Michela Mapelli
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Dynamics of Stars and Black Holes in Dense Stellar Systems:

Lecture V:

STELLAR & INTERMEDIATE-MASS BLACK HOLES

0. stellar black holes (BHs) from star evolution
1. BHs as members of binary systems
2. dynamical formation of BH binaries
3. formation of intermediate-mass BHs (IMBHs)

0. stellar black holes (BHs) from star evolution

Mass of stellar BHs?

Very complicated
(see Marco Limongi's lectures)

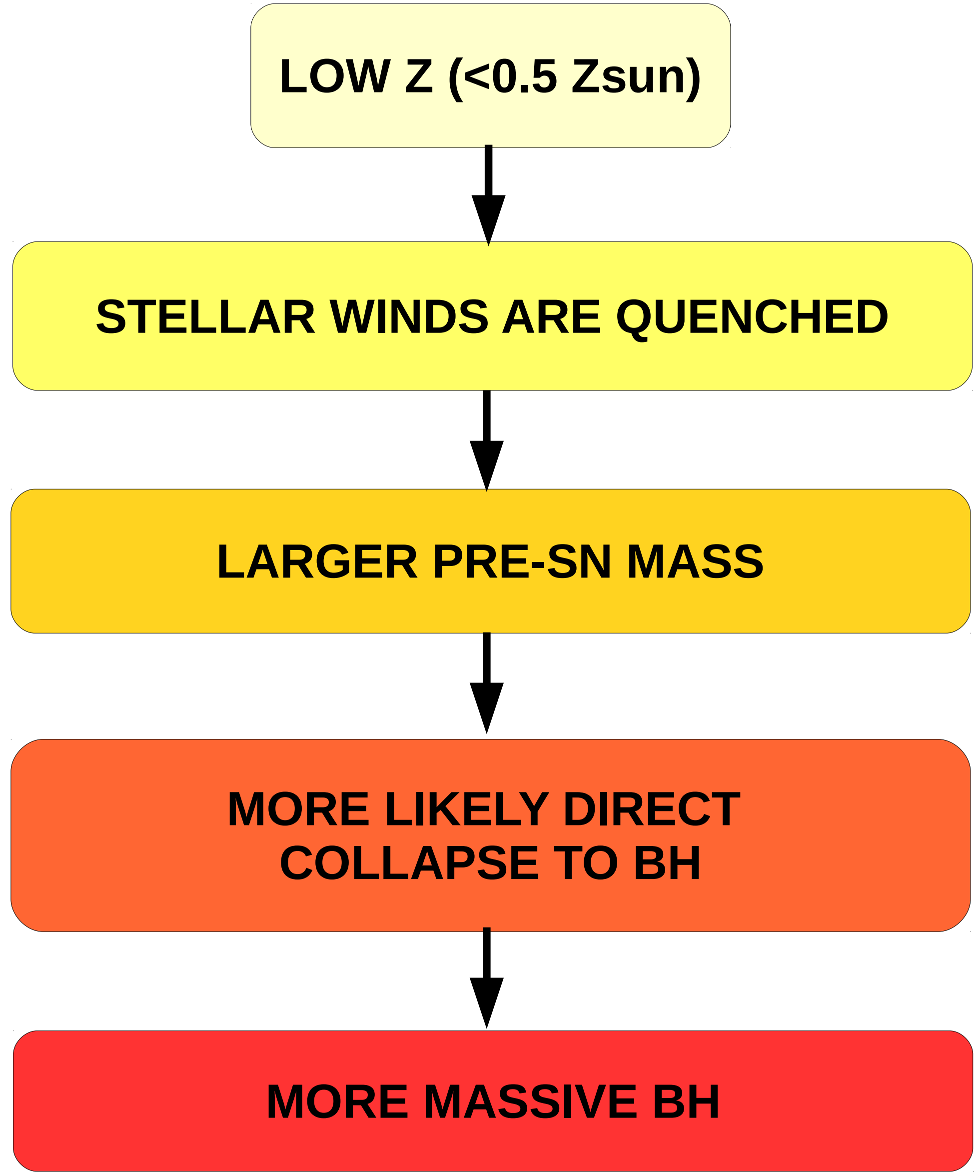
WHAT DO YOU THINK ABOUT
THE MASS OF STELLAR-BORN
BLACK HOLES?

0. stellar black holes (BHs) from star evolution

Mass of stellar BHs?

Very complicated
(see Marco Limongi's lectures)

However, as a rule of thumb:



MM+ 2009, MNRAS, 395, L71
Belczynski+ 2010, ApJ, 714, 1217
Fryer+ 2012, ApJ, 749, 91
MM+ 2013, MNRAS, 429, 2298
Spera, MM & Bressan 2015, MNRAS, 451, 4086

0. stellar black holes (BHs) from star evolution

Back-of-the-envelope calculation to connect direct collapse and pre-supernova mass:

Supernova shock stops if BOUND MASS is too LARGE (Fryer 1999; Fryer & Kalogera 2001)

Back-of-the-envelope calculation:

envelope mass

proto-NS ~ 1 Msun

$$E_{\rm SN} = \frac{G\, M_{\rm env}\, (M_{\rm env} + M_{\rm core})}{R_{\rm env}}$$

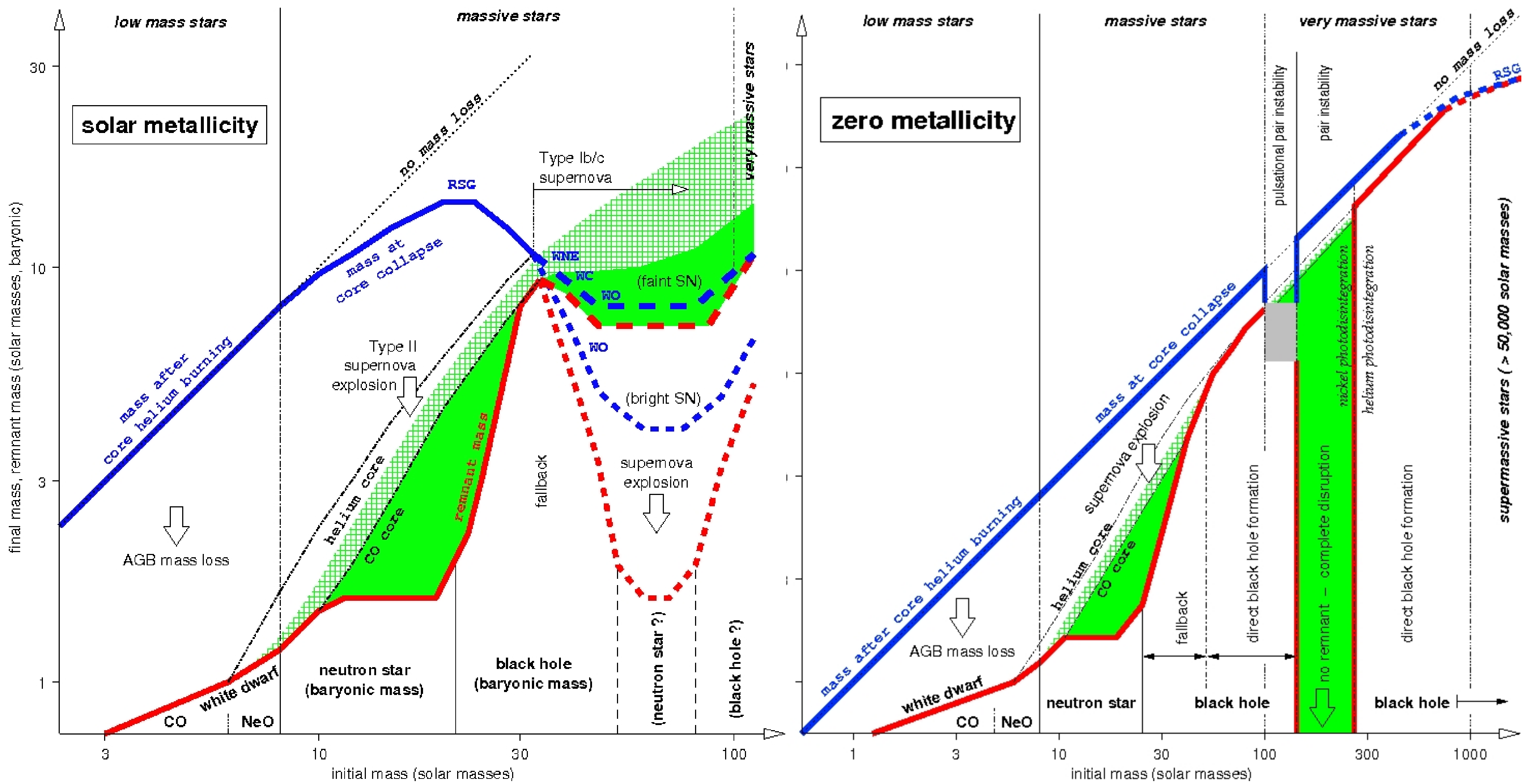
envelope radius

Star cannot explode if envelope binding energy > ~ SN energy

$$M_{\rm env} \sim 50 M_\odot \left(\frac{E_{\rm SN}}{10^{51} {\rm erg\, s^{-1}}}\right)^{\frac{1}{2}} \left(\frac{R_{\rm env}}{10\, R_\odot}\right)^{\frac{1}{2}}$$

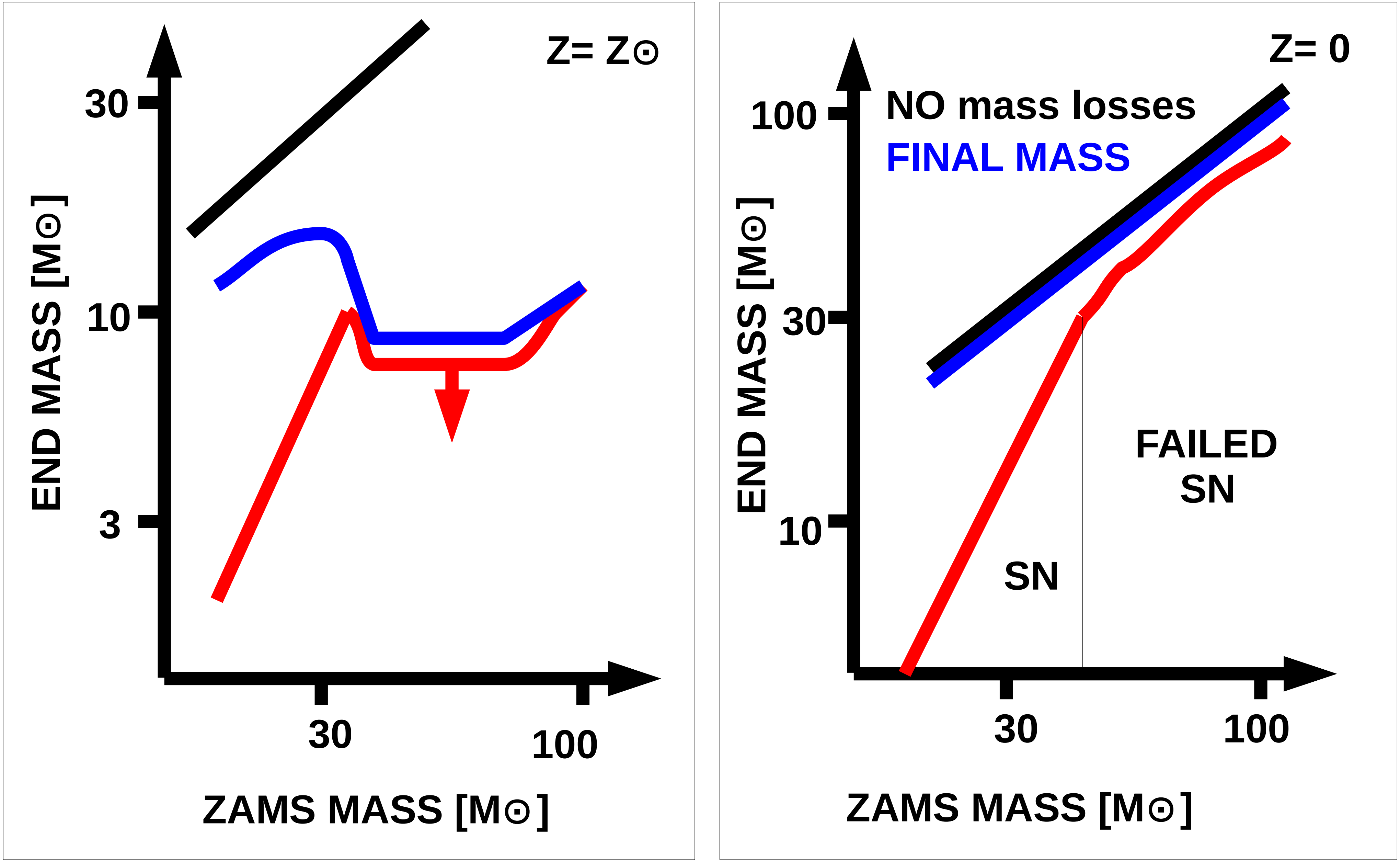
If Mfin>50 Msun this SN fails and star collapses to a BH!

