
→ The only SMBH candidate for which we can exclude almost every other nature (thanks to S02's orbit):

$m_{BH}=4.30 \pm 0.20|_{stat} \pm 0.30|_{sys} \times 10^6 M_\odot$

Gillessen et al. (2009)

Credits: Prof. Andrea Ghez, UCLA

Very young stars (aka clockwise disc)



- between 0.04 and 0.5 pc

~ 10^4 Msun

- age ~ 2 – 8 Myr

- eccentricity ~ 0.3 +/- 0.1

- top-heavy MF (slope~1.5)

- between 0.04 and 0.15 pc lie in a disc (clockwise disc)

Figure from Yelda et al. 2014, ApJ, 783, 131

Bartko et al. 2009; Lu et al. 2009, 2013; Do et al. 2013; Yelda et al. 2014

1. dynamics around super-massive black holes (SMBHs)

A lot of dynamics close to a SMBH:

- Newtonian precession
- Relativistic precession
- Two-body relaxation
- Resonant relaxation
- Kozai-Lidov (same as star cluster case but with SMBH)
- Tidal capture or Binary break-up (exchange with SMBH)

+ GAS physics:

- Disruption of molecular clouds
- Accretion on SMBH
- SMBH feedback
- Star formation

REVIEW: Mapelli & Gualandris 2016, LNP, Springer,
https://arxiv.org/abs/1505.05473

1. dynamics around super-massive black holes (SMBHs)

* NSCs are COLLISIONAL SYSTEMS
BUT ONLY IF SMBH is not included (increases local velocity field)

* If there are SMBHs, nuclear star clusters are still COLLISIONAL OUT OF SMBH INFLUENCE RADIUS (inside SMBH dominates gravity)

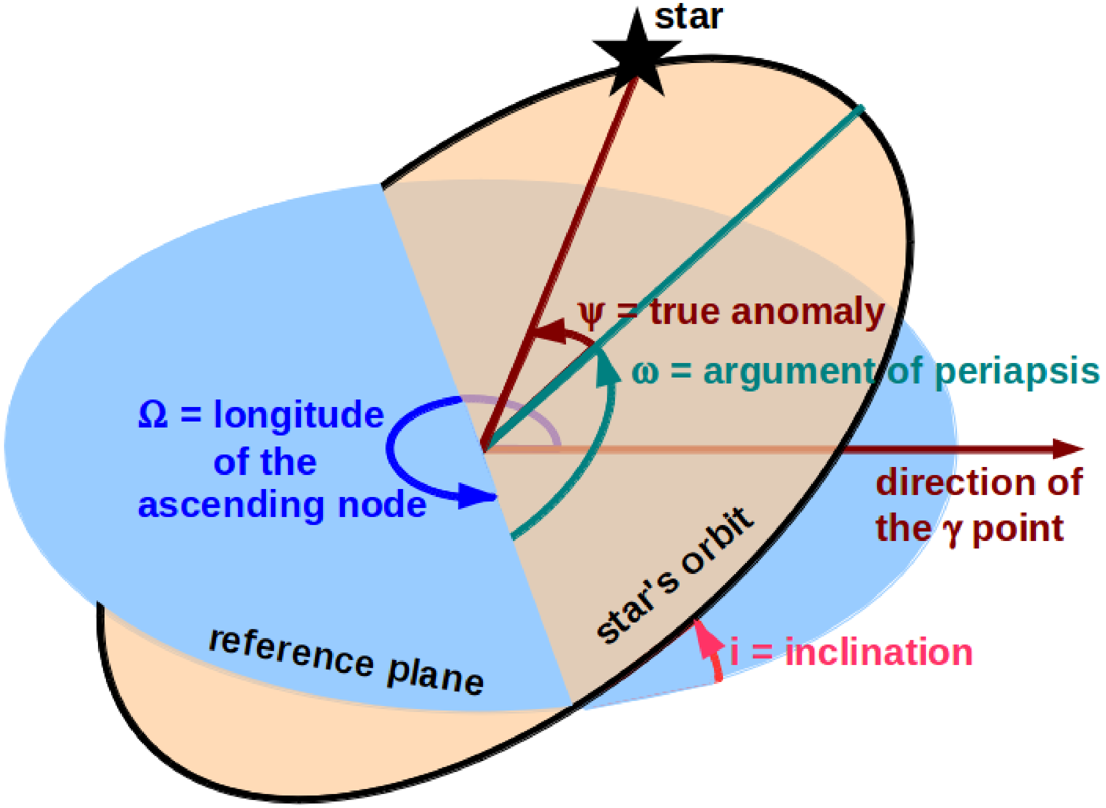
$$r_{BH} = \frac{G\,m_{BH}}{\sigma^2} = 1.7\,\mathrm{pc}\,\left(\frac{m_{BH}}{10^6\,M_\odot}\right)\,\left(\frac{50\,\mathrm{km\ s}^{-1}}{\sigma}\right)^2$$

→ in off-nuclear star clusters two-body relaxation
is most important effect by far

in NSCs there are other effects at least as
important as two-body relaxation

NEWTONIAN PRECESSION(s)

A star orbiting the SMBH can be described as in Keplerian motion around the SMBH plus an EXTERNAL POTENTIAL (= the old stellar cusp, the other young stars, the CNR)
The external induces PRECESSION

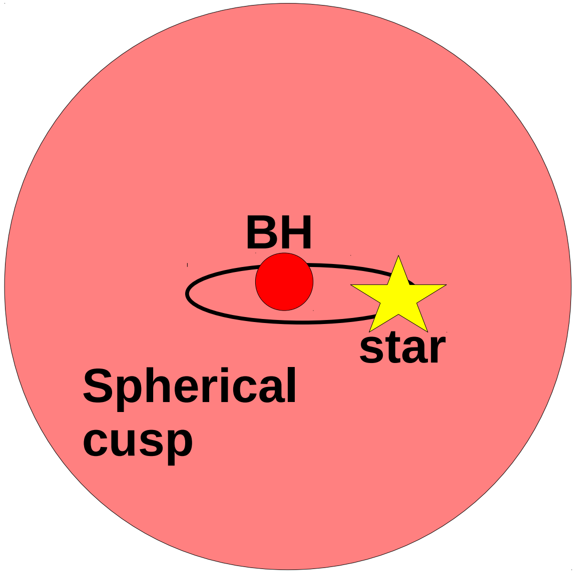


Precession can affect:

- argument of periapsis
- longitude of asc. node
- inclination
- eccentricity

Depending on the structure of the external potential

1. dynamics around super-massive black holes (SMBHs)

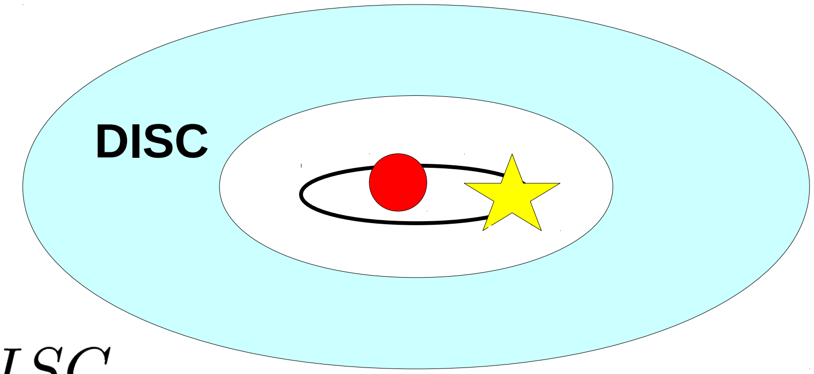


- NEWTONIAN PREC. in SPHERICAL POTENTIAL (e.g. spherical stellar cusp):

Timescale

$$T_{cusp} = \frac{M_{BH}}{M_{cusp}(a)} P_{orb}\, f(e)$$

Only argument of pericentre



- in AXISYMMETRIC POTENTIAL (e.g. stellar or gas ring)

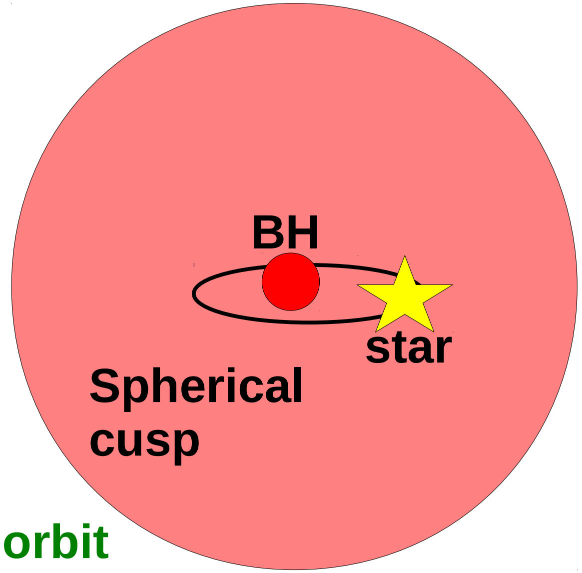
Timescale

$$T_K = \frac{M_{BH}}{M_{DISC}} \frac{R^3_{DISC}}{a^{3/2}\sqrt{G\, M_{BH}}}$$

- if i~0 only longitude of ascending node
- if i>>0 also inclination and eccentricity are affected

1. dynamics around super-massive black holes (SMBHs)

- NEWTONIAN PREC. in SPHERICAL POTENTIAL (e.g. spherical stellar cusp):



Timescale

SMBH mass

orbital period of star

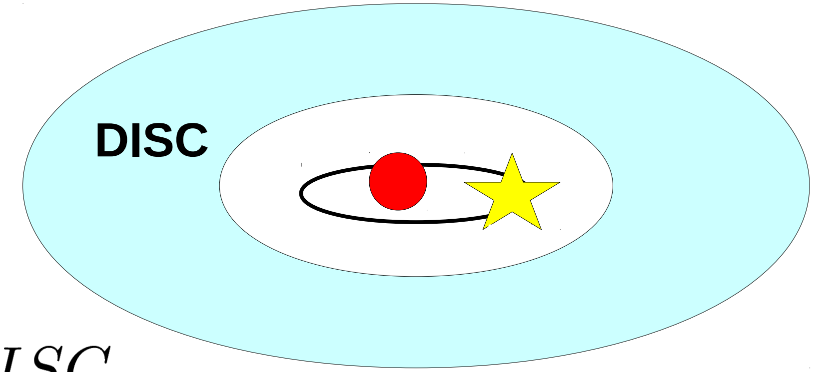
function of eccentricity

$$T_{cusp} = \frac{M_{BH}}{M_{cusp}(a)} P_{orb} \, f(e)$$

cusp mass enclosed within semi-major axis a of stellar orbit

Only argument of pericentre

- in AXISYMMETRIC POTENTIAL (e.g. stellar or gas ring)



disc size

Timescale

$$T_K = \frac{M_{BH}}{M_{DISC}} \frac{R^3_{DISC}}{a^{3/2} \sqrt{G \, M_{BH}}}$$

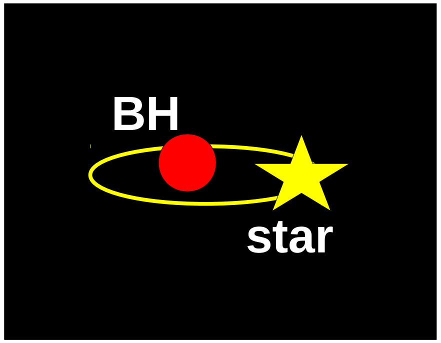
disc mass

semi-major axis a of stellar orbit

- if i~0 only longitude of ascending node
- if i>>0 also inclination and eccentricity are affected

RELATIVISTIC PRECESSION:

precession of orbits in general relativity



Caused by the SMBH mass, even if there are no external potentials
Three types (Schwarzschild prec. + 2 precession effects that depend on spin)

Schwarzschild precession (lowest order correction to Newton):

$$T_{\rm RP} = 1.3 \times 10^3 \ {\rm yr} \ \left(1 - {\rm ecc}^2\right) \left(\frac{r}{0.001 \ {\rm pc}}\right)^{5/2} \left(\frac{4 \times 10^6 \ {\rm M}_\odot}{M_{\rm BH}}\right)^{3/2}$$

orbital eccentricity

star – BH distance

SMBH mass

- **affects only argument of pericentre**
- **efficient for very small semi-major axis**
- **more efficient for high eccentricity**
- **more efficient for large SMBH mass**

1. dynamics around super-massive black holes (SMBHs)