0. stellar black holes (BHs) from star evolution



Heger et al. (2003)

0. stellar black holes (BHs) from star evolution

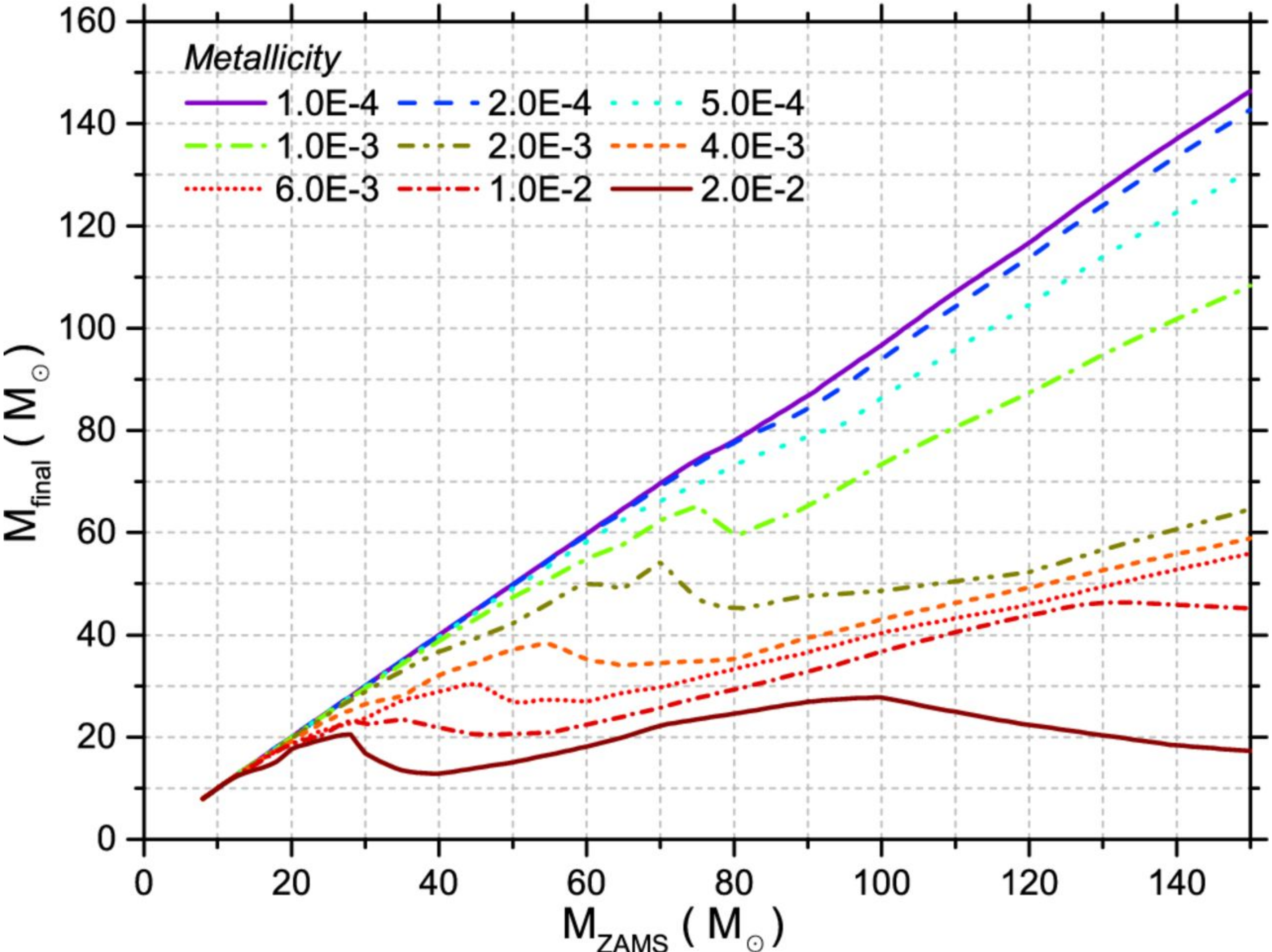


My cartoon from
Heger et al. (2003)

0. stellar black holes (BHs) from star evolution

What about intermediate metallicities between 0 and solar?

- more difficult because stellar winds are uncertain
- importance of final mass: pre-supernova mass of the star (CO core)



Spera, MM, Bressan 2015

0. stellar black holes (BHs) from star evolution

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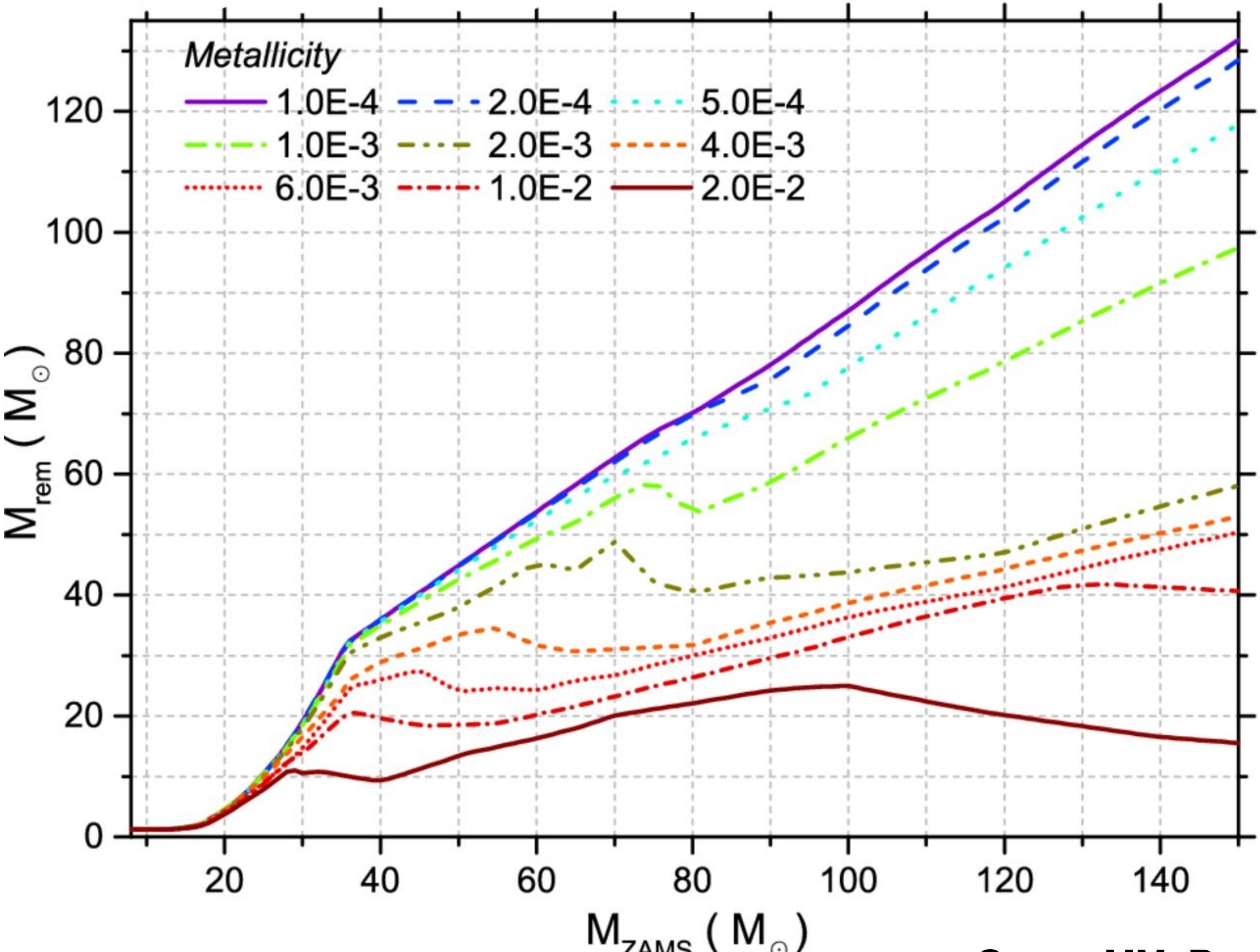


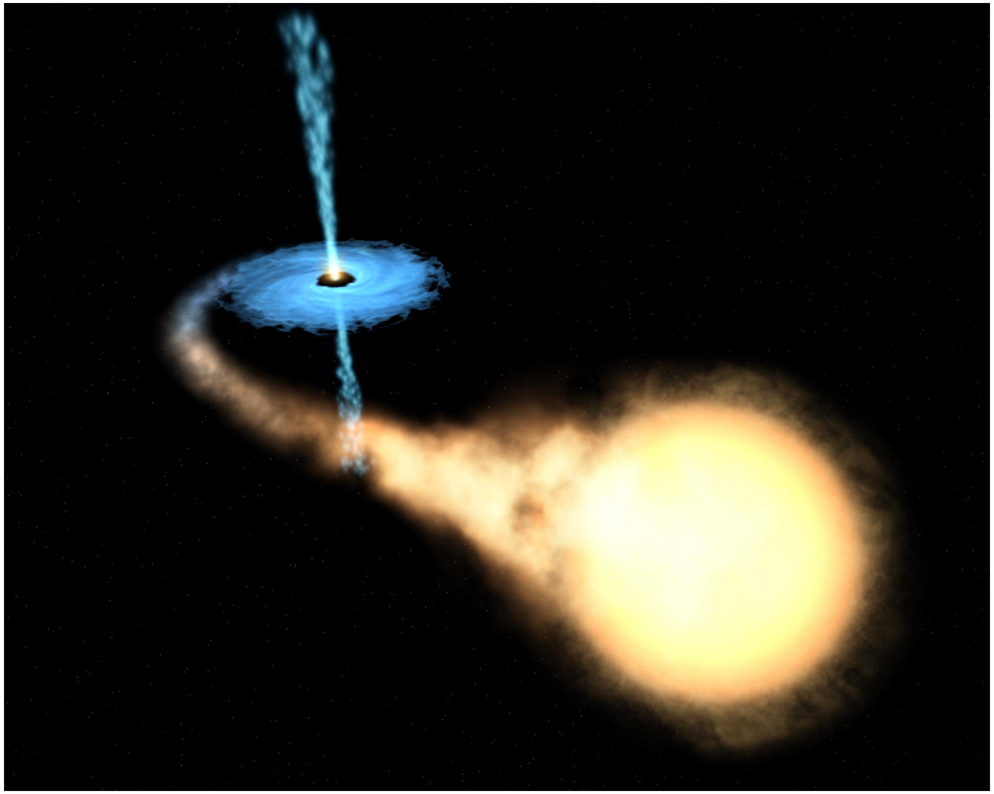
Spera, MM, Bressan 2015

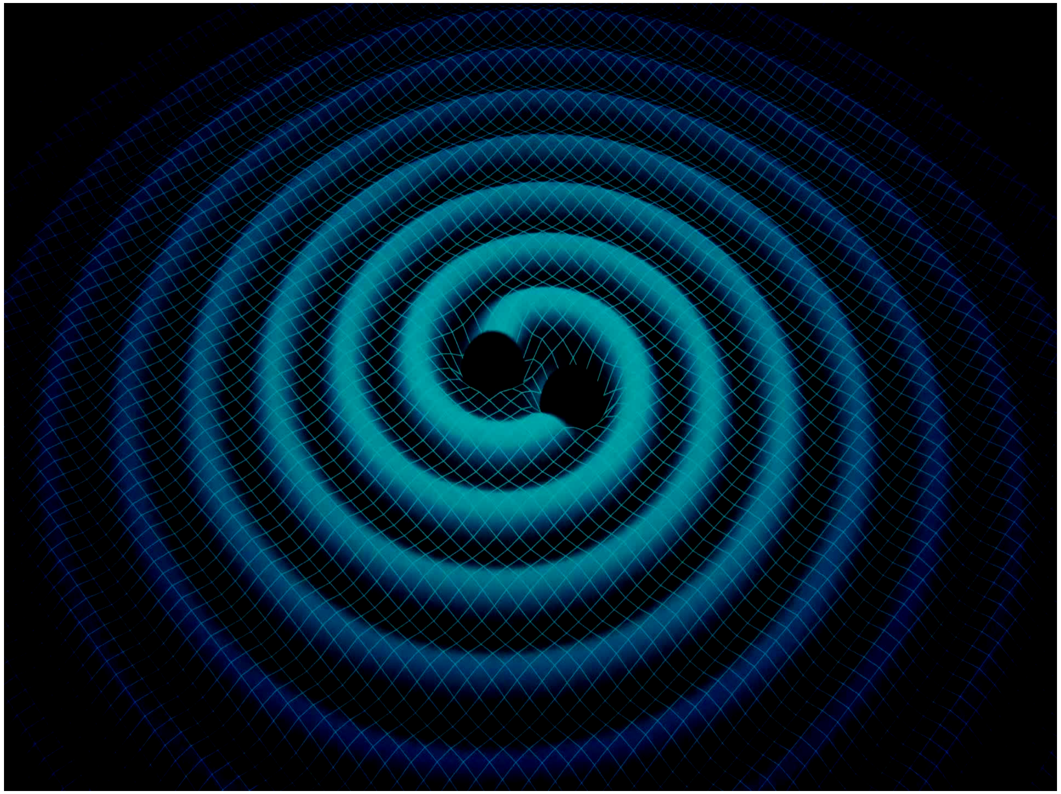
1. BHs as members of binary systems:

WHY are BH binaries IMPORTANT?

* Compact object binaries (with a stellar companion) can emit X-rays

* Double compact object binaries can emit detectable gravitational waves (GWs)





1. BHs as members of binary systems:

WHY are BH binaries IMPORTANT?

* Compact object binaries (with a stellar companion) can emit X-rays

* Double compact object binaries can emit detectable gravitational waves (GWs)

Compact-object binaries lose energy and angular momentum by GW emission

→ requires adding new timescale to the picture: timescale for the system to merge by GW emission

From Peters (1964, Gravitational radiation and the motion of two point masses, Phys. Rev. B136, 1224) the timescale of orbital decay by GWs is

$$t_{GW} = \frac{5}{256}\,\frac{c^5\,a^4\,\left(1-e^2\right)^{7/2}}{G^3\,m_1\,m_2\,(m_1+m_2)}$$

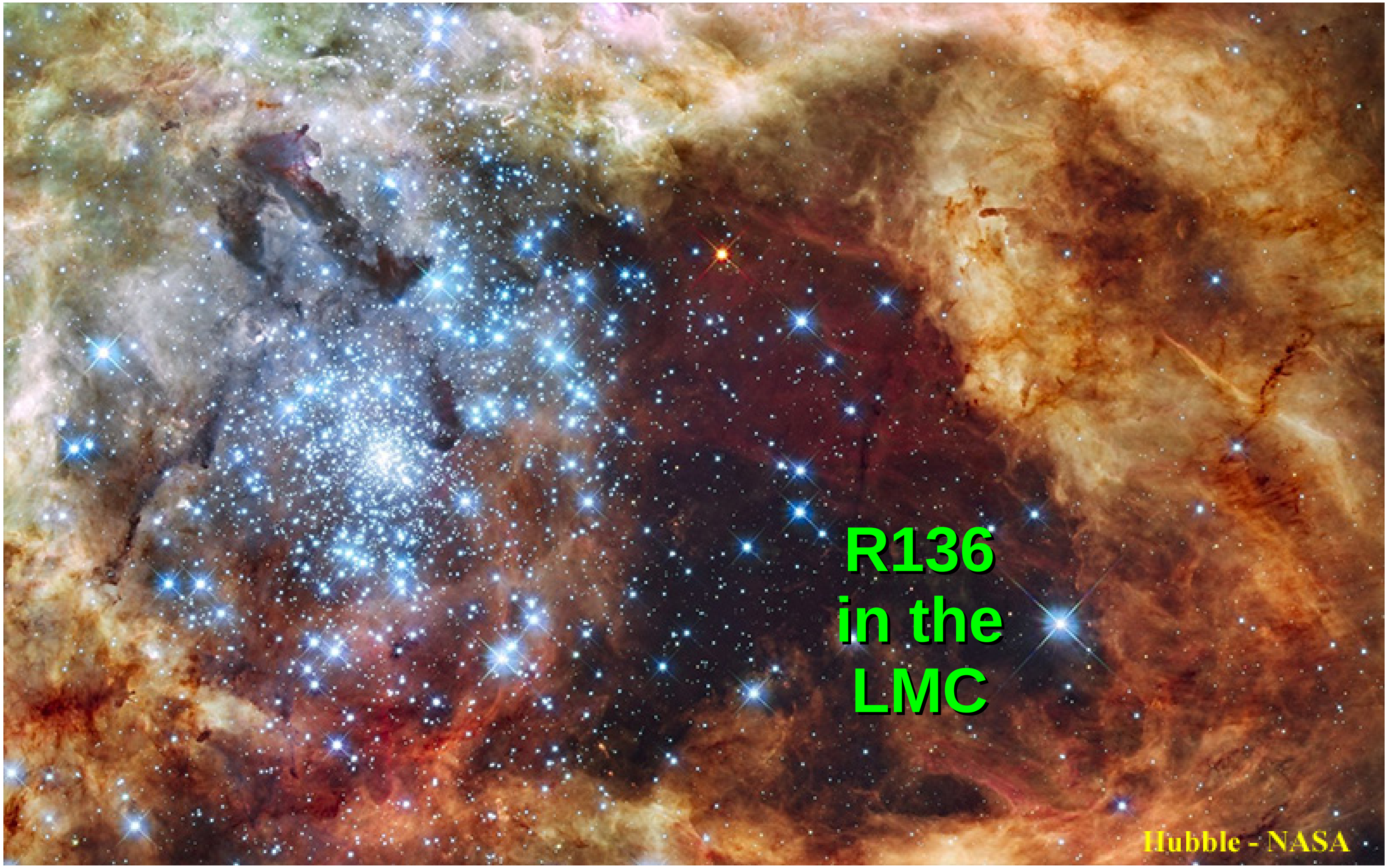
2. dynamical formation of BH binaries

WHY should we care about DYNAMICS when studying BH binaries?

2. dynamical formation of BH binaries

WHY DYNAMICS????????

Massive stars (BH progenitors) form in STAR CLUSTERS: dynamically 'ACTIVE' places (Lada & Lada 2003)



2. dynamical formation of BH binaries

WHY DYNAMICS???????

Massive stars
(BH progenitors)
form in
STAR CLUSTERS

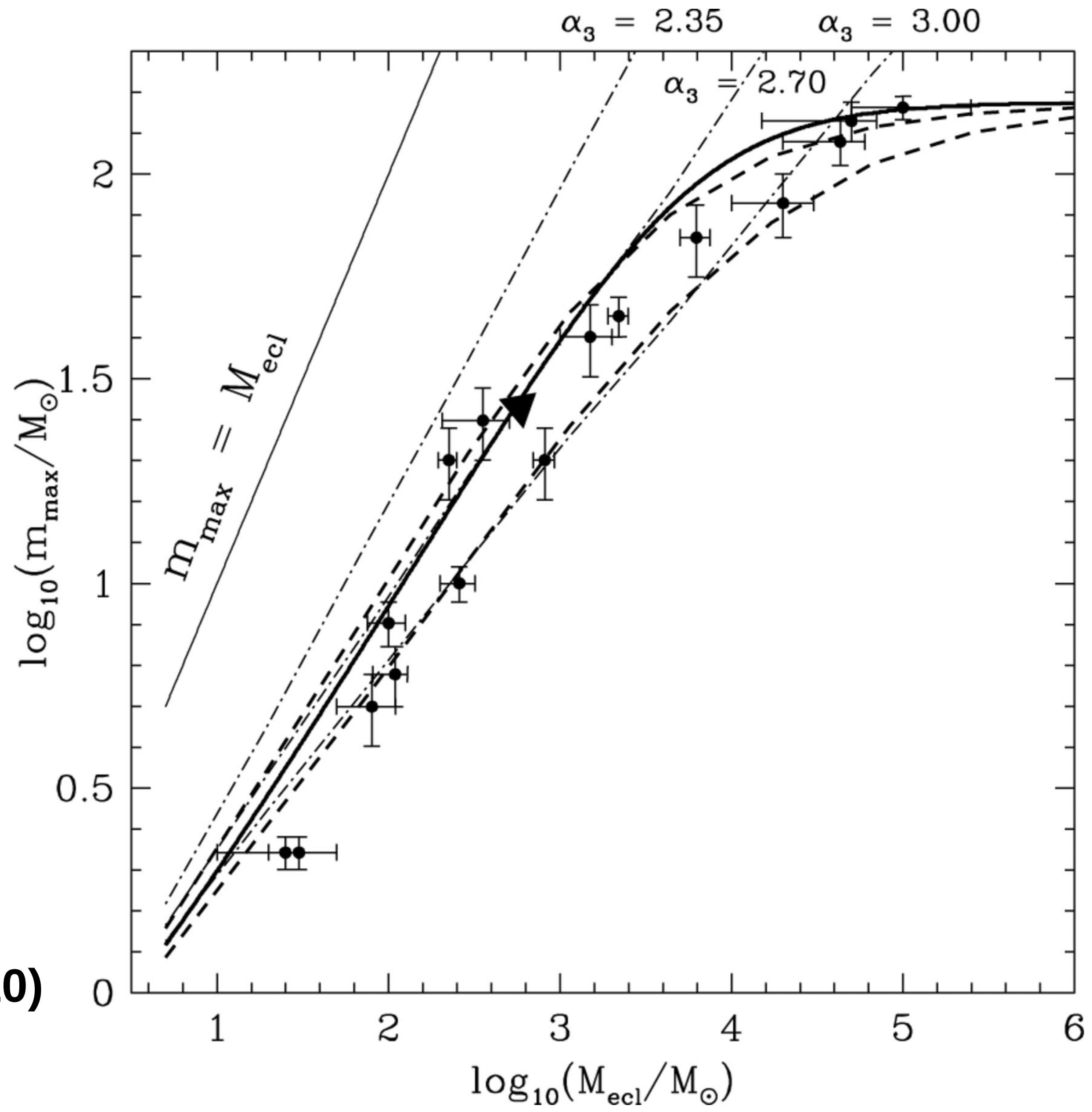


Figure from
Weidner & Kroupa (2006)

Data points:
observed star clusters

Lines: theoretical fits

See also
Weidner, Kroupa & Bonnell (2010)

2. dynamical formation of BH binaries

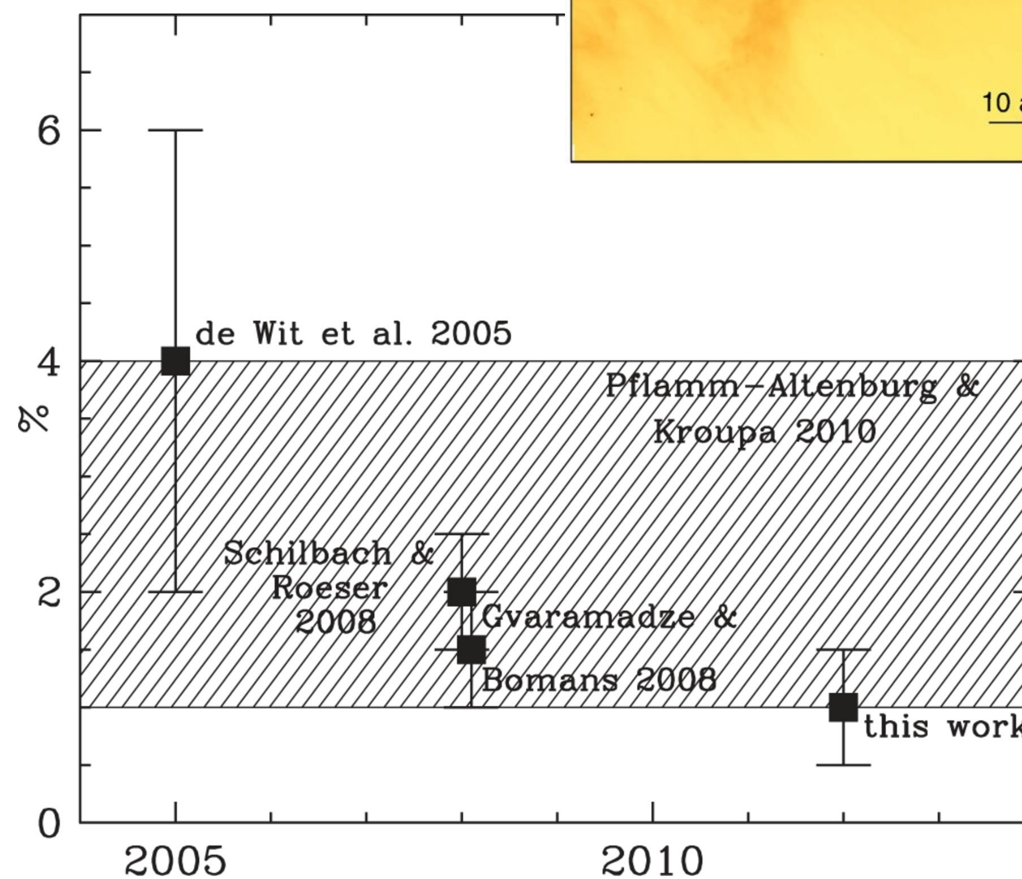
WHY DYNAMICS???????