In the case of Galactic centre

- relativistic precession important only if a<<0.1 pc

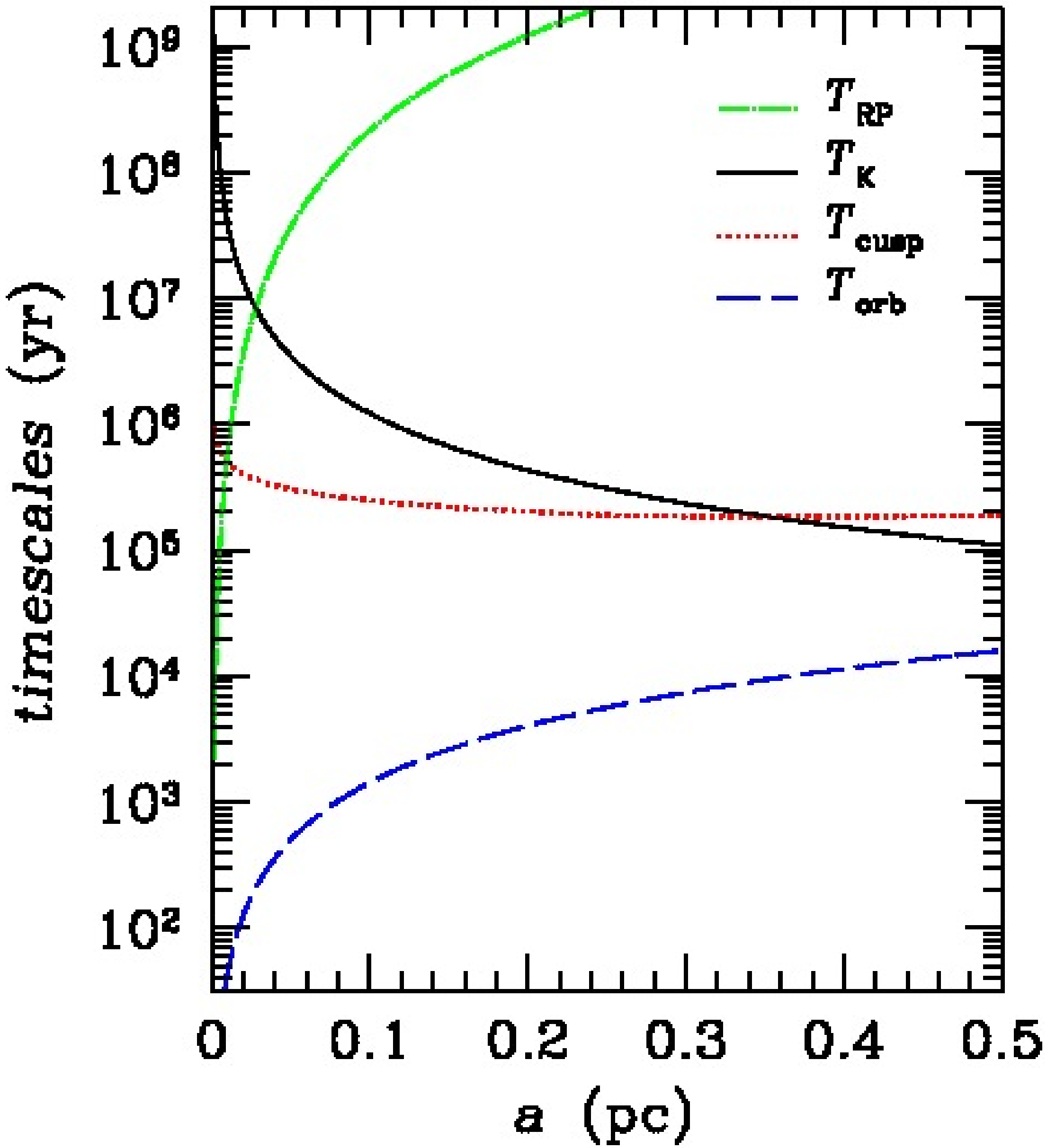
- spherical cusp important at <0.3 pc

- circumnuclear disc important at >0.3 pc

↓

IF SPHERICAL POTENTIAL DOMINATES over AXISYMMETRIC ($T_{cusp} << T_K$),

then only precession of argument of pericentre and of longitude of ascending node are not damped



1. dynamics around super-massive black holes (SMBHs)

TWO-BODY RELAXATION: changes ENERGY

$$T_{\rm rlx} = 0.34 \, \frac{\sigma^3(r)}{G^2 \, m_* \, \rho_*(r) \, \ln \Lambda}$$

RESONANT RELAXATION: changes ECCENTRICITY
NO ENERGY

$$T_{\rm RR} = 10^4 \ {\rm yr} \ \left(\frac{r}{0.001 \ {\rm pc}}\right)^{3/2} \sqrt{\frac{M_{\rm BH}}{3 \times 10^6 \ {\rm M}_\odot}} \left(\frac{10 \ {\rm M}_\odot}{m_*}\right) \sqrt{\frac{10^3}{N_*}}$$

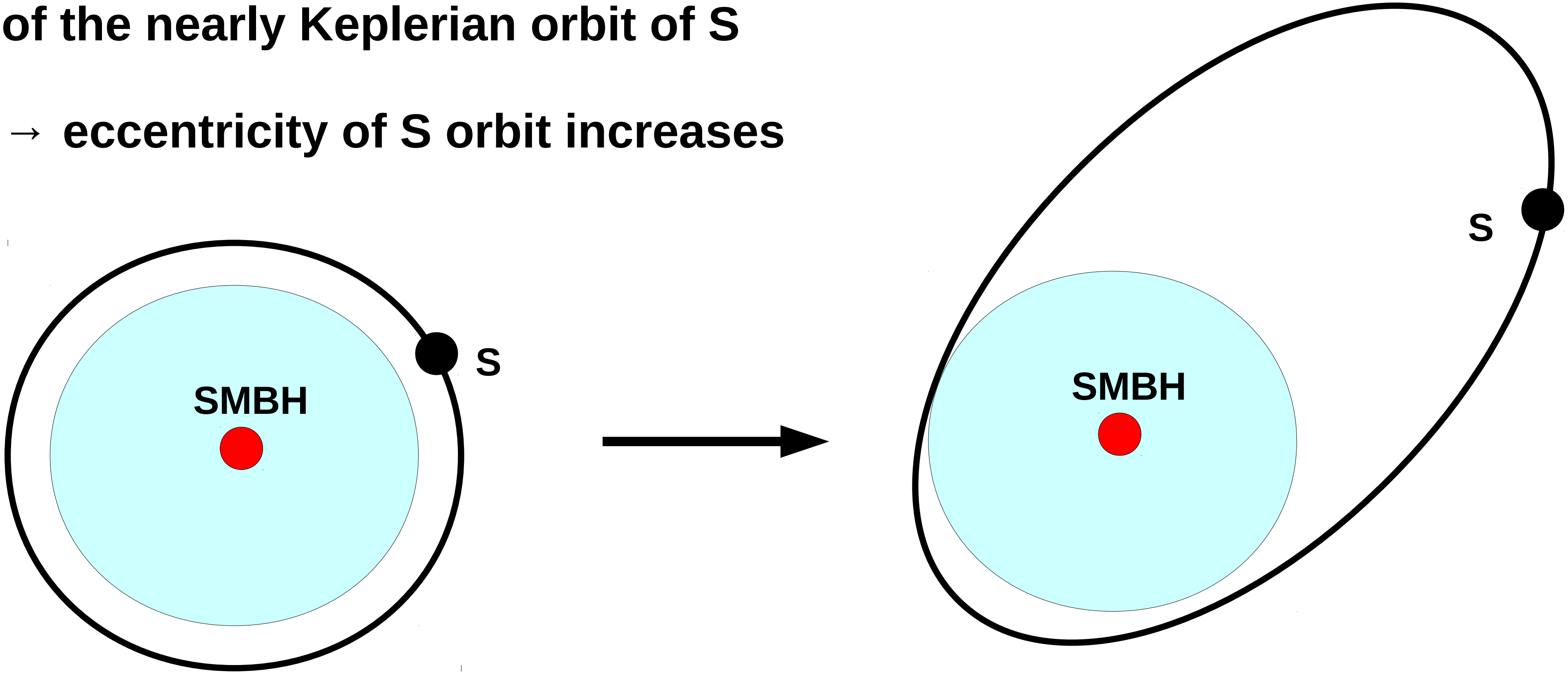
1. dynamics around super-massive black holes (SMBHs)

RESONANT RELAXATION

Stars orbiting between the SMBH and the star S exert TORQUES on S, if they belong to a flattened structure (disc)

Such torques REDUCE ANGULAR MOMENTUM, not energy of the nearly Keplerian orbit of S

→ eccentricity of S orbit increases



Rauch & Tremaine 1996, NewA, 1, 149
Hopman & Alexander 2006, ApJ, 645, 1152

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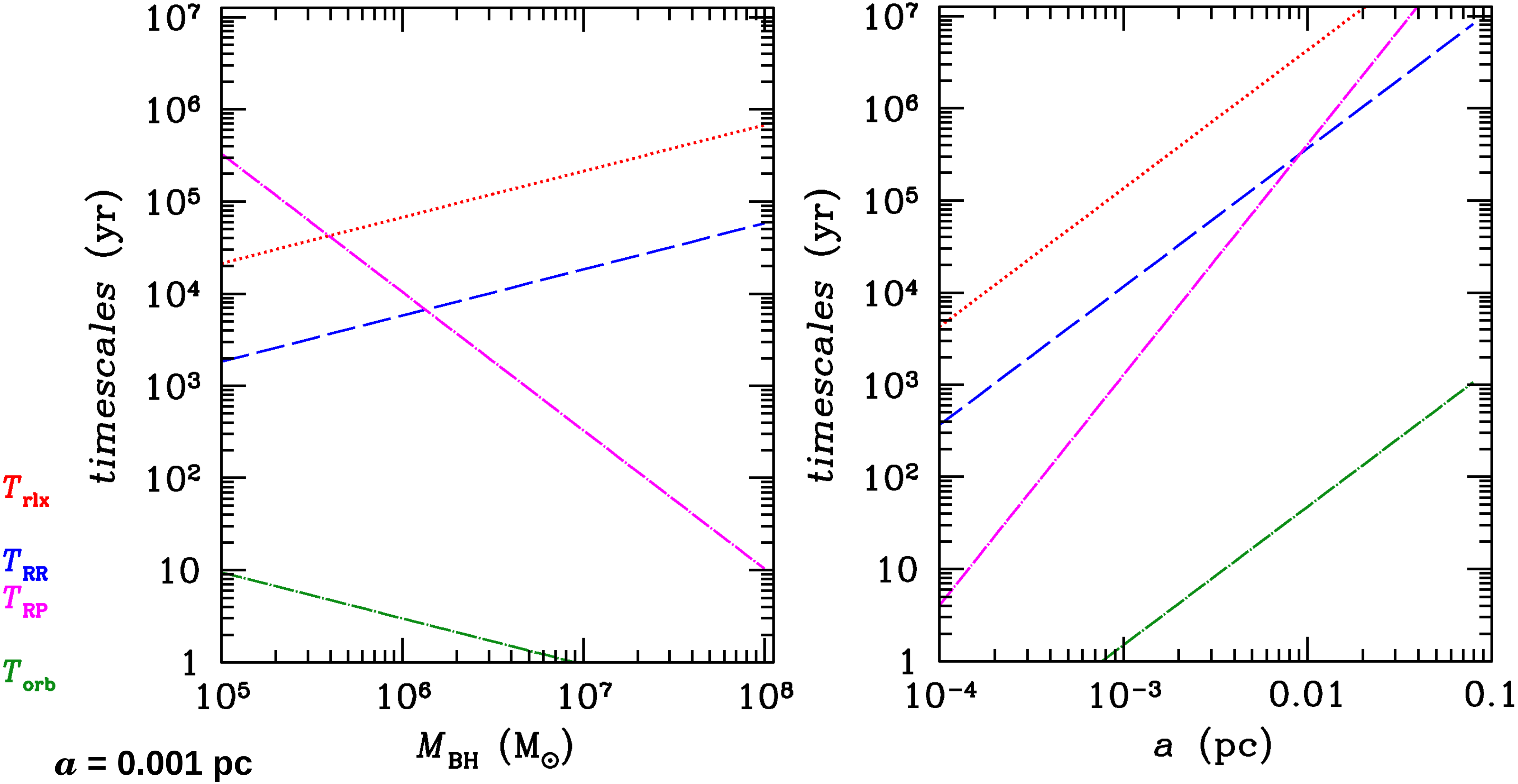


M* = 10 Msun
ecc=0.1
N* = 10^3

2. formation and dynamics of stars in the central parsec

What is puzzling about the young stars in the central pc?

They should not be there!