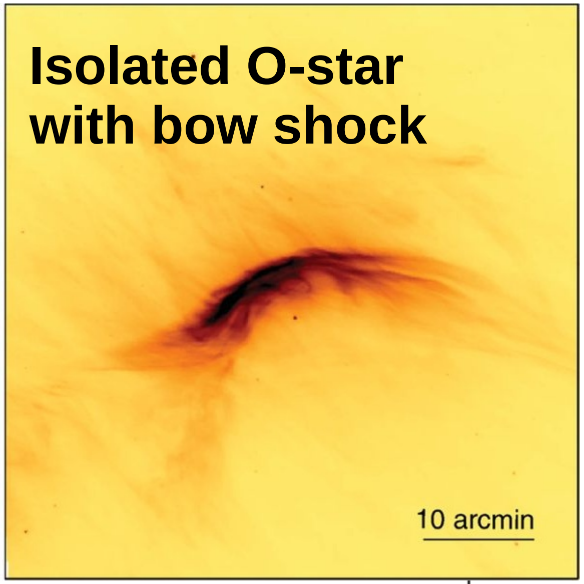
O-type stars in the field are mostly RUNAWAY from star clusters (as we see from bow shocks)

Figures from Gvaramadze et al. (2012)

See also
De Wit et al. (2004, 2005)
Schilbach & Roeser (2008)

Percentage of genuine field O stars





2. dynamical formation of BH binaries

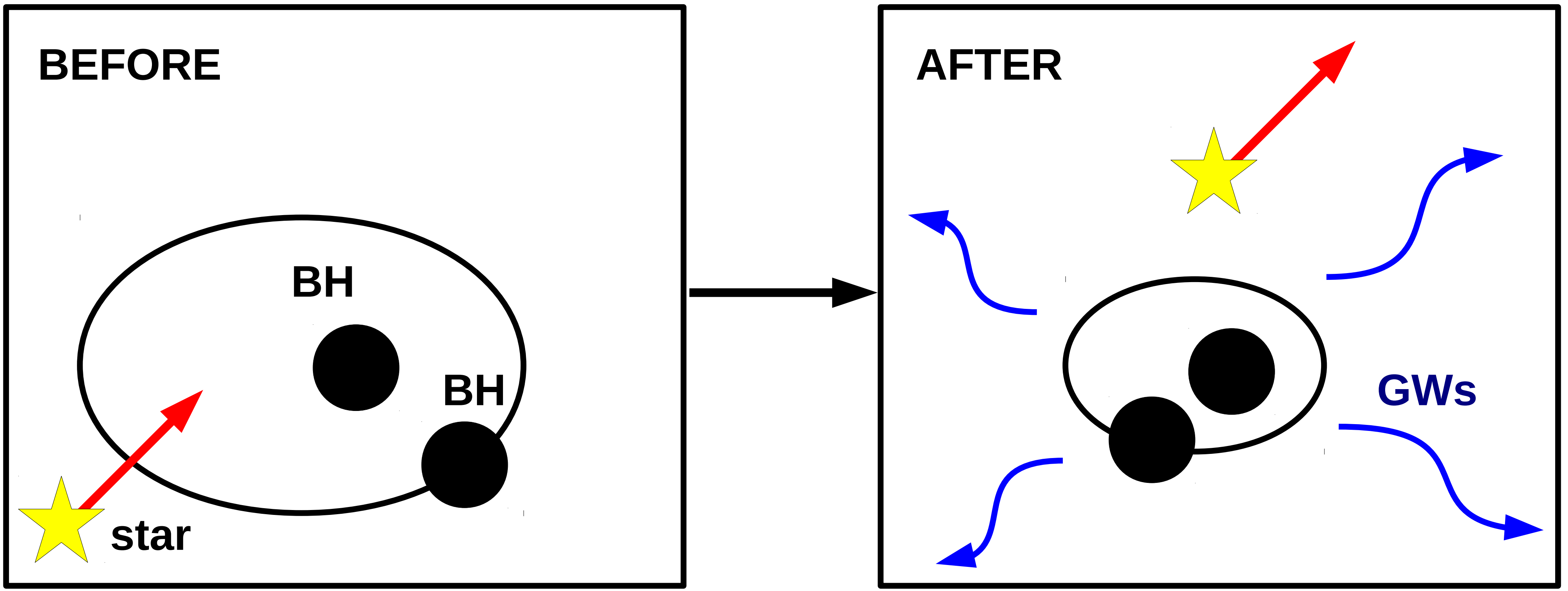
WHY DYNAMICS???????

Massive stars (BH progenitors) form in STAR CLUSTERS: dynamically 'ACTIVE' places (Lada & Lada 2003)

BASED on PREVIOUS LECTURES, WHAT IS THE EFFECT OF DYNAMICS ON BH BINARIES?

2. dynamical formation of BH binaries

1. HARDENING:



After 3-body encounters, the semi-major axis shrinks and the BH-BH (or BH-NS or NS-NS) binary becomes important as gravitational wave (GW) source

2. dynamical formation of BH binaries

HARDENING TIMESCALE

$$t_h = \left|\frac{a}{\dot{a}}\right| = \frac{1}{2\,\pi\,G\,\xi}\,\frac{\sigma}{\rho}\,\frac{1}{a}$$

GRAVITATIONAL WAVE (GW) TIMESCALE

$$t_{GW} = \frac{5}{256}\,\frac{c^5\,a^4\,(1-e^2)^{7/2}}{G^3\,m_1\,m_2\,(m_1+m_2)}$$

Combining 1) and 2) we can find the maximum semi-major axis for GWs to dominate evolution

$$a_{GW} = \left[\frac{256}{5}\,\frac{G^2\,m_1\,m_2\,(m_1+m_2)\,\sigma}{2\,\pi\xi\,(1-e^2)^{7/2}\,c^5\,\rho}\right]^{1/5}$$

2. dynamical formation of BH binaries

* blue
m_1=200 $M_\odot$ m_2=10 $M_\odot$

* green
m_1=50 $M_\odot$ m_2=10 $M_\odot$

* red
m_1=30 $M_\odot$ m_2=3 $M_\odot$

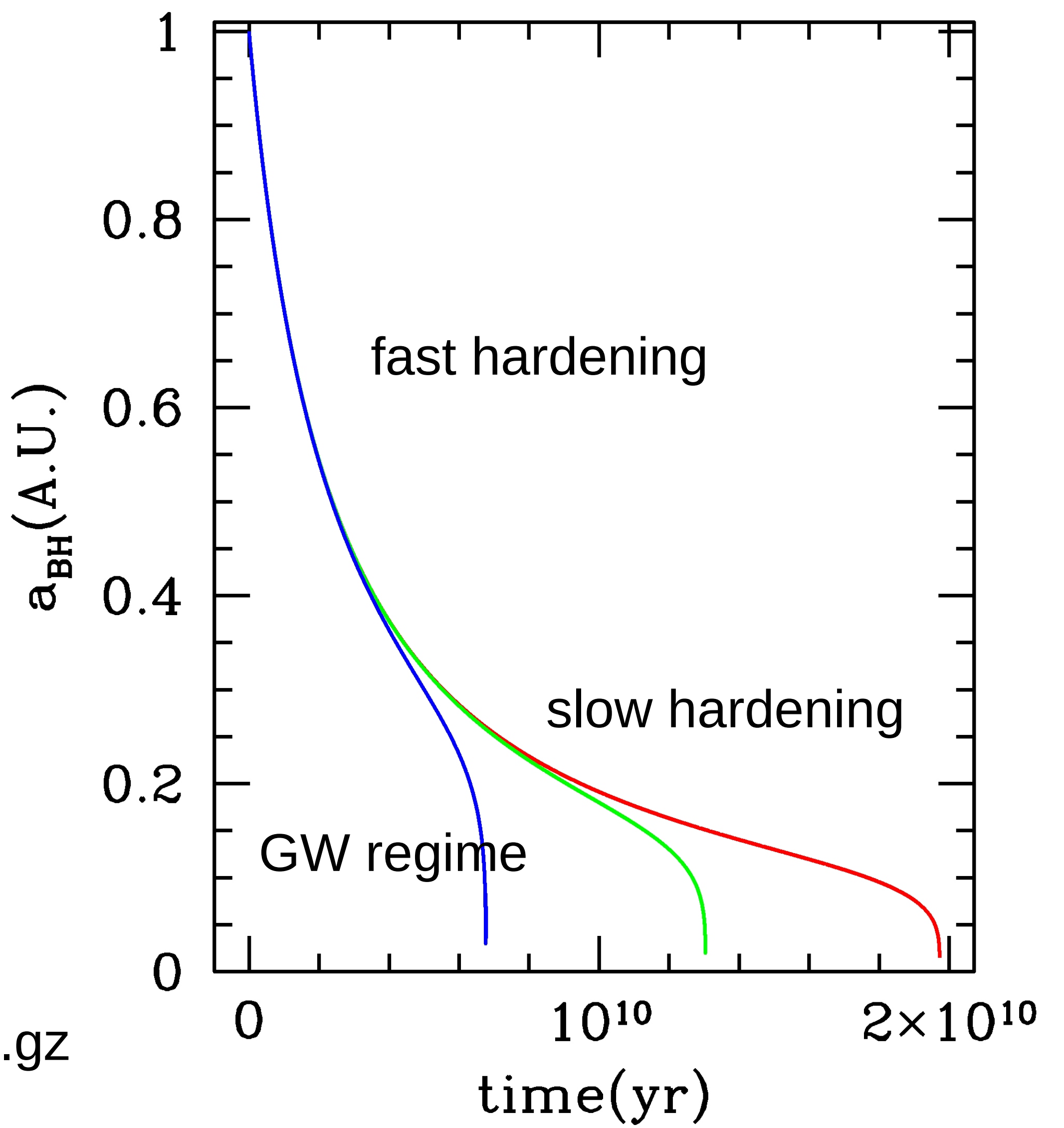


Figure and calculation
from page 200 of
M. Mapelli's thesis
http://web.pd.astro.it/mapelli/images/tesi.ps.gz

$$\frac{da}{dt} = \underbrace{-2\,\pi\,\xi\,\frac{G\,\rho}{\sigma}\,a^2}_{\text{Binary shrinking by hardening}} - \underbrace{\frac{64}{5}\,\frac{G^3\,m_1\,m_2\,(m_1+m_2)}{c^5\,(1-e^2)^{7/2}}\,a^{-3}}_{\text{Binary shrinking by GWs (Peters 1964)}}$$

Binary shrinking by hardening

Binary shrinking by GWs (Peters 1964)

Number of encounters before GW regime:

$$N_{int} = \int_0^t R\,dt = \int_0^t \frac{2\,\pi\,G\,m_T\,n\,a}{\sigma}\,dt$$

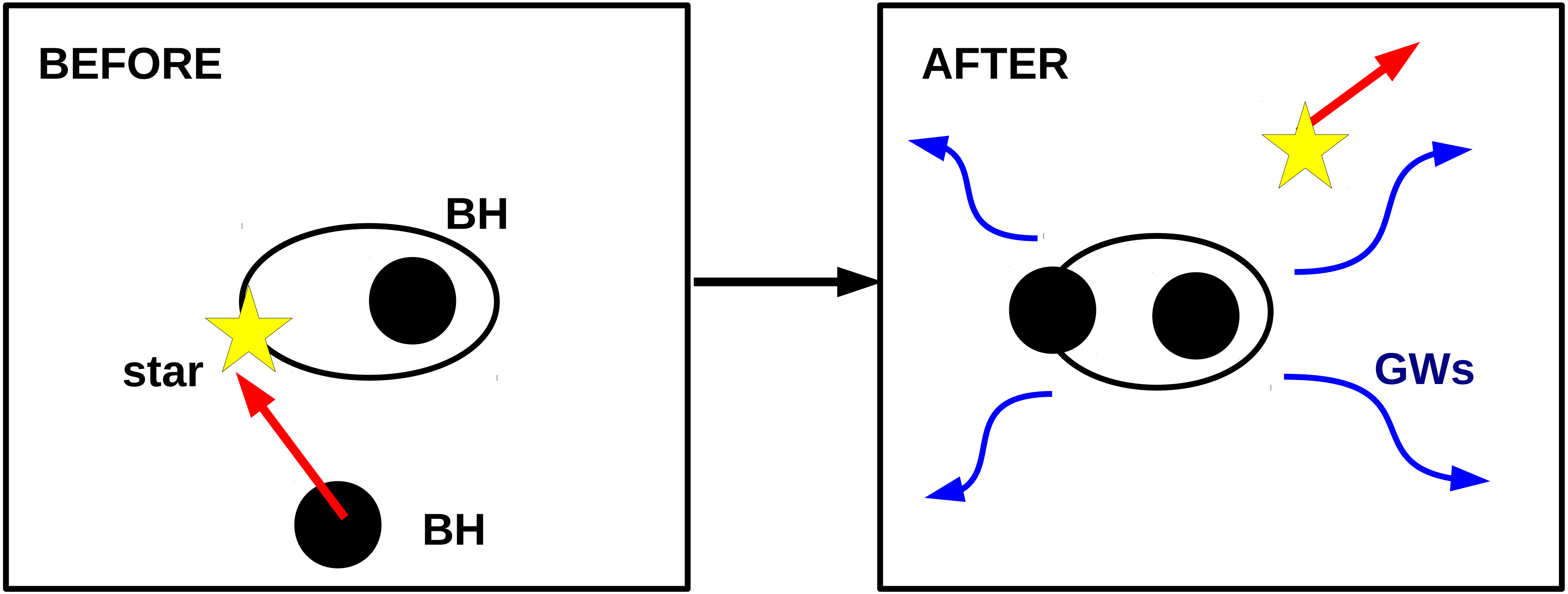
$$= \int_{a_0}^{a(t)} \frac{2\,\pi\,G\,m_T\,n\,a}{\sigma}\,\frac{\sigma\,da}{-2\,\pi G\,\xi\rho\,a^2} = \int_{a(t)}^{a_0} \frac{1}{\xi}\,\frac{m_T}{\langle m\rangle}\,\frac{da}{a}$$

↑

$$\frac{da}{dt} = -2\,\pi\,G\,\xi\,\frac{\rho}{\sigma}\,a^2$$

$$= \frac{1}{\xi}\,\frac{m_T}{\langle m\rangle}\,\ln\left(\frac{a_0}{a(t)}\right)$$

2. EXCHANGE:

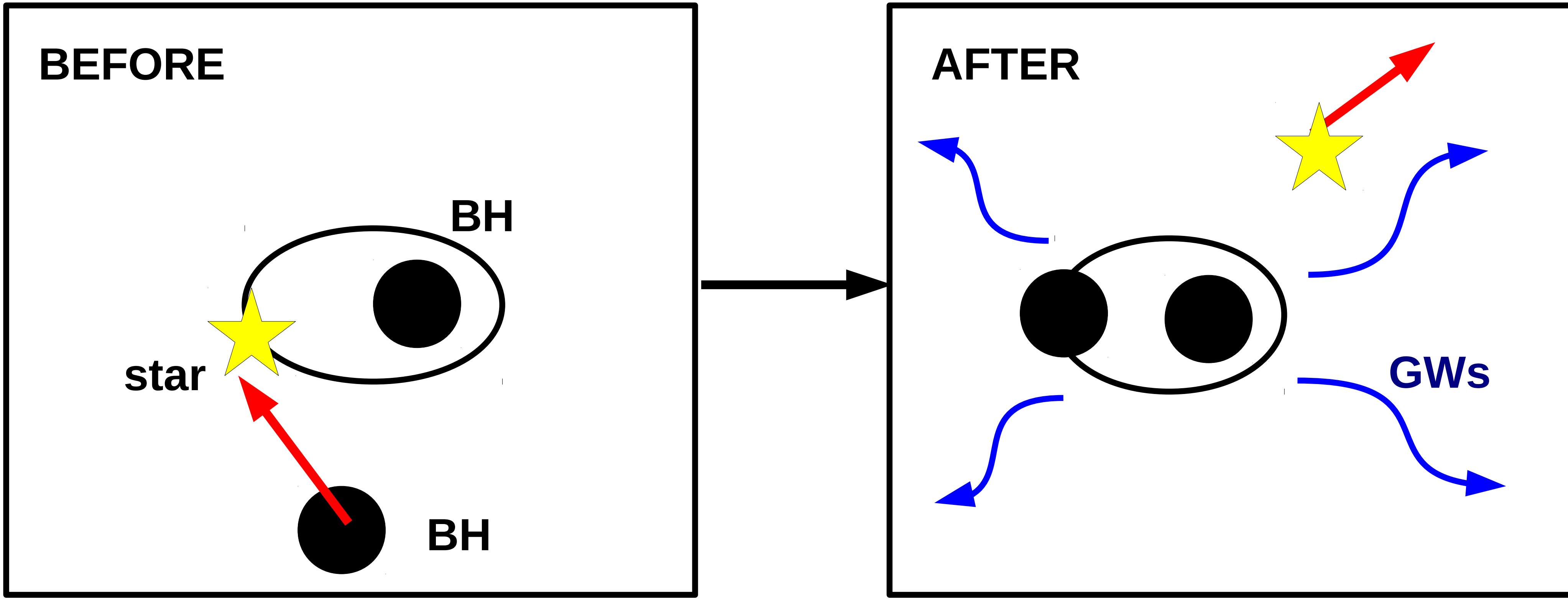


Exchanges are very important: bring BHs in binaries

BHs are FAVOURED BY EXCHANGES BECAUSE THEY ARE MASSIVE!

BH born from single star in the field never acquires a companion
BH born from single star in a sc likely acquires companion from dynamics

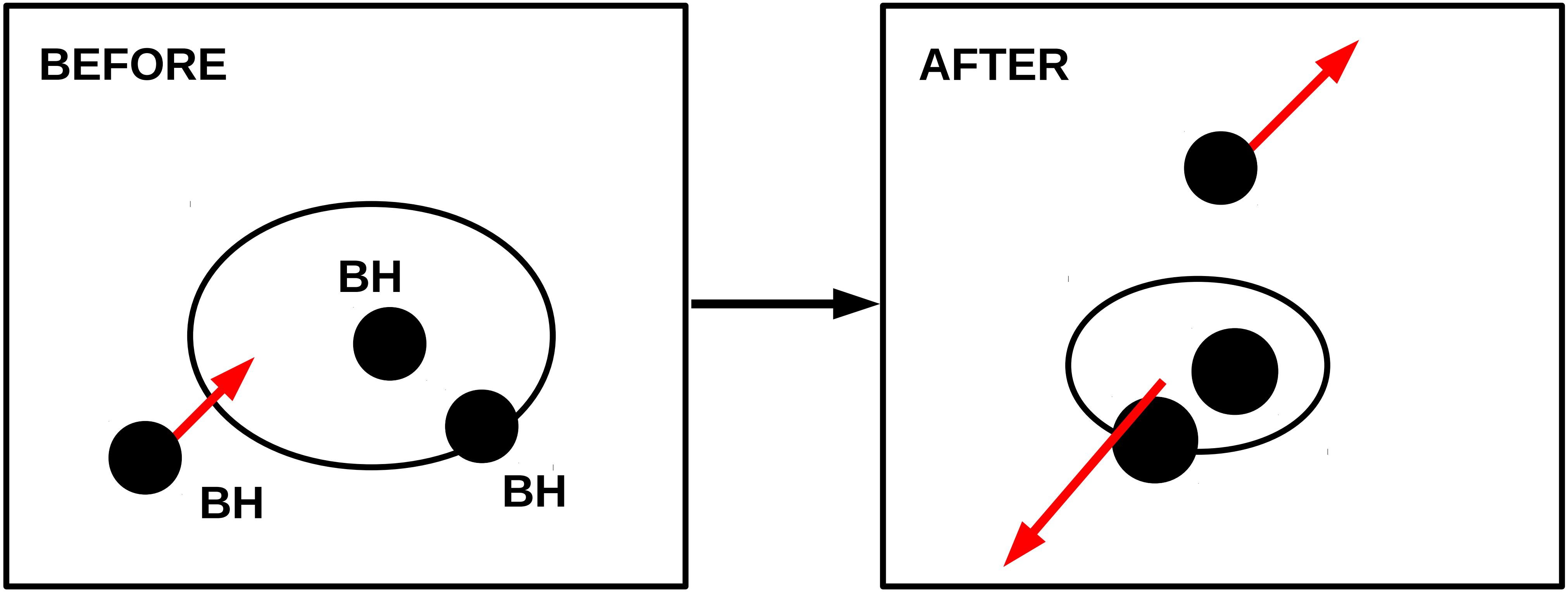
2. EXCHANGE:



>90% BH-BH binaries in young star clusters form by exchange (Ziosi+ 2014)

EXCHANGES FAVOUR THE FORMATION of BH-BH BINARIES WITH
- *THE MOST MASSIVE BHs*
- *HIGH ECCENTRICITY*
- *MISALIGNED BH SPINS*

3. EJECTION:



Internal energy is extracted from the binary

→ **converted into KINETIC ENERGY of the INTRUDER AND of the centre-of-mass of the BINARY**

→ **BOTH RECOIL and can be ejected from star cluster**

4. KOZAI-LIDOV RESONANCE:

Discovered by Kozai and Lidov in ~ 1962

I need a HIERARCHICAL TRIPLE:

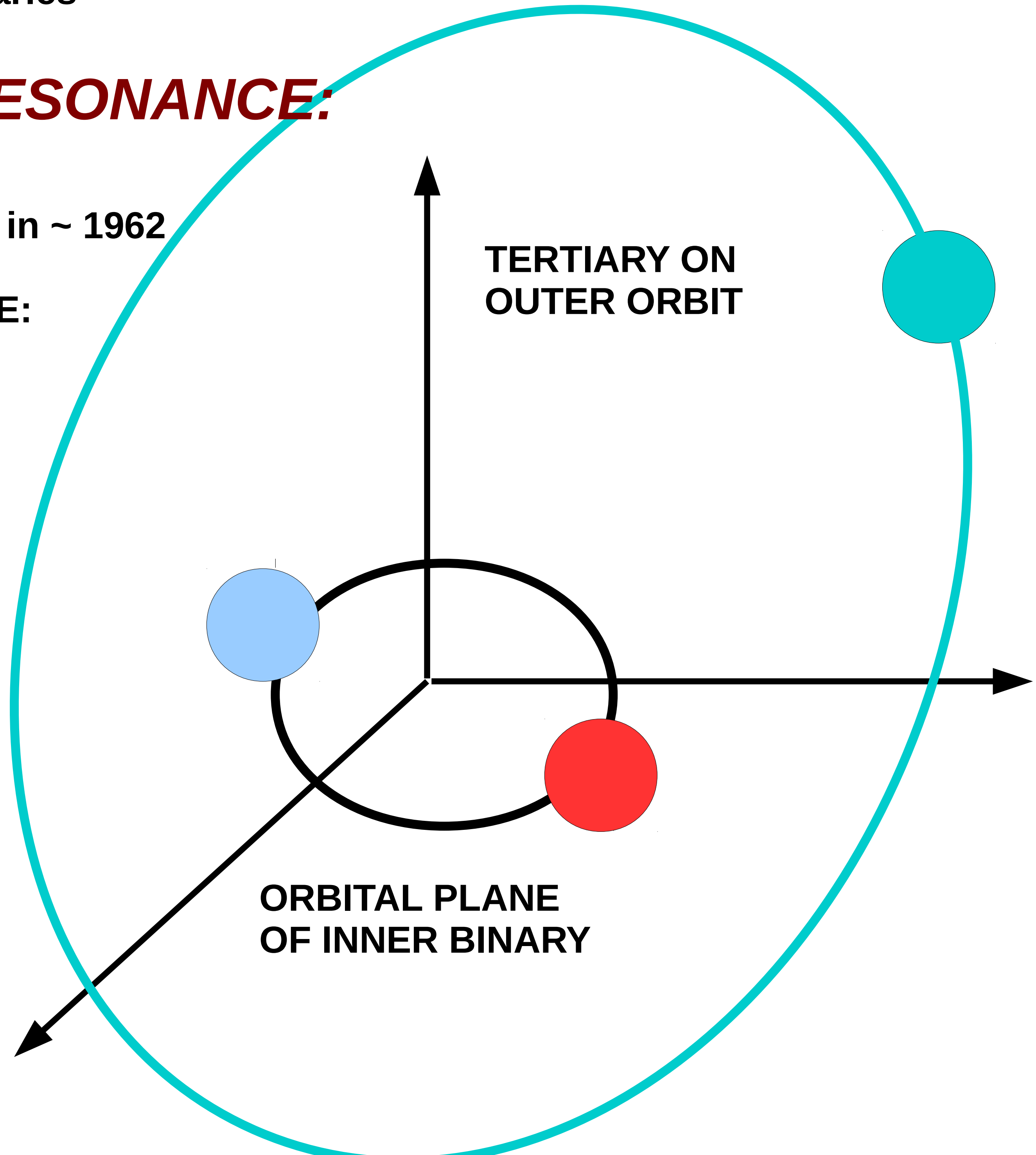
- Inner tight binary

- Outer body whose motion about the binary can be approximated with outer binary (CM + 3rd body)