STARS FORM from gravitational collapse of dense cold gas in the cores of MOLECULAR CLOUDS

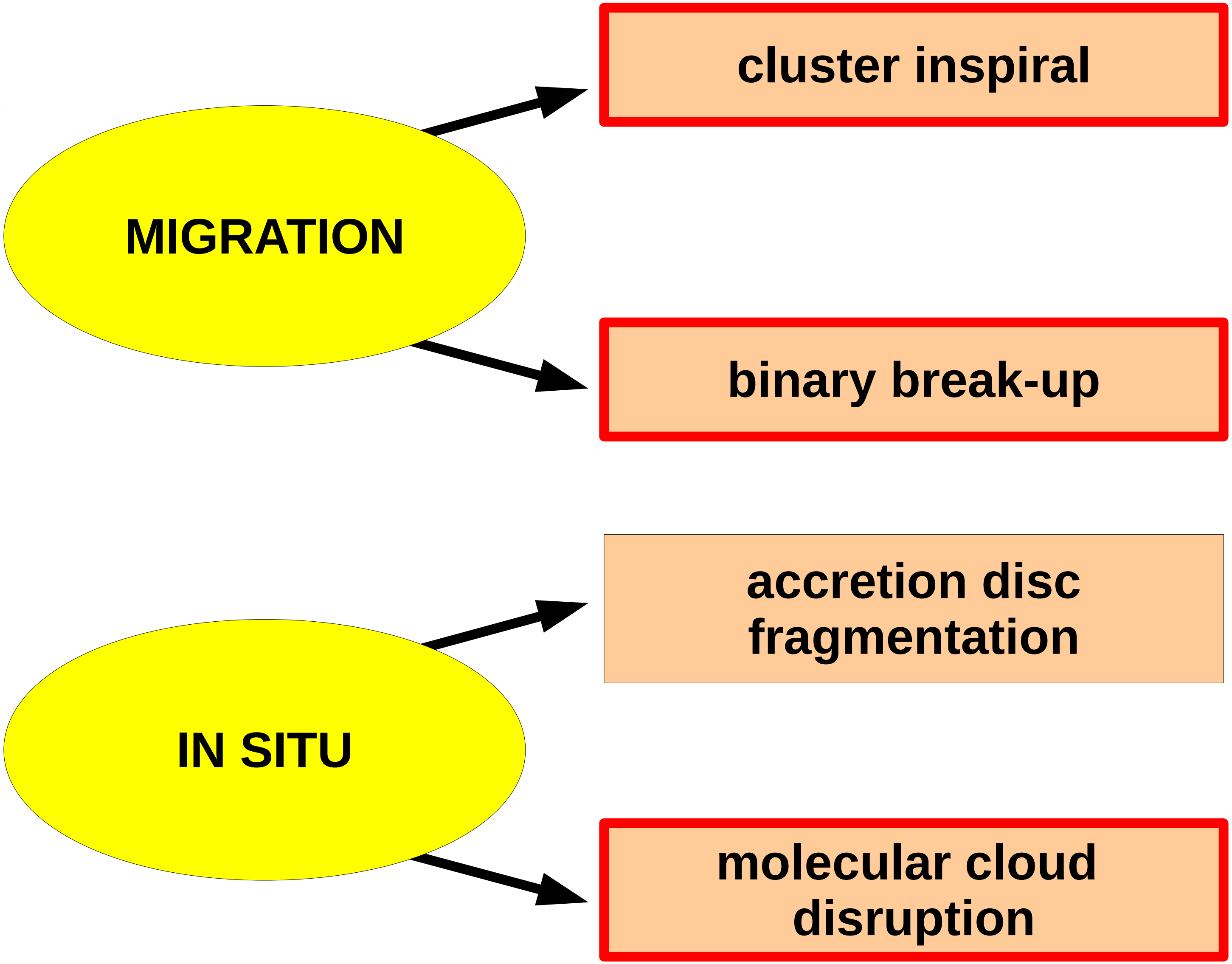
A molecular cloud is disrupted by the tidal field exerted by the SMBH if its density is lower than the Roche density

$$n_{\mathrm{RL}} \sim 10^{7}\,\mathrm{cm}^{-3} \left(\frac{m_{\mathrm{BH}}}{3\times 10^{6}\,M_{\odot}}\right) \left(\frac{\mathrm{pc}}{r}\right)^{3}$$

Typical molecular cloud core density < 10^{6} cm^{-3}

The stars cannot form in 'normal conditions' if the cloud is disrupted (Phinney 1989).

Scenarios to explain the young stars (and in general the formation of NSCs)



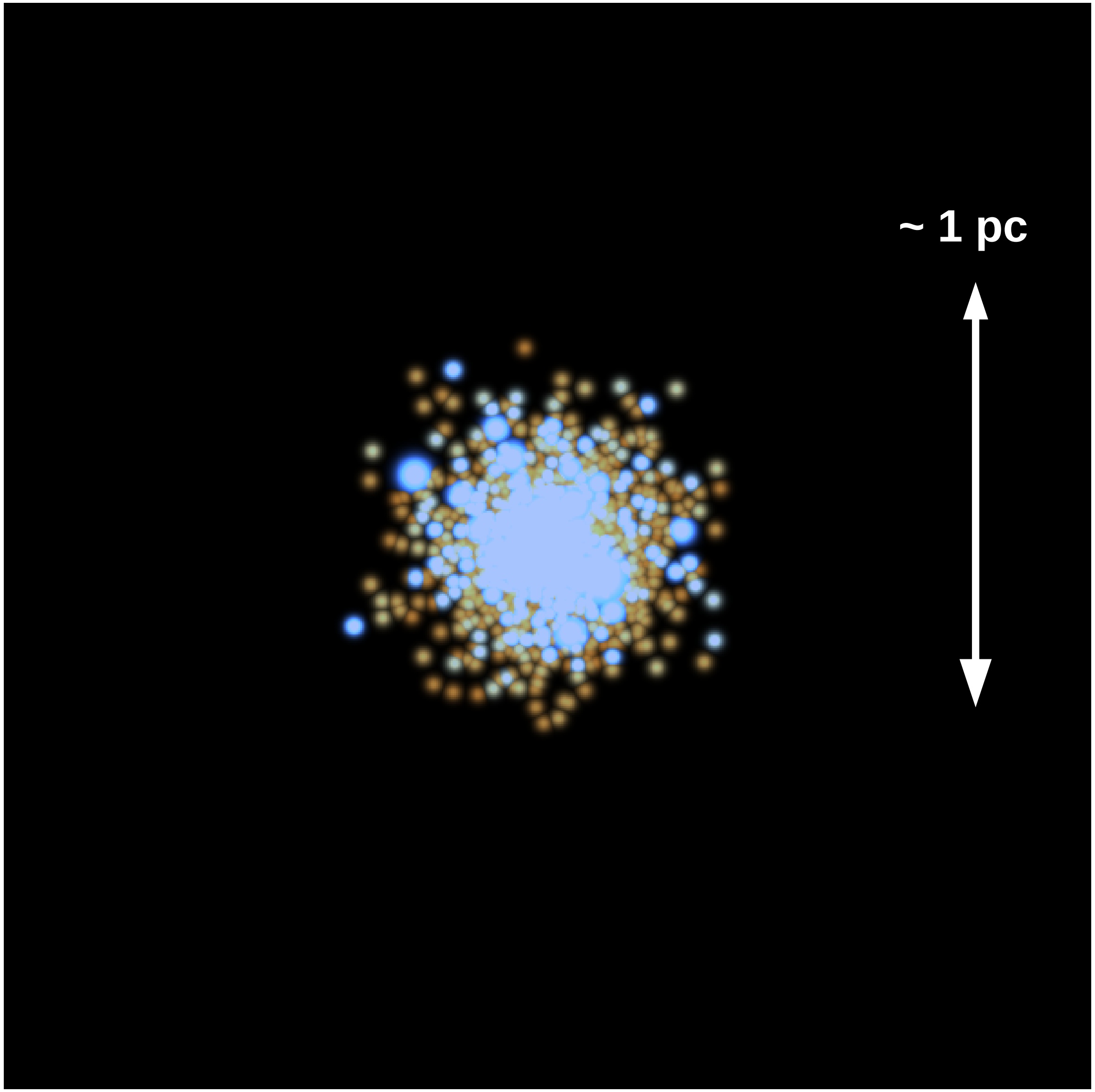
Cluster inspiral:

A star cluster spirals toward the SMBH by DYNAMICAL FRICTION and is disrupted by the SMBH tidal field

(Gerhard 2001, Portegies Zwart et al. 2003, Gurkan & Rasio 2005, Fujii et al. 2008; Petts & Gualandris 2017)

STAR CLUSTER SINKS BY DYNAMICAL FRICTION

WHILE DISRUPTED BY SMBH SHEAR



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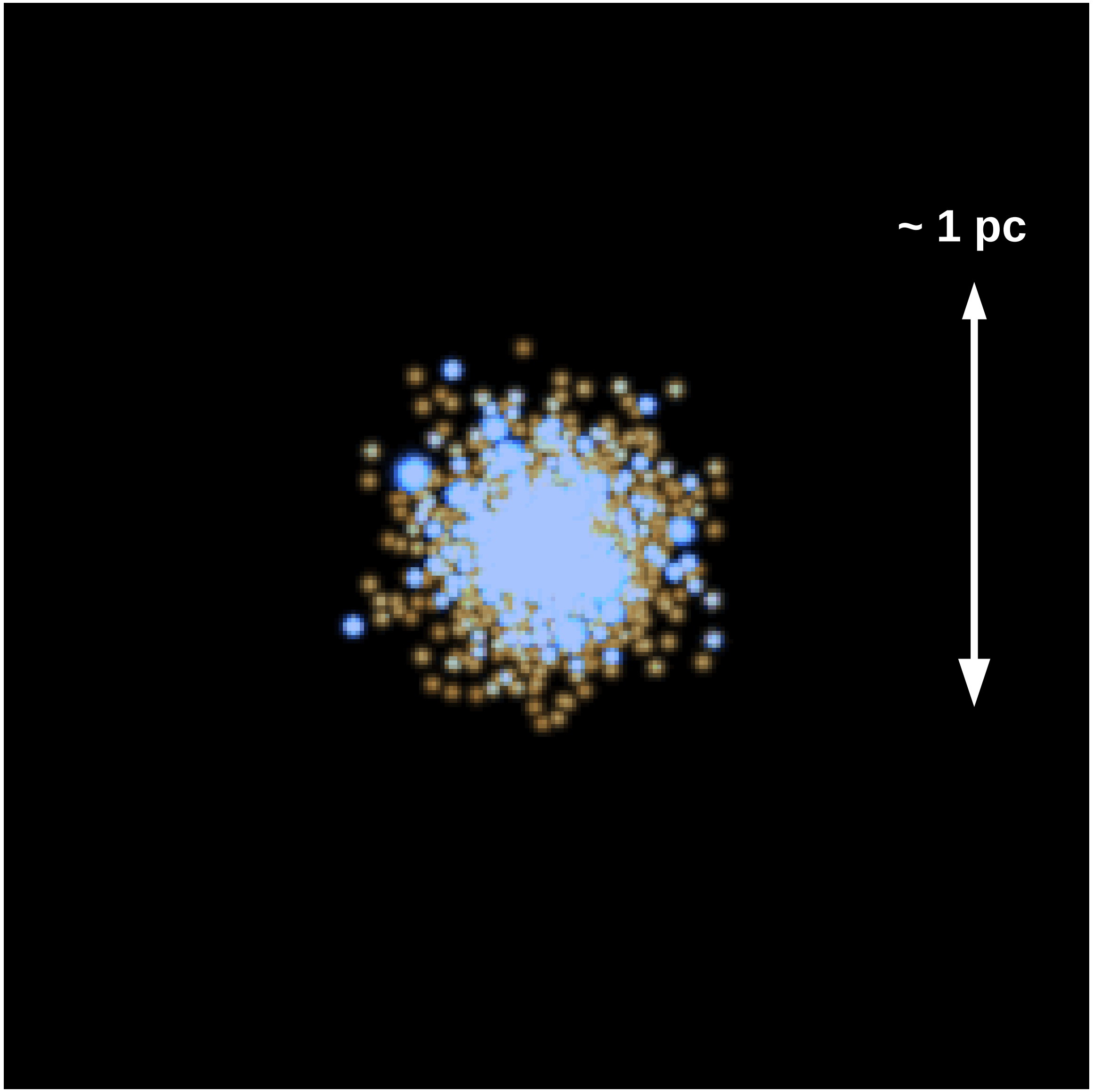

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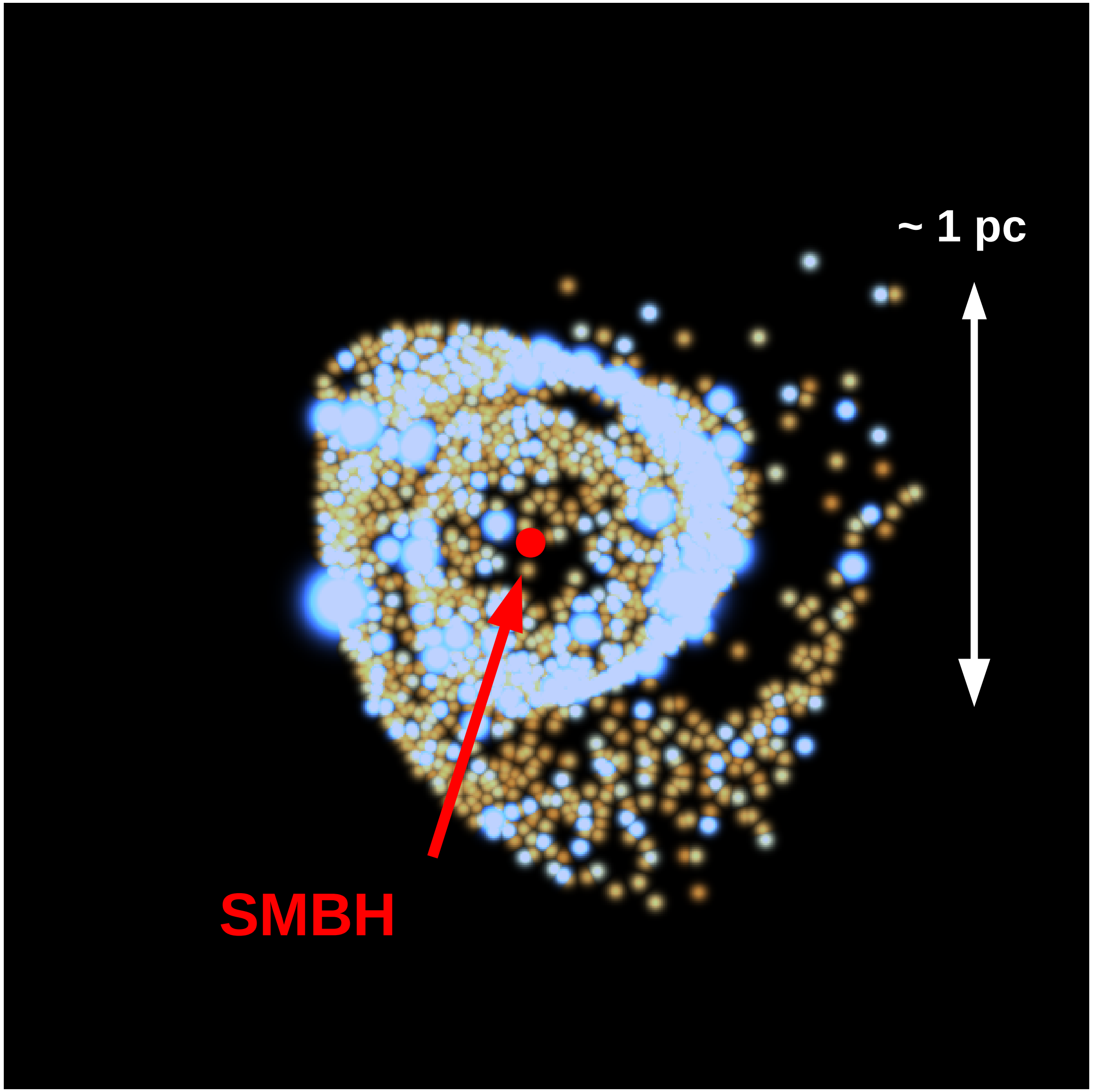


Molecular cloud disruption:

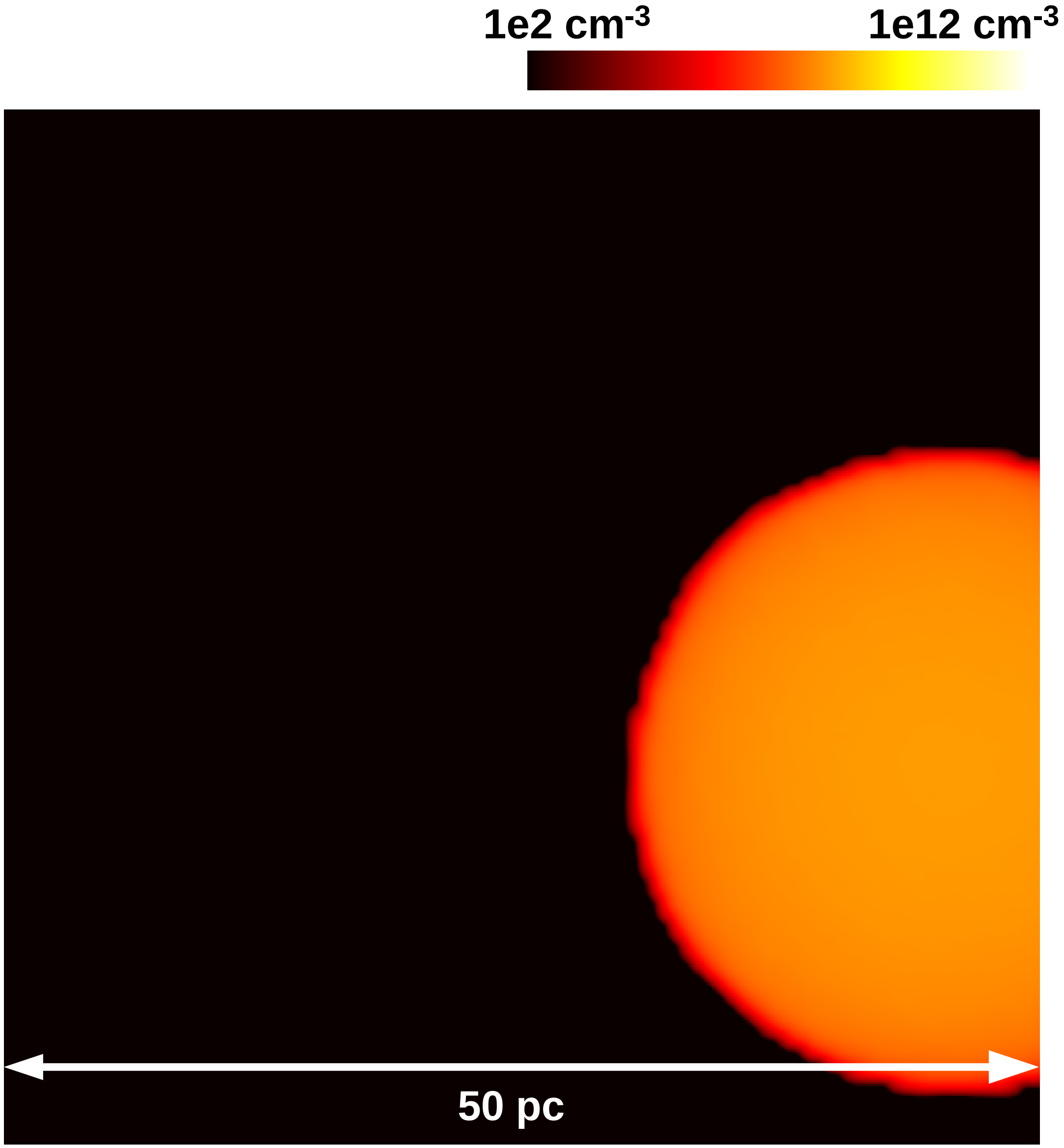
A molecular cloud is disrupted by the SMBH, but

(i) the residual angular momentum,
(ii) the shocks that take place in gas streams

might lead to the formation of

a DENSE DISC,

denser than Roche density



Bonnell & Rice 2008; MM et al. 2008; Hobbs & Nayakshin 2009;
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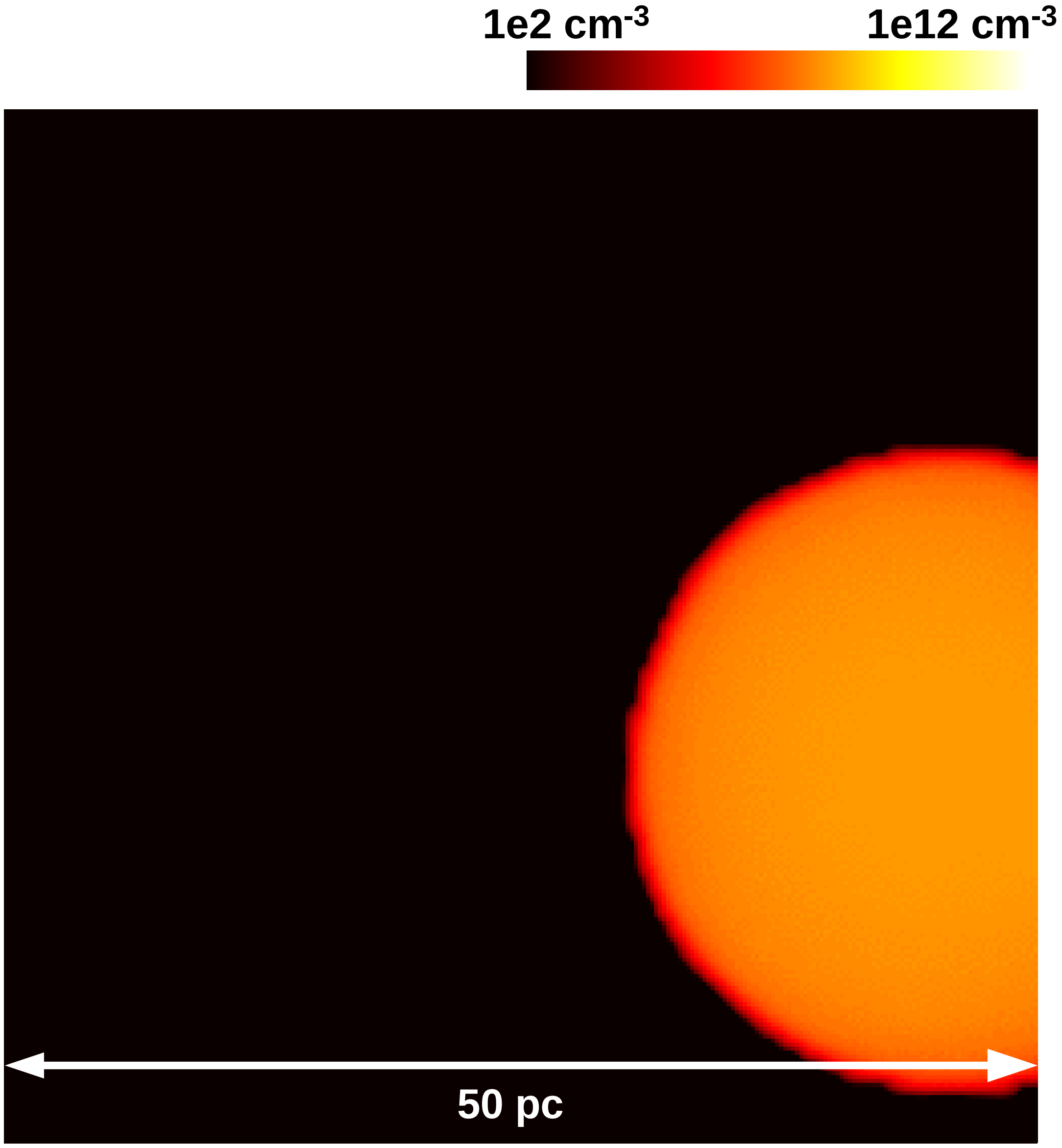