- if inclination between 2 orbital planes is not 0

→ KOZAI RESONANCE

ECCENTRICITY AND INCLINATION OSCILLATE

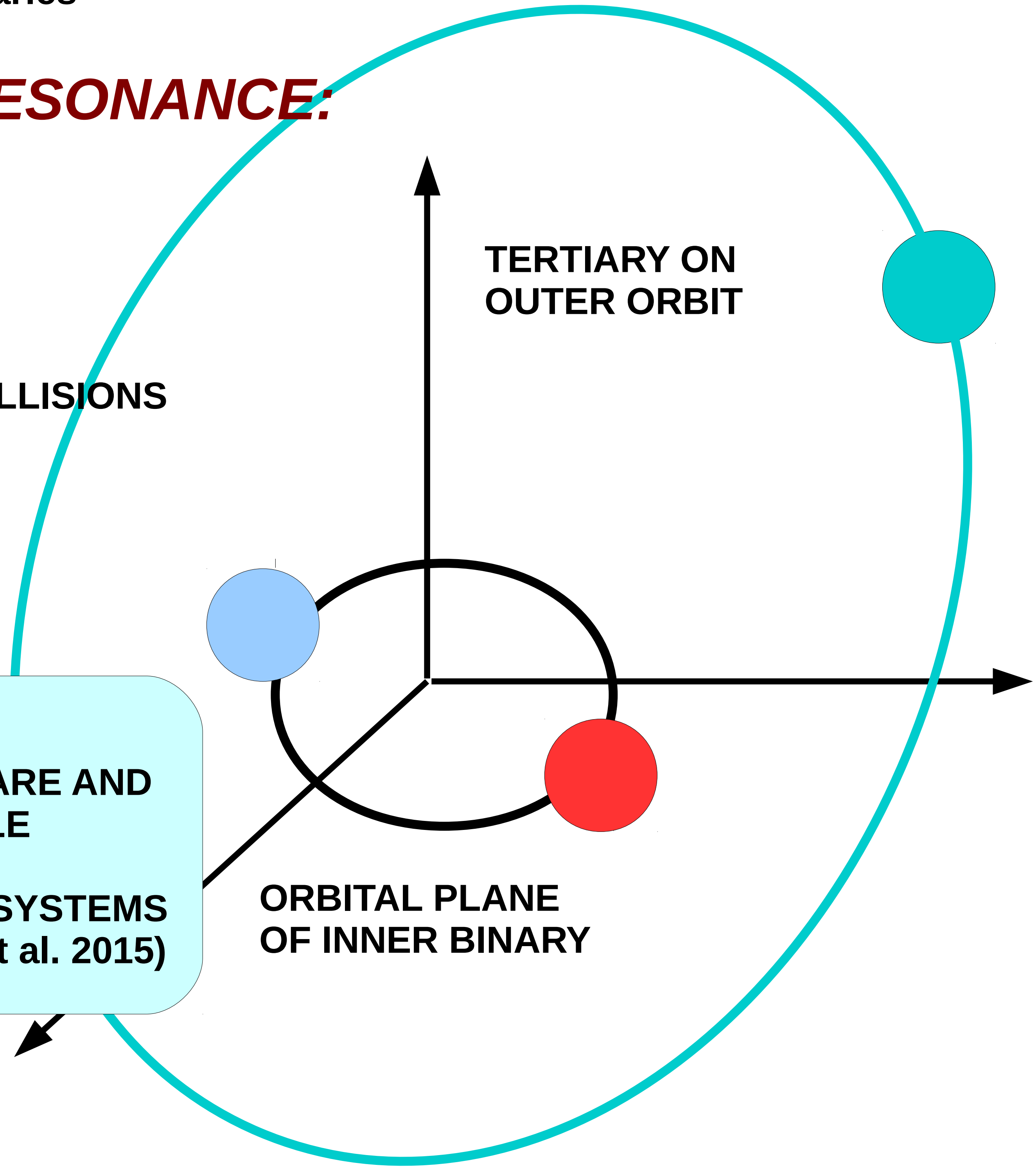


4. KOZAI-LIDOV RESONANCE:

ECCENTRICITY AND INCLINATION OSCILLATE

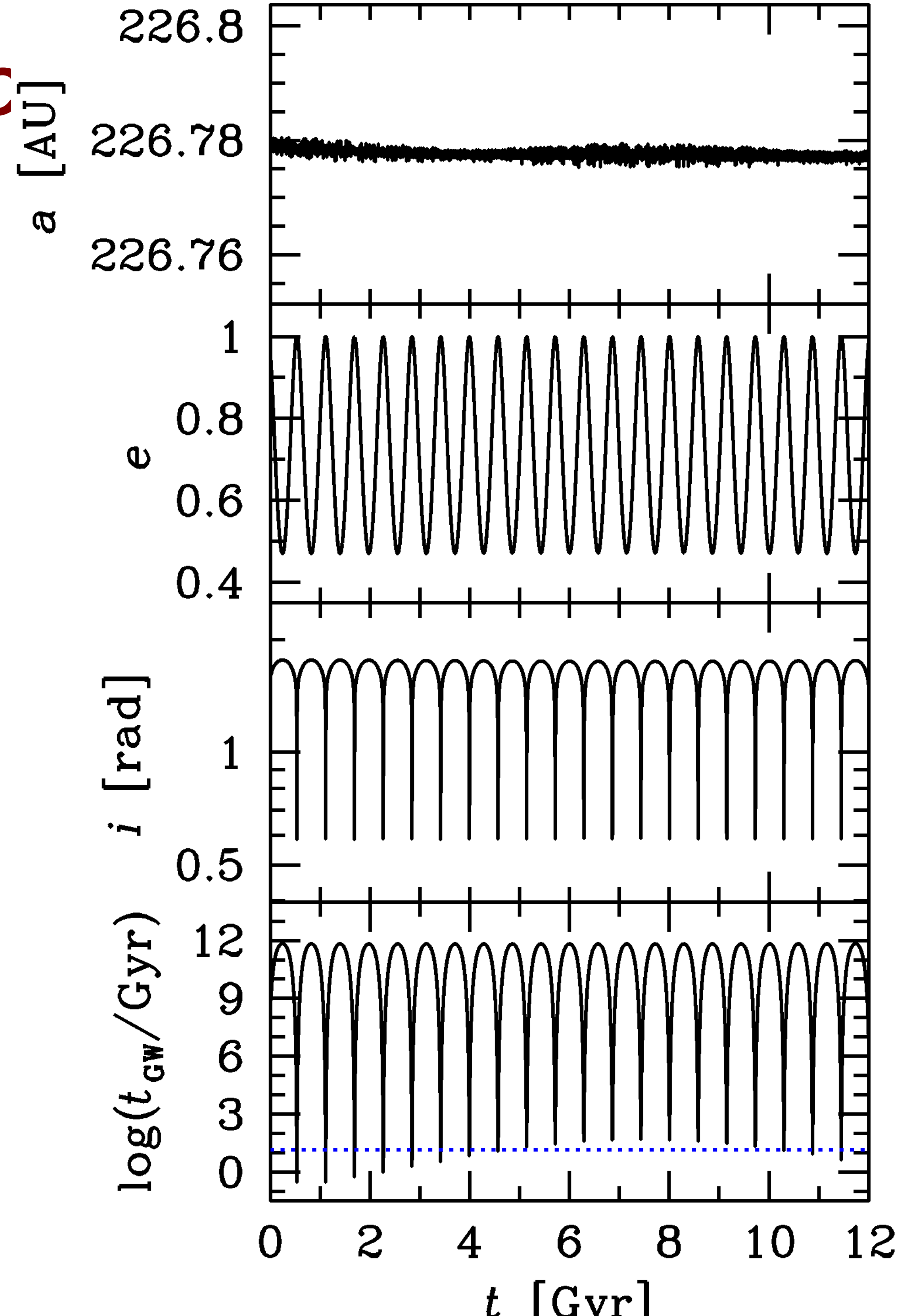
TRIGGERING MERGERS / COLLISIONS between binary members

→ IMPORTANT FOR GRAVITATIONAL WAVES



YOU MIGHT SAY:
TRIPLES SHOULD BE VERY RARE AND KL RESONANCE IS NEGLIGIBLE INSTEAD
~ 10 % STARS ARE IN TRIPLE SYSTEMS (Raghavan et al. 2010; Riddle et al. 2015)

4. KOZAI-LIDOV RESONANC



ECCENTRICITY AND INCLINATION OSCILLATE

TRIGGERING MERGERS between binary members

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Kimpson+ 2016

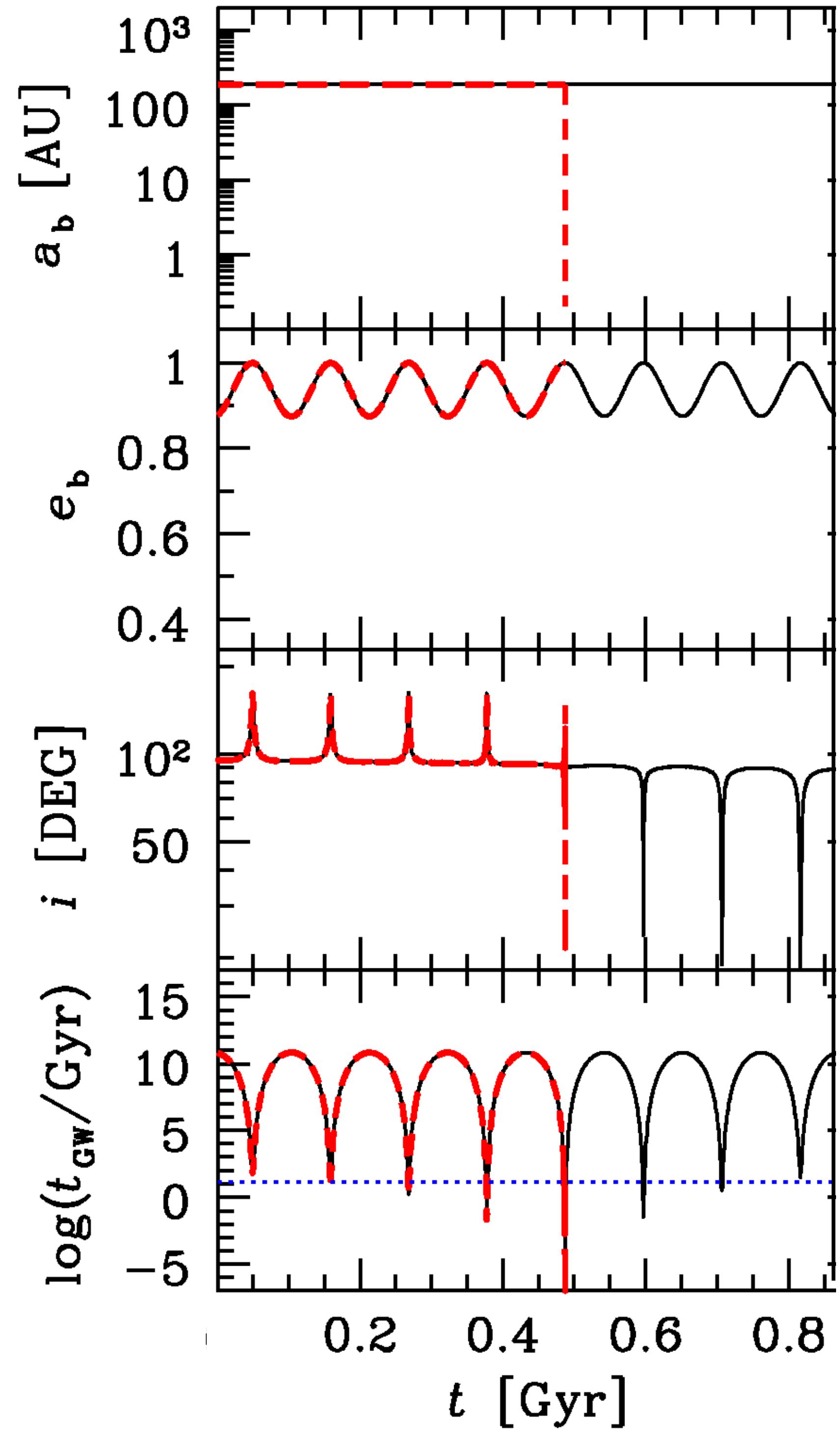
4. KOZAI-LIDOV RESONANCE:

No post-Newtonian (PN) — solid line

With 2.5 PN term — dashed line

PN: treatment of Einstein's non-linear equations as lowest-order deviations from Newton's equation

~ 50% more MERGERS
of BH-BH binaries
in young dense star clusters
If Kozai accounted for



Kimpson, Spera, MM, Ziosi 2016

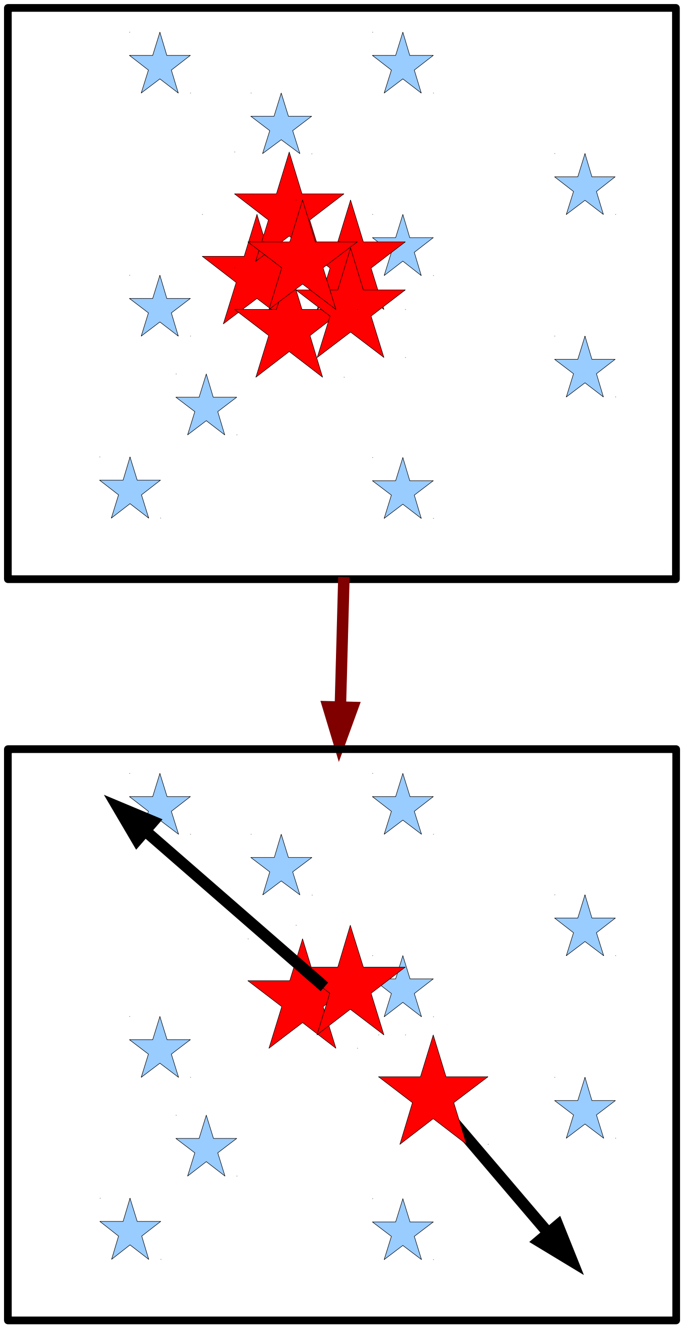
5. SPITZER'S INSTABILITY

see lecture 2017dynamics4.pdf

Spitzer's instability triggers:

*** formation and ejection of (massive) BH binaries**

*** runaway collision of very massive BH (formation of IMBHs)**



6. RUNAWAY COLLISION OF STARS

Similar to Spitzer's instability

See formation of IMBHs (next slides)

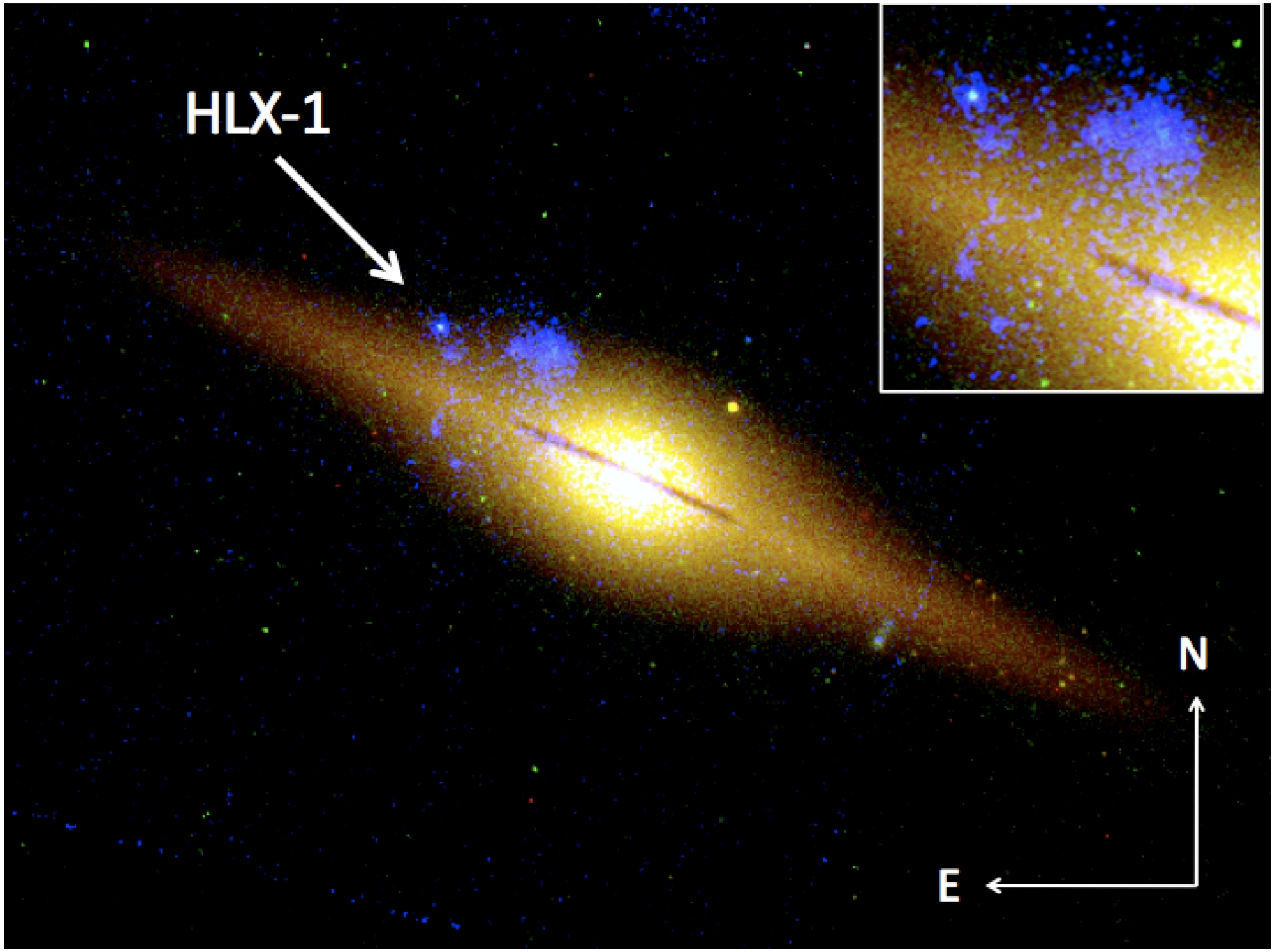
3. intermediate-mass BHs (IMBHs)

DEFINITION of IMBHs: BHs with mass 10^{2-5} M⊙

OBSERVATIONAL EVIDENCES: none, just hints

 # 1 Hyperluminous X-ray source HLX-1 close to ESO 243-49

peak L_X~10^{42} ergs,
X-ray VARIABILITY,
redshift consistent
with ESO 243-49
(not a background object)
→ BH mass~10^4 M⊙



Farrell+ 2009, 2012;
Soria+ 2010, 2012;
MMi+ 2012, 2013

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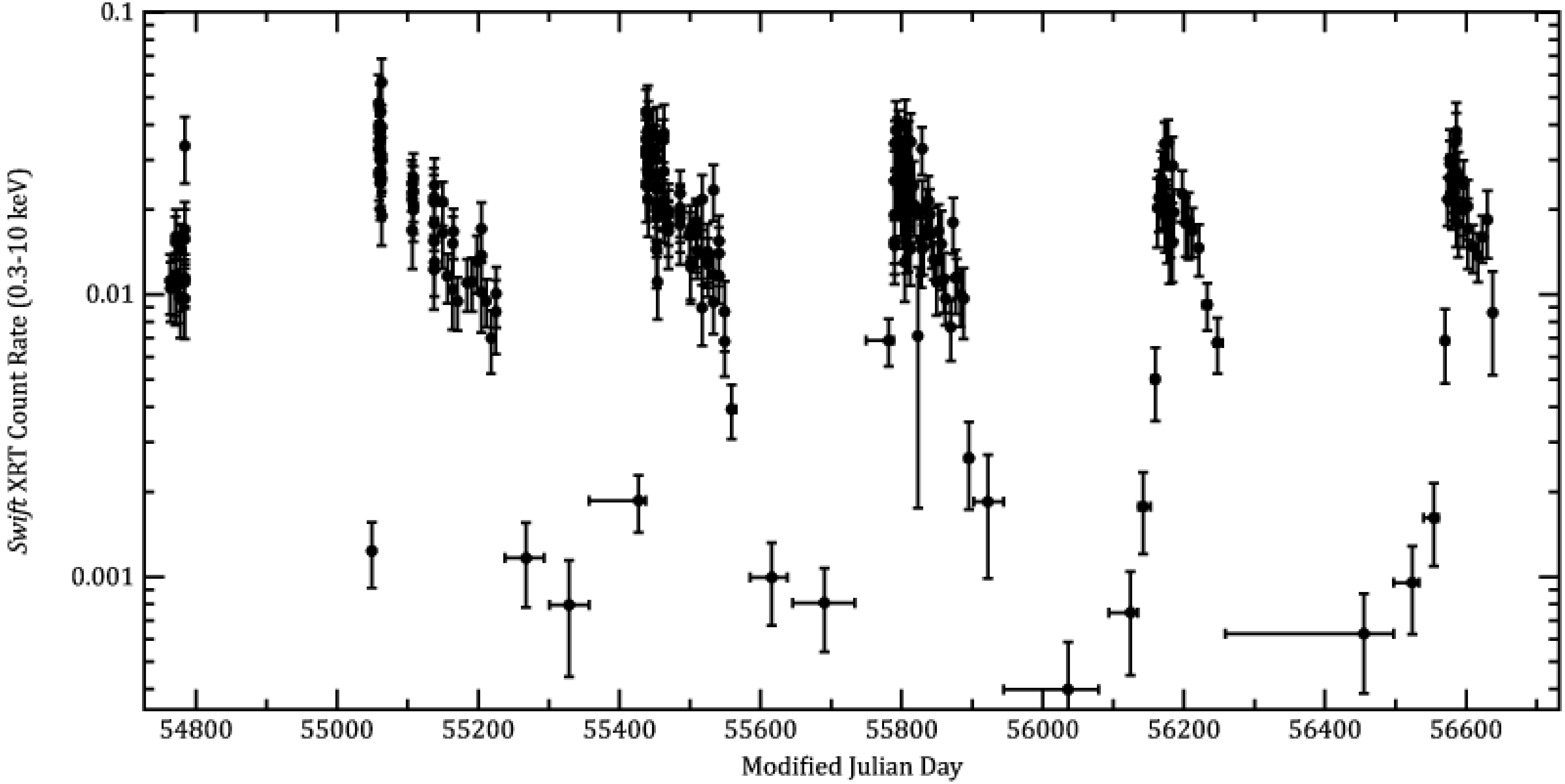
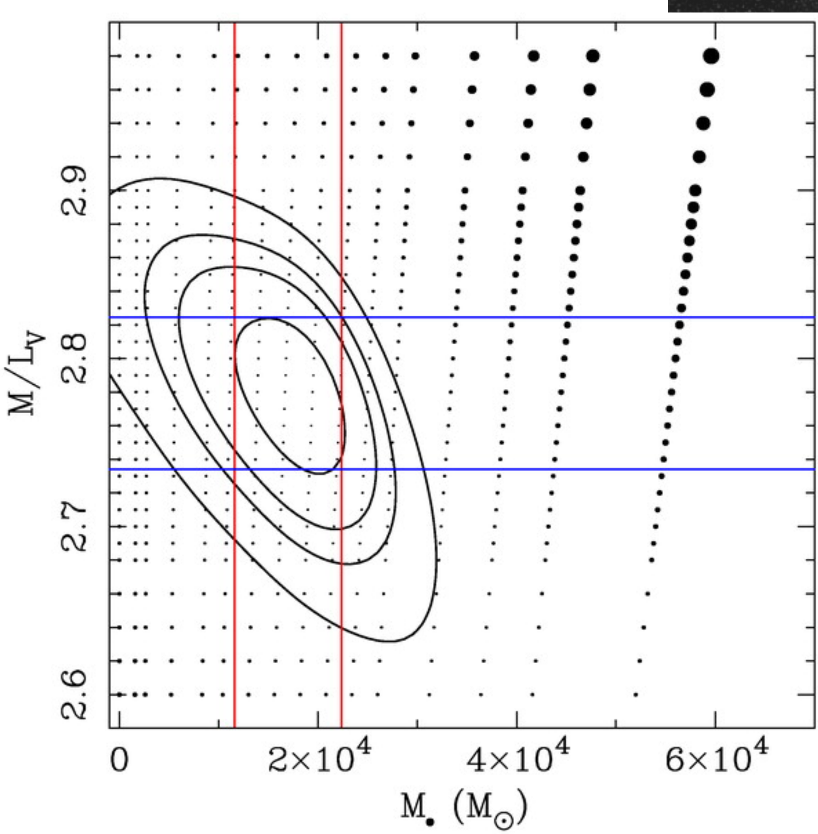