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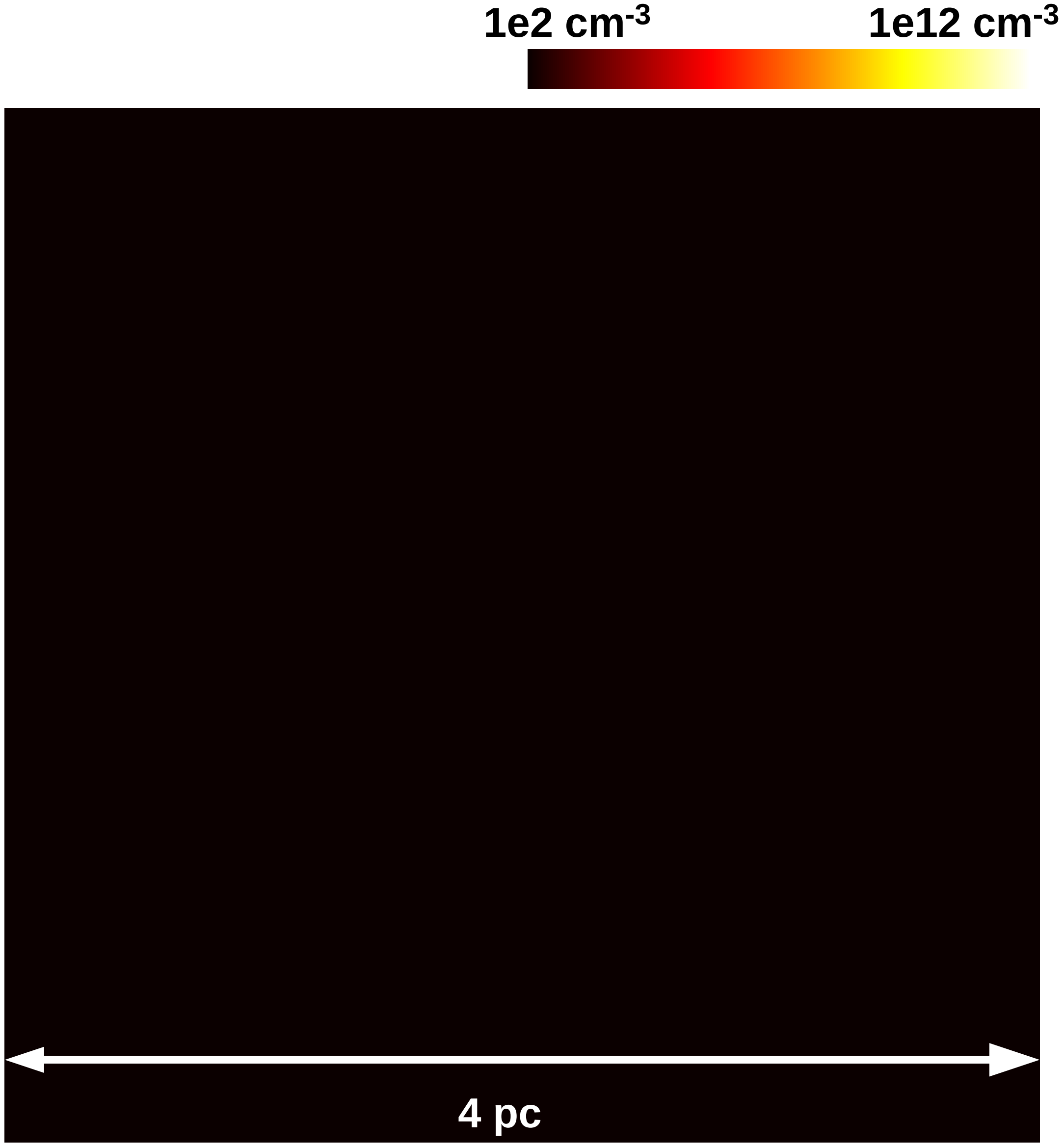

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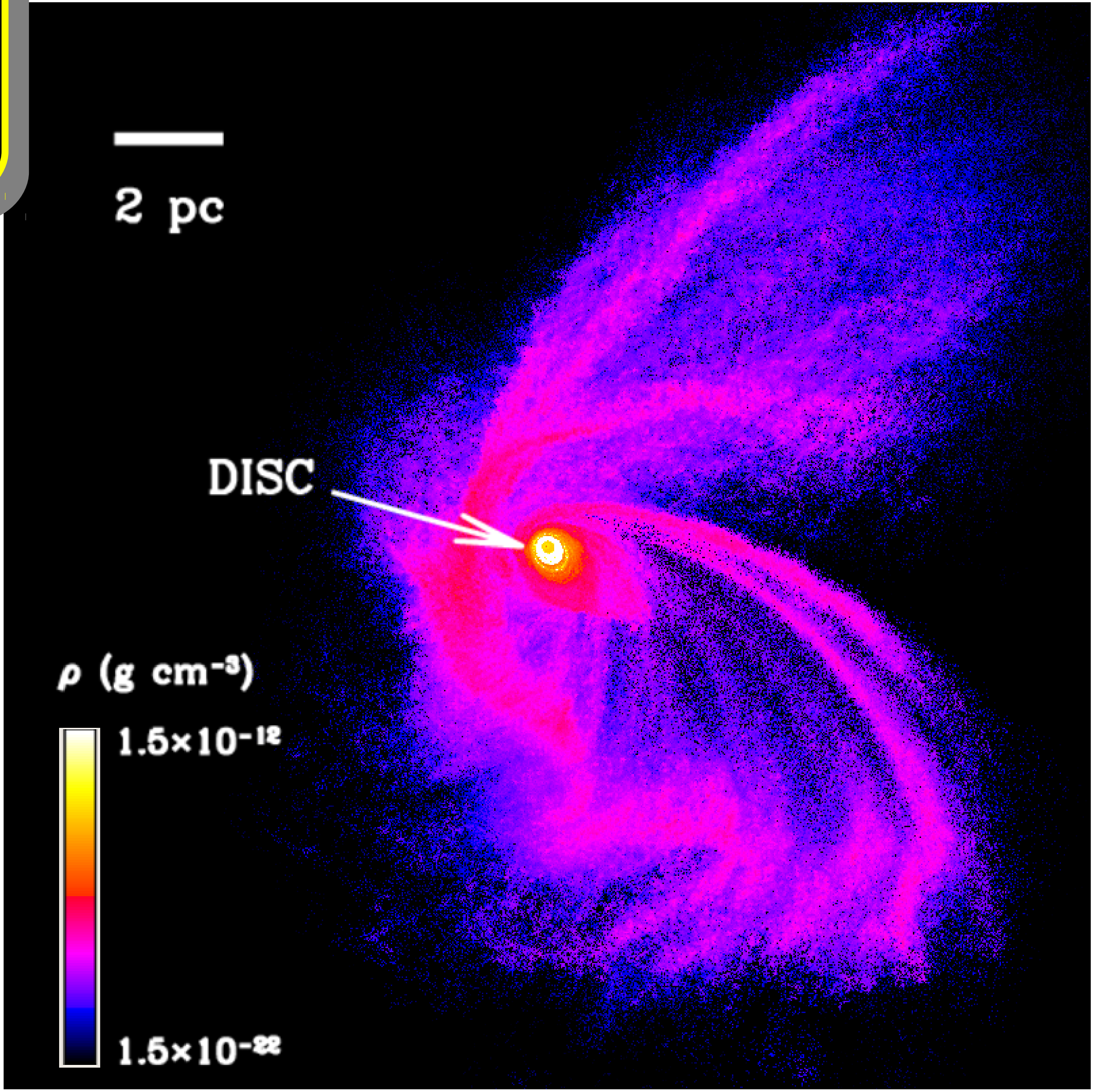
2. formation and dynamics of stars in the central parsec

Stars can form in a gas disc, born from the disruption of a molecular cloud



INGREDIENTS:

* A turbulent molecular cloud R~15 pc, M~10^5 M$_\odot$

* a SMBH sink particle

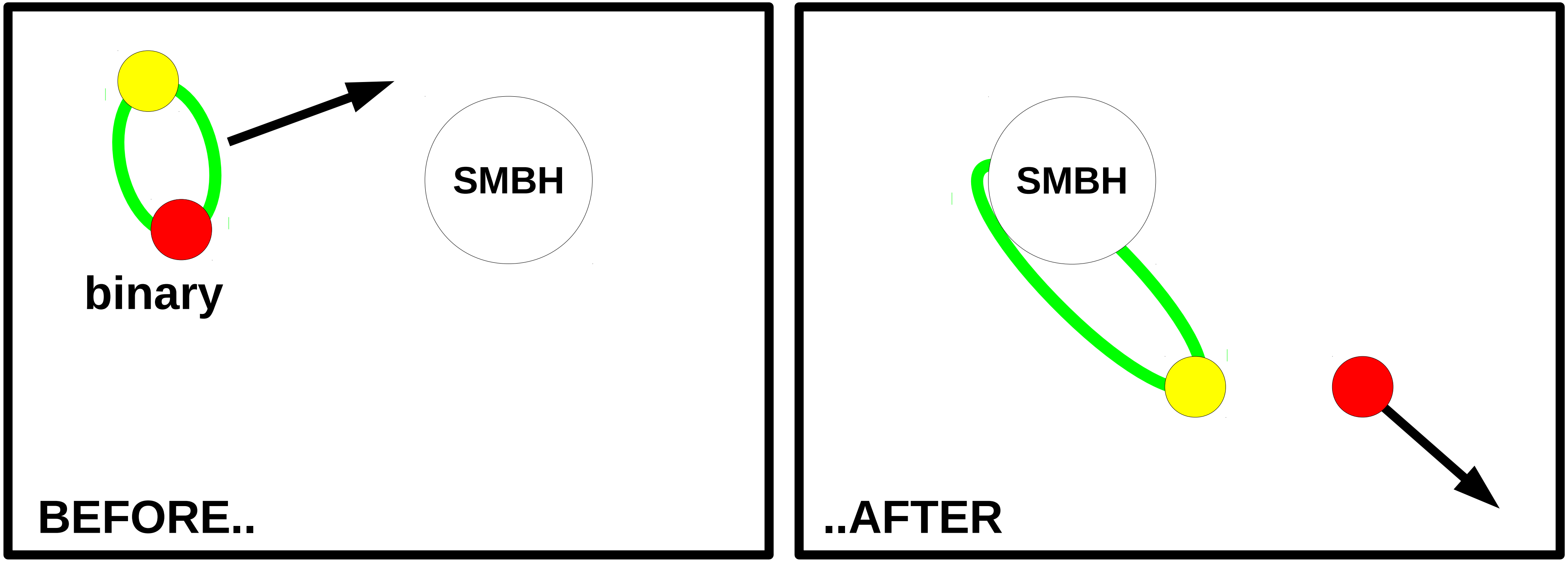
* integration with SPH or AMR

* cooling + Planck & Rosseland opacities

MM et al. 2012

Binary break-up:

Several binaries are captured and disrupted by the SMBH



$$a_c \sim 0.6 \left(\frac{m_{\rm BH}}{M_{\rm bin}}\right)^{1/3} r_{\rm tid}$$

$$e_c \sim 1 - \frac{r_{\rm tid}}{a_c} \sim 0.97$$

Hills 1991, Perets et al. 2007, Perets & Gualandris 2010

2. formation and dynamics of stars in the central parsec

Scenario	Star distribution	Eccentricity	Age
Cluster inspiral	disc (*r*~1 pc)	depends on cluster orbit	> 6 Myr
Binary break-up	random	>0.9	depends on stellar population
Accretion disc	disc (*r*~0.5 pc)	~ 0	1 – 6 Myr
Molecular cloud disruption	disc (*r*~0.5 pc)	~ 0.3 (depends on cloud orbit)	1 – 6 Myr

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GOOD FOR S-cluster

GOOD FOR CLOCKWISE DISC

3. gravitational waves involving NSCs

i- mergers of stellar BH binaries in NSCs

ii- mergers of stellar BHs with SMBHs

iii- mergers of SMBH binaries

i- mergers of stellar BH binaries in NSCs

* Same effects as lecture 2017dynamics5.pdf :
exchanges, hardening

* Potential well makes it difficult for BH binaries
to be ejected after 3body encounters

* Potential well makes it difficult for BH binaries
to be ejected by GRAVITATIONAL WAVE RECOIL
(Campanelli et al. 2007, ApJ, 659, L5;
Lousto & Zlochower 2009, Phys Rev D, 79, 4018)
→ more massive BH binaries in galactic nuclei?

* Kozai-Lidov of stellar BH binary with SMBH (tertiary)
enhances mergers (Antonini & Rasio 2016, ApJ, 831, 187)

3. gravitational waves involving NSCs

i- mergers of stellar BH binaries in NSCs

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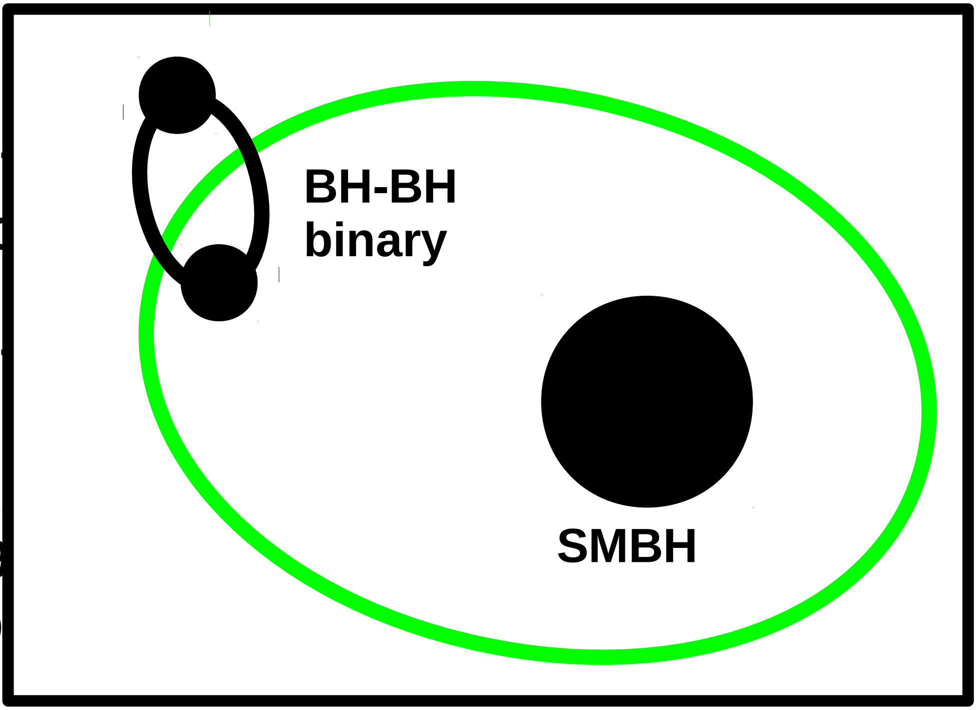

* Kozai-Lidov of stellar BH binary with SMBH (tertiary)
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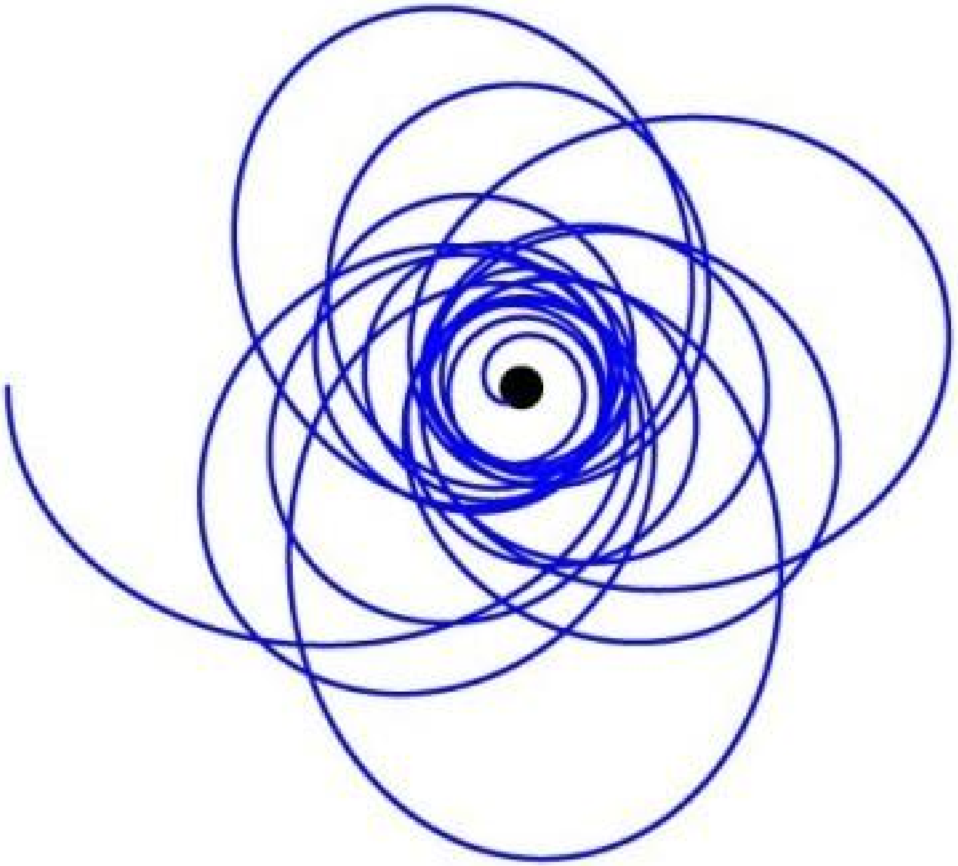
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

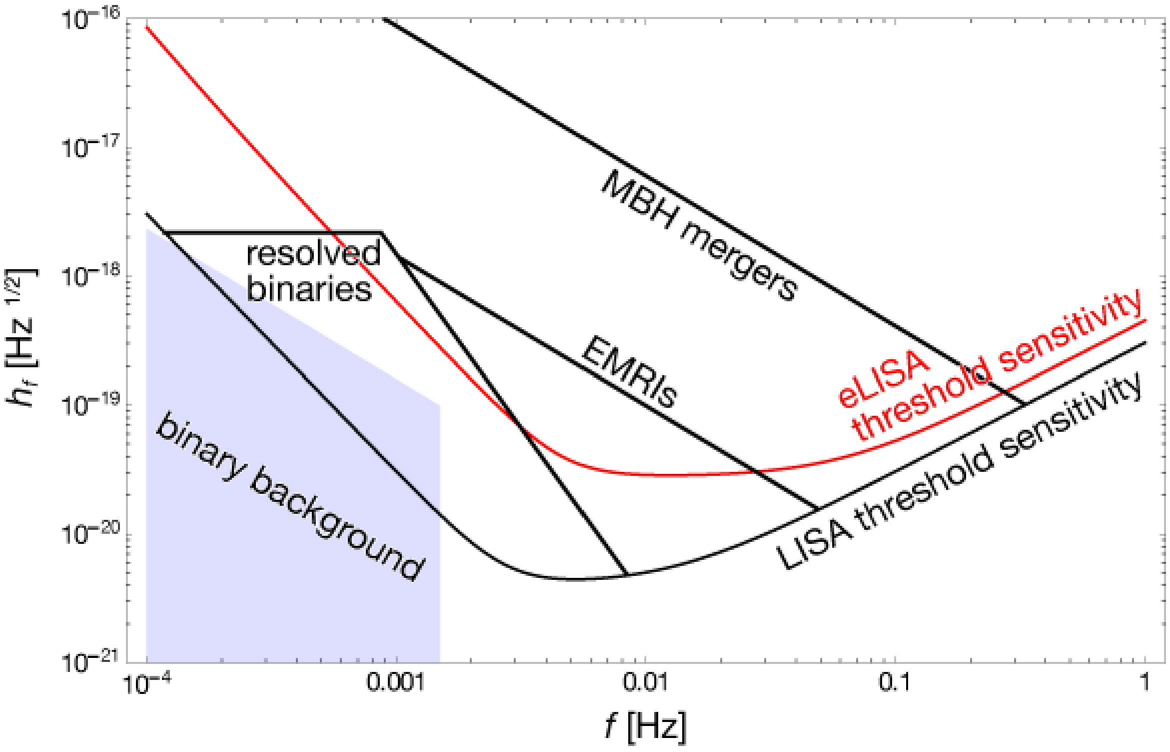
* Extreme mass-ratio inspirals (EMRIs)



ii- mergers of stellar BHs with SMBHs

*** Extreme mass-ratio inspirals (EMRIs)**

*** Frequency of GWs never in LIGO-Virgo range but in space-borne detectors range (LISA)**



Jonathan R. Gair and Michele Vallisneri and Shane L. Larson and John G. Baker, "Testing General Relativity with Low-Frequency, Space-Based Gravitational-Wave Detectors",

Living Rev. Relativity, 16 (2013), 7, doi:10.12942/lrr-2013-7, URL (accessed <date>): http://www.livingreviews.org/lrr-2013-7

ii- mergers of stellar BHs with SMBHs

* Extreme mass-ratio inspirals (EMRIs)

* Frequency of GWs never in LIGO-Virgo range
but in space-borne detectors range (LISA)

* Extremely unlikely event:

depends on interplay between
- relativistic precession (RP)
- GW decay timescale (GW)
- resonant relaxation (RR)

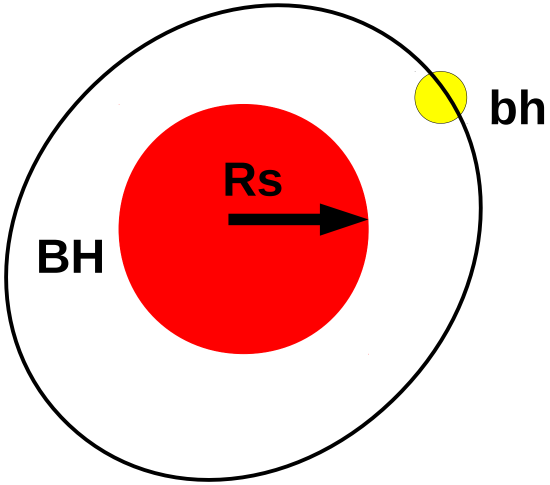
HOW?

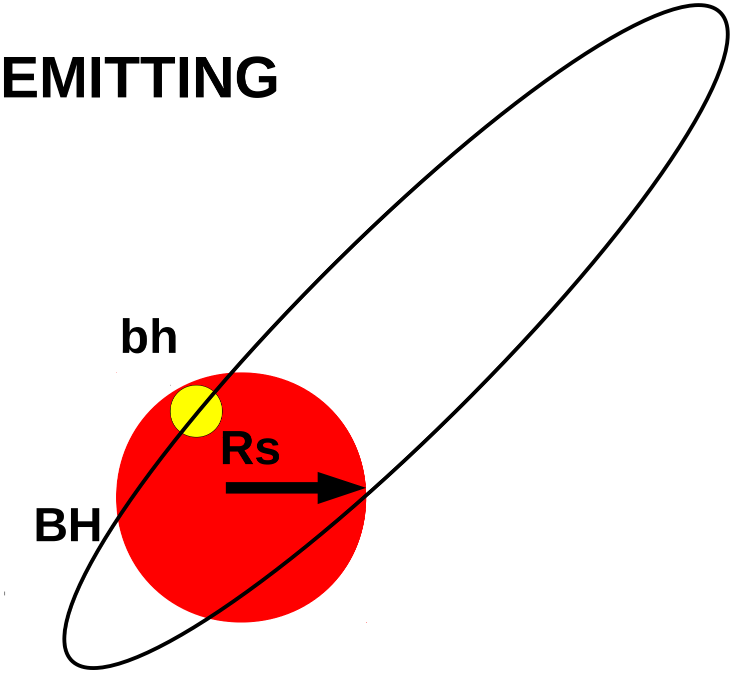
ii- mergers of stellar BHs with SMBHs

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

- RR tends to increase eccentricity of stellar BH orbit

If RR more efficient than GW decay
the stellar BH PLUNGES into
Schwarzschild radius BEFORE EMITTING
observable GWs



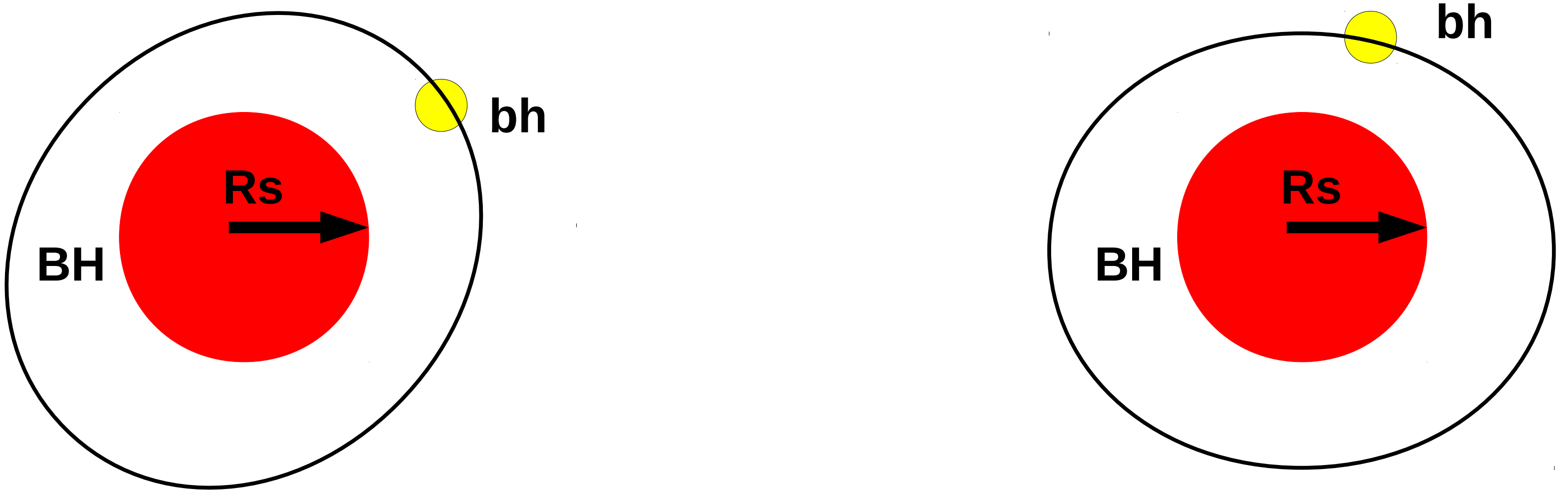


WE HAVE NO EMRI, because no GW

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

- RP tends to change the pericentre of the BH orbit

If RP more efficient than RR
the fast changes of BH pericentre
nullify the effect of RR torques
and BH orbit remains ~ the same



The shielding effect of RP on RR is called
<u>Schwarzschild barrier (Merrit+ 2011, PhRvD, 84, 4024)</u>:
RP keeps orbits stable against plunge

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

- If RR more efficient than RP
  → BH plunges into SMBH without observable GW event

- If RP more efficient than RR
  → BH avoids plunge-in but remains on a stable circular orbit: it does not merge

- A fine-tuning is needed for GW decay to be more efficient than both RR and RP
  → the BH spiral-in into the SMBH and emits observable GWs

LET US LOOK AT THE TIMESCALES

ii- mergers of stellar BHs with SMBHs

We must calculate the three timescales:

(1) RELATIVISTIC PRECESSION (RP) TIMESCALE

$$T_{\rm RP} = 1.3 \times 10^3 {\rm yr}\,(1-e^2)\left(\frac{a}{10^{-3}{\rm pc}}\right)^{5/2}\left(\frac{4\times 10^6 M_\odot}{m_{\rm BH}}\right)^{3/2}$$

(2) GW DECAY TIMESCALE

$$T_{\rm GW} \sim 6 \times 10^{12} {\rm yr}\,(1-e^2)^{7/2}\left(\frac{a}{1~{\rm mpc}}\right)^4\left(\frac{3\times 10^6 M_\odot}{m_{\rm BH}}\right)^2\left(\frac{10~M_\odot}{m_{\rm bh}}\right)$$

(3) RESONANT RELAXATION (RR) TIMESCALE

$$T_{\rm RR} \sim 10^4 {\rm yr}\left(\frac{a}{1~{\rm mpc}}\right)^{3/2}\left(\frac{m_{\rm BH}}{3\times 10^6 M_\odot}\right)^{1/2}\left(\frac{10~M_\odot}{m_{\rm bh}}\right)\left(\frac{10^3}{N_*}\right)^{1/2}$$

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

In summary,

(1) If $T_{GW} < \min(T_{RR}, T_{RP})$

GW emission is efficient and leads to an EMRI

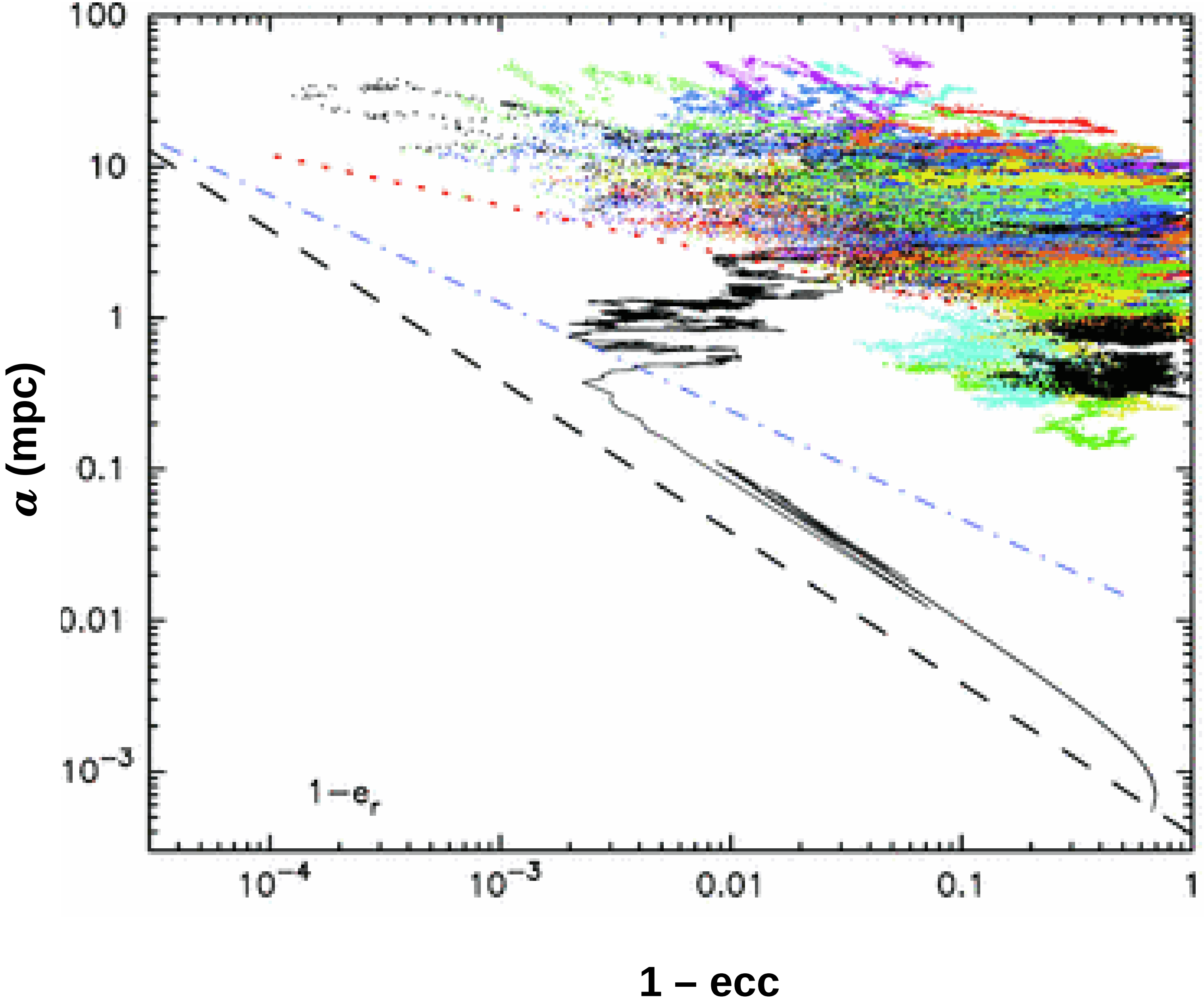
(2) If $T_{RR} < \min(T_{RP}, T_{GW})$

RR increases the eccentricity till the BH PLUNGES into the Schwarzschild radius of the SMBH, WITHOUT EMRI

(3) If $T_{RP} < \min(T_{RR}, T_{GW})$

RP prevents RR from plunging the BH into the Schwarzschild radius, but the BH is 'stuck' in its orbit and cannot merge
Another perturbation is needed to produce an EMRI or a plunge!

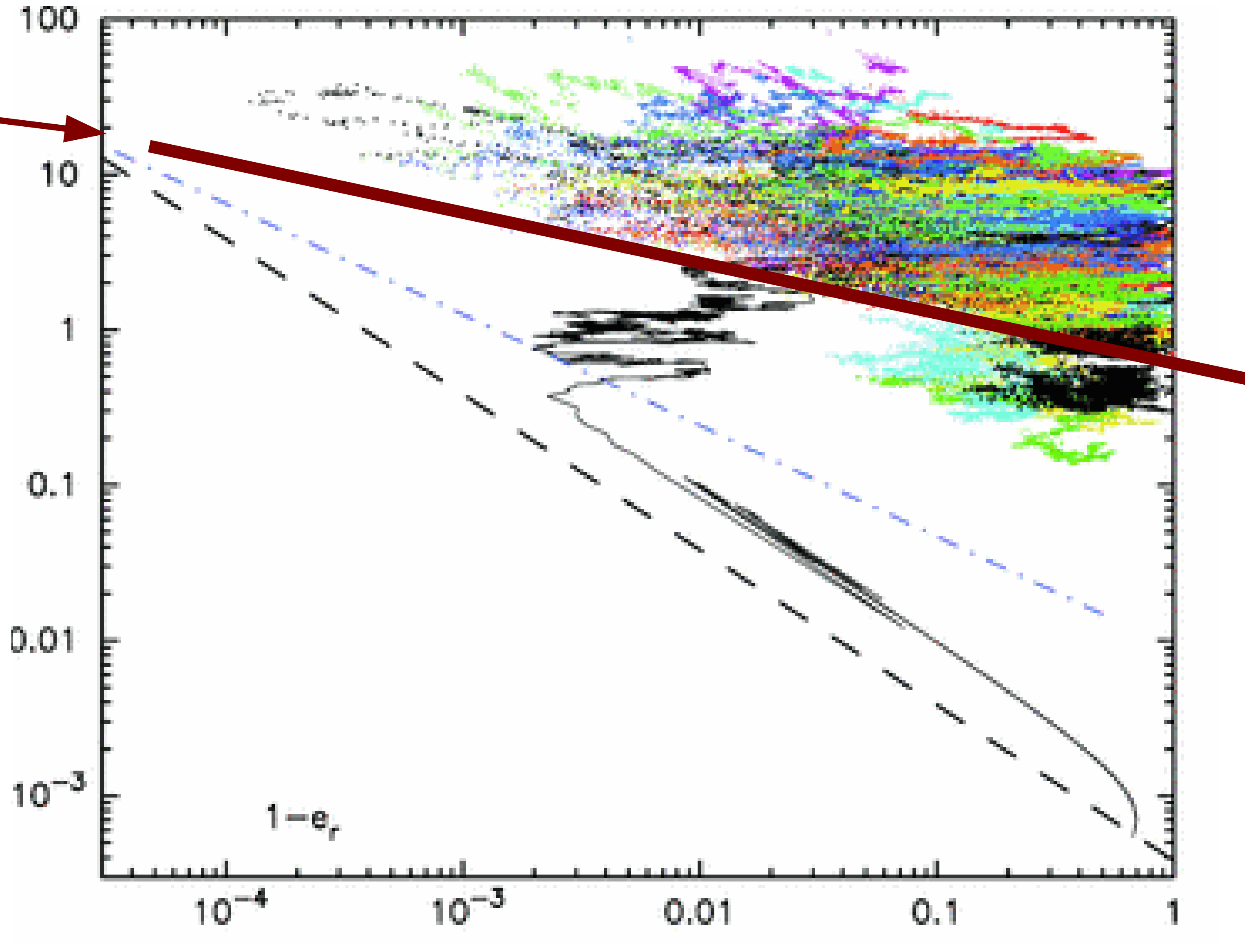
The Schwarzschild barrier according to Merritt+ 2011 calculations



ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier:
$t_{RP}<t_{RR}$

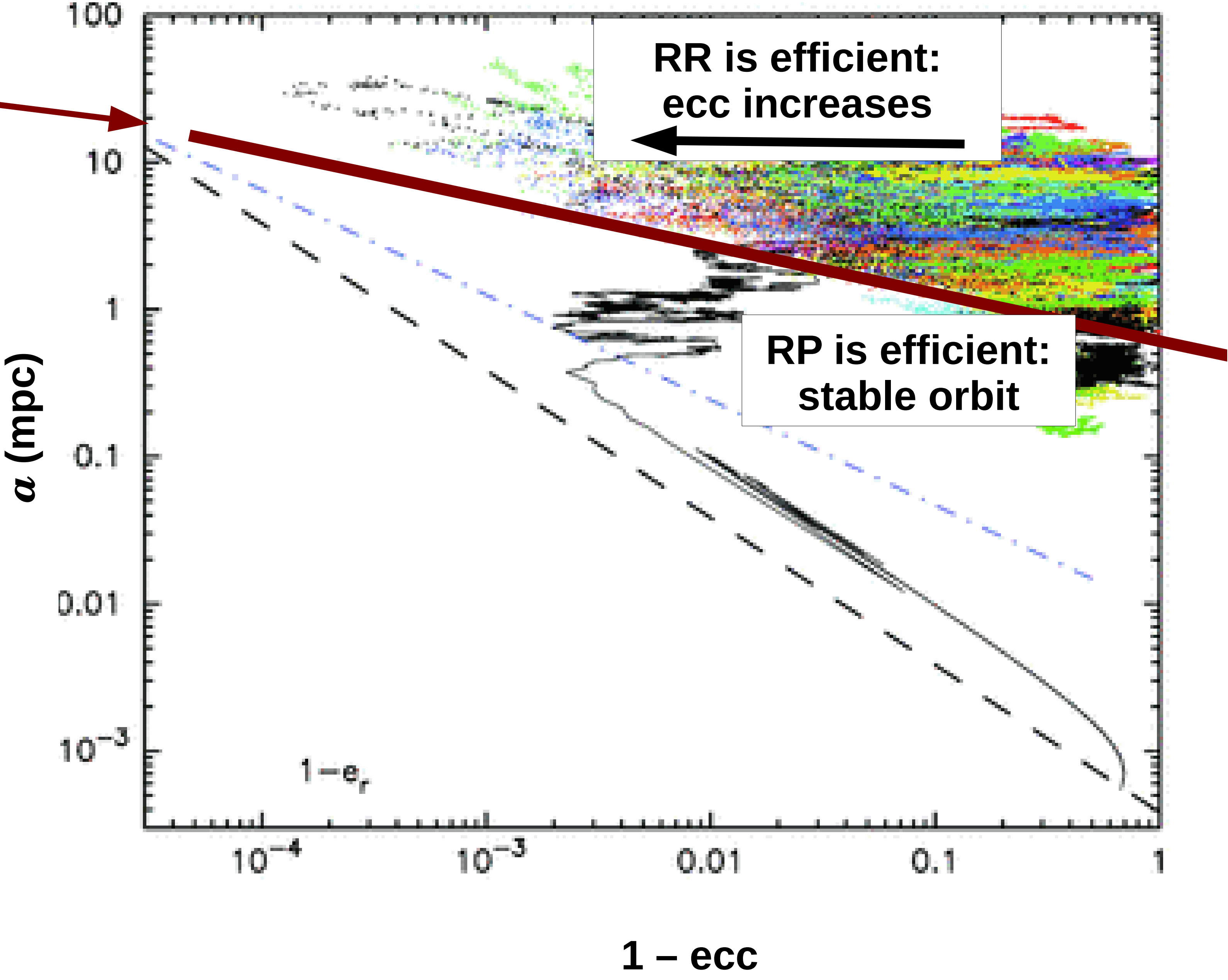


1 – ecc

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier:
$t_{RP} < t_{RR}$

*** Most orbits stall when encounter the barrier**



RR is efficient: ecc increases

RP is efficient: stable orbit

a (mpc)

1 – ecc

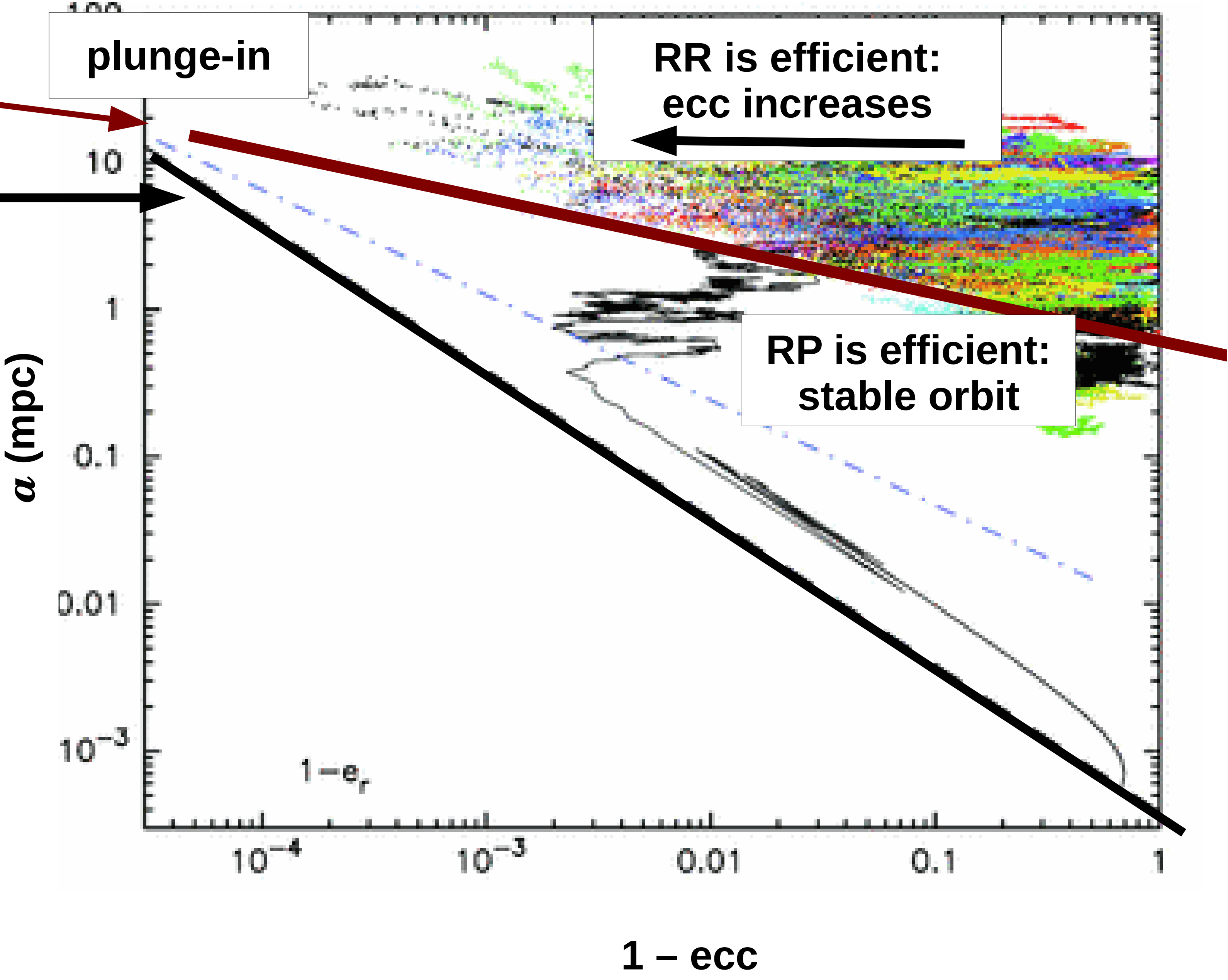
The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier:
$t_{RP} < t_{RR}$

Capture by SMBH (loss cone)

*** Most orbits stall when encounter the barrier**

*** If orbit hits the capture radius when ecc ~1 we have a plunge-in (no significant GW emission)**



plunge-in

RR is efficient: ecc increases

RP is efficient: stable orbit

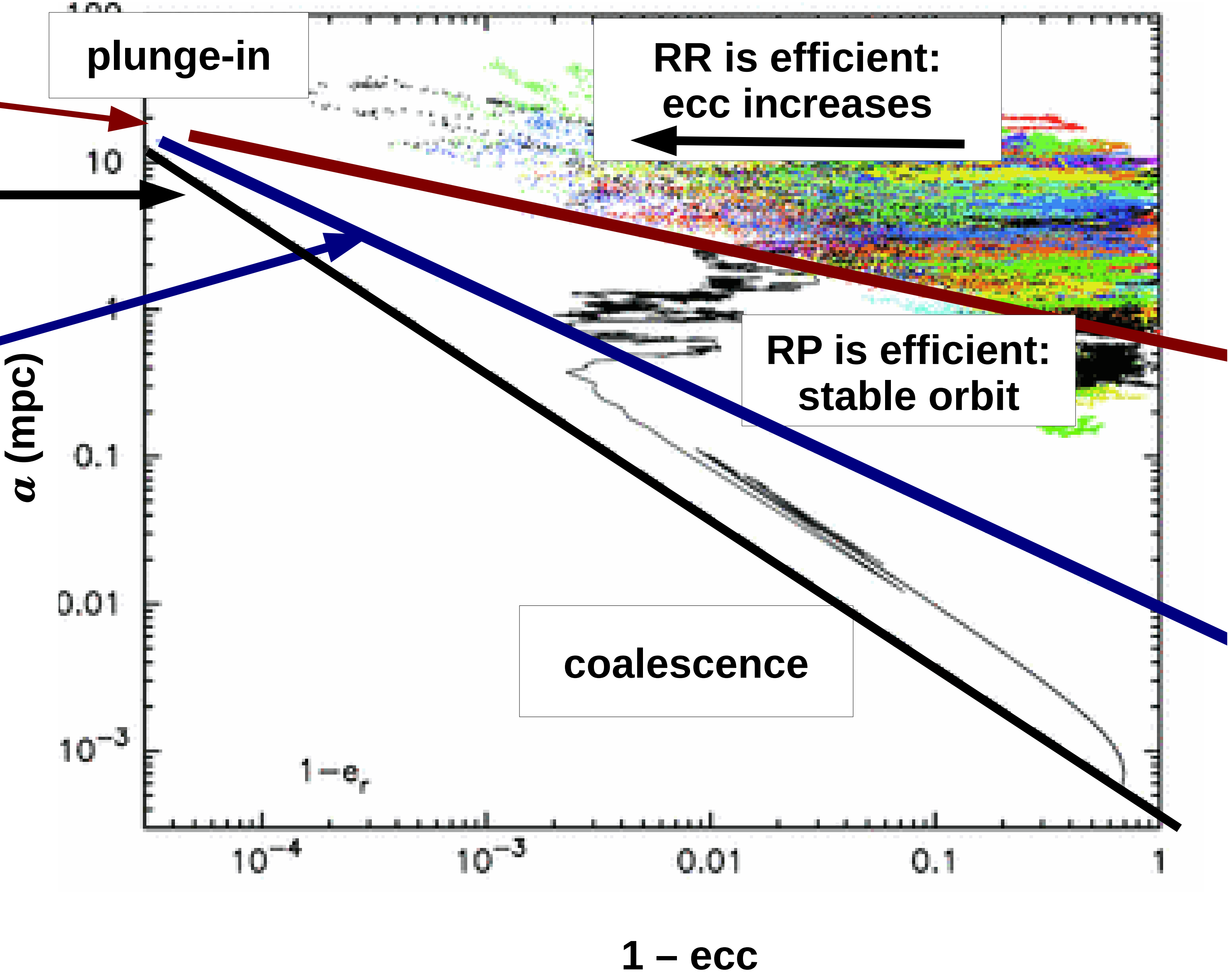
The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier:
$t_{RP}<t_{RR}$

Capture by SMBH (loss cone)

Line for coalescence:
$t_{GW}<t_{RP}$

objects entering this line coalesce in Hubble time and EMIT GWs (EMRIs)



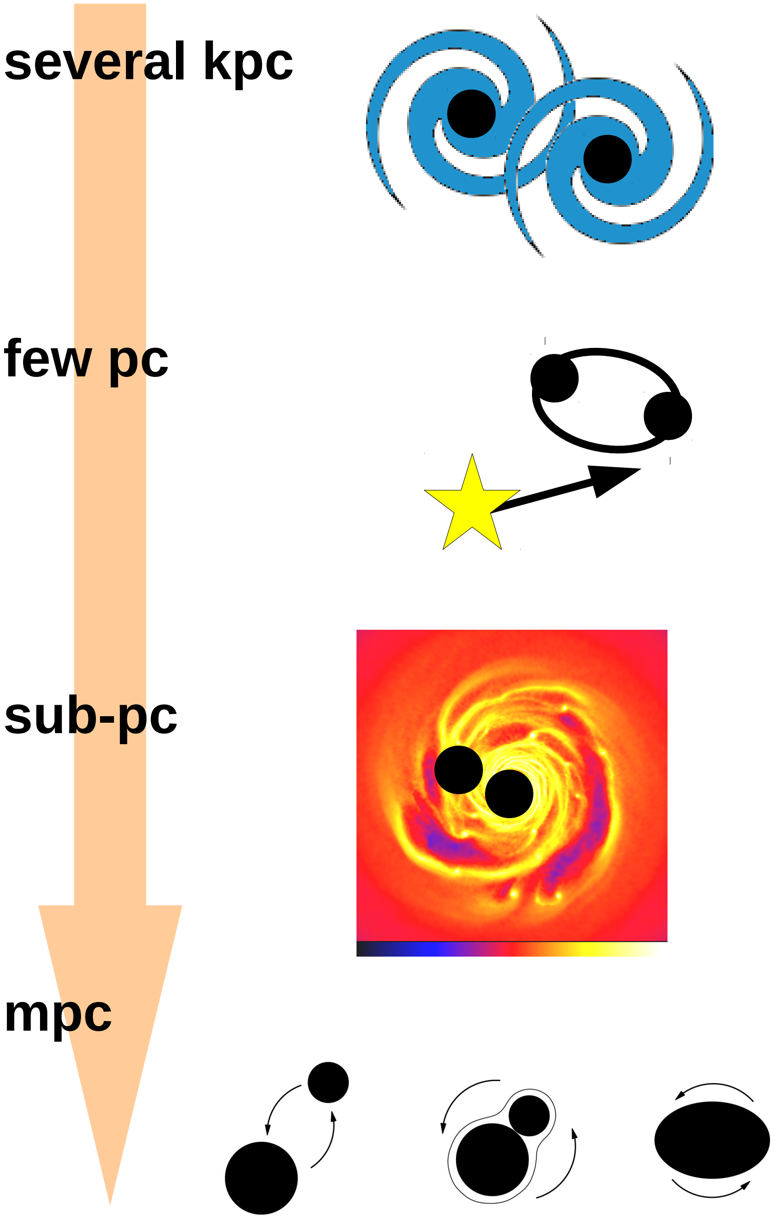
plunge-in

RR is efficient: ecc increases

RP is efficient: stable orbit

coalescence

iii- mergers of SMBH binaries



several kpc

* SMBHs might form binaries
(thanks to dynamical friction)
after their host galaxies merged

few pc

* a SMBH-SMBH binary shrinks
by 3-body encounters
(same as stellar binaries)
until 3-body encounters are
no longer efficients because $\frac{da}{dt} \propto -a^2$

sub-pc

* further physics needed to
shrink semi-major axis
(eg dynamical friction in a gas disc)

mpc

* if the SMBH-SMBH enters
the GW regime it merges by GW decay

References:

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- Mapelli & Gualandris, 2016, Lecture Notes in Physics, Springer, 905, 205, https://arxiv.org/abs/1505.05473

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- Rauch & Tremaine 1996, NewA, 1, 149

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