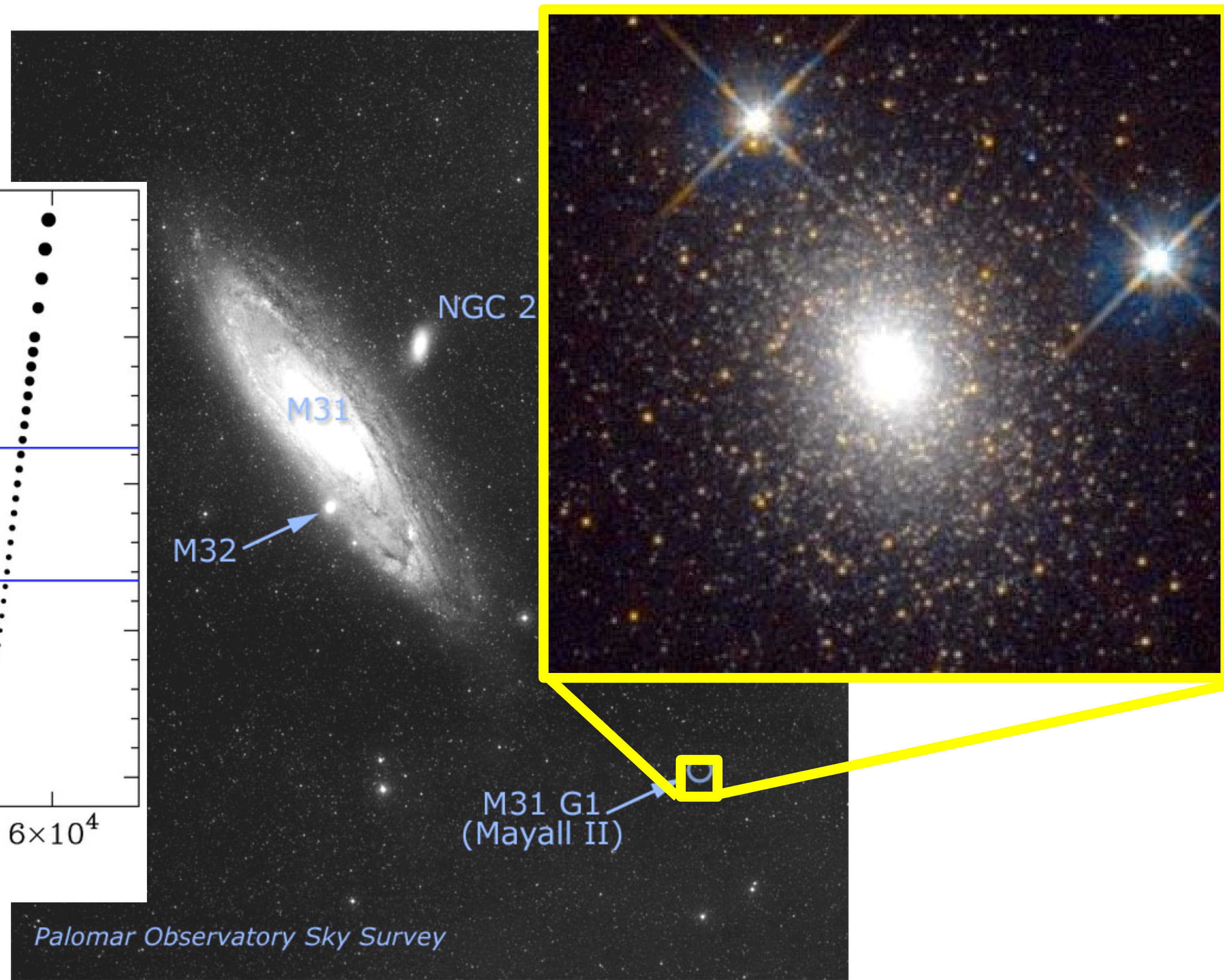
Webb+ 2014, arXiv:1401.1728

3. intermediate-mass BHs (IMBHs)

#2 centre of G1 globular cluster (dwarf nucleus?) in Andromeda

Central velocity distribution + central M/L ratio suggest BH mass~10^4 M⊙





Gebhardt+ 2005

3. intermediate-mass BHs (IMBHs)

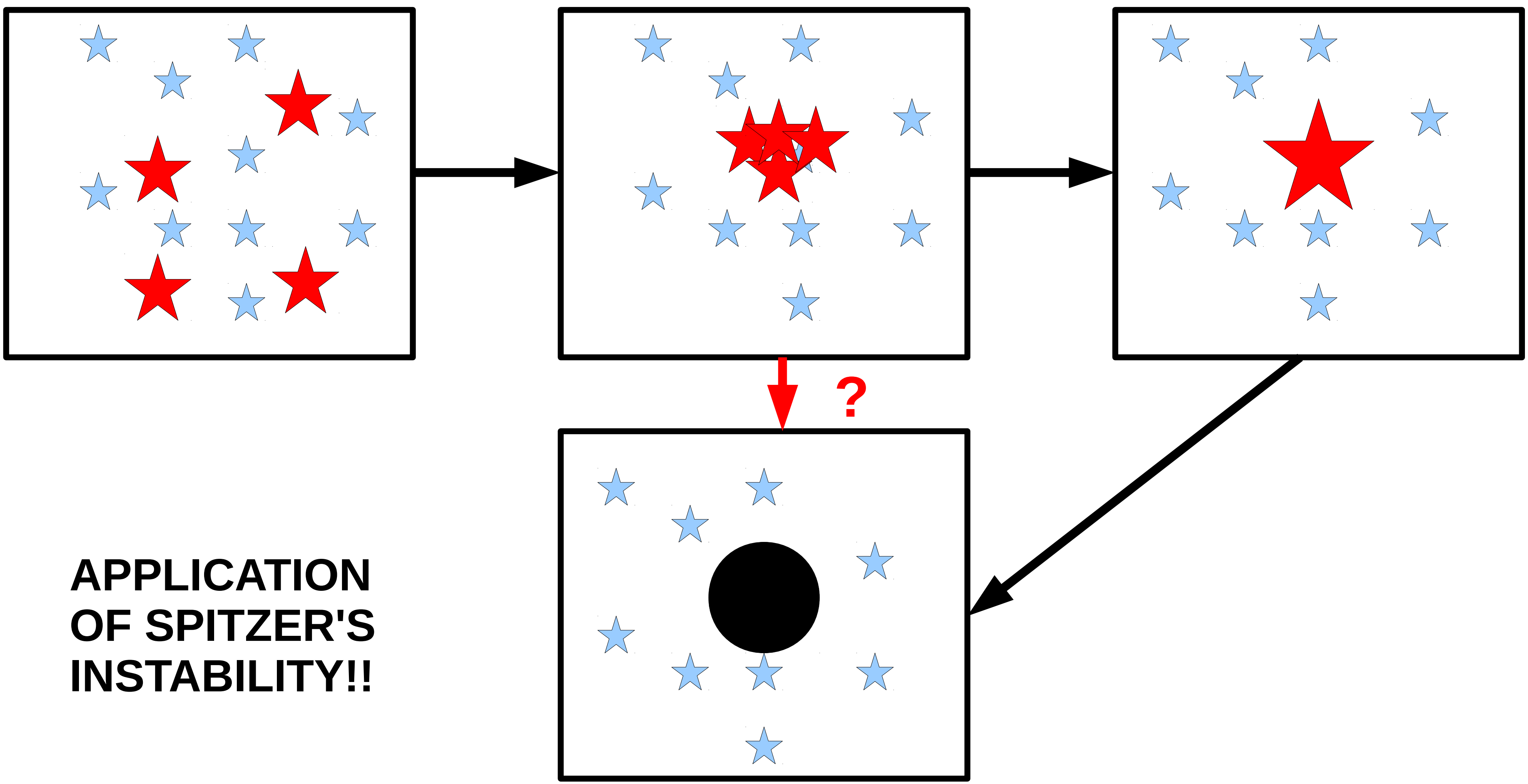
How do IMBHs form?	***Requires dynamics?***
1- runaway collisions of stars	***yes***
2- repeated mergers of BHs	***yes***
3- remnants of very massive (>260 Msun) extremely metal-poor stars (stellar BHs)	***No*** ***(unless very massive star was dynamically formed)***
4- low mass end of super-massive BHs (not part of this course)	***maybe***

3. IMBHs: runaway collisions

IDEA: mass segregation brings very massive stars to the centre

If timescale for mass segregation < timescale for SN explosion
+ encounter rate sufficiently high

→ massive stars collide, merge and form a super-massive star, which collapses to a BH



APPLICATION OF SPITZER'S INSTABILITY!!

3. IMBHs: runaway collisions

"Analytic" formalism by Portegies Zwart & McMillan 2002, ApJ, 576,899

IDEA: hard binaries sink to the centre and likely collide with other stars/binaries unless they are ejected.

The product of the first collisions is SO MASSIVE that it triggers other collisions (=is the main collision target) starting a RUNAWAY PROCESS

→ Maximum mass that can be grown in a dense star cluster If all collisions involve the same star

$$\frac{dM_{raway}}{dt} = R_{coll}\,\delta m_{coll}$$

Where R_{coll} = collision rate,

δm_{coll} = mass transferred per collision on average

3. IMBHs: runaway collisions

(i) ESTIMATE of R_{coll}

Maximum recoil velocity for a binary not to be ejected

$$v_{rec} = v_{esc} = 2\,\sigma$$

Definition of v_{rec}

$$v_{rec} = \frac{m_3}{m_T}\,v_{fin} = \frac{m_3}{m_T}\sqrt{\frac{m_3\,(m_1+m_2)}{m_e\,(m_a+m_b)}\,{v_\infty}^2 + \frac{2\,m_T}{m_e\,(m_a+m_b)}\,\Delta E_b}$$

$$= \frac{m}{3\,m}\sqrt{\frac{m\,(2\,m)}{m\,(2\,m)}\,\sigma^2 + \frac{2\,(3\,m)}{m\,(2\,m)}\,\Delta E_b} = \frac{1}{3}\sqrt{\sigma^2 + \frac{3}{m}\,\frac{m}{2\,m}\,E_b}$$

Nearly equal mass cluster

$$\Delta E_b = \frac{m}{2\,m}\,E_b$$

3. IMBHs: runaway collisions

$$v_{rec} = v_{esc} = 2\,\sigma \qquad (1)$$

$$v_{rec} = \frac{1}{3}\sqrt{\sigma^2 + \frac{3}{2\,m}\,E_b} \qquad (2)$$

Combining (1) and (2) $36\,\sigma^2 = \sigma^2 + \dfrac{3}{2\,m}\,E_b$

$$E_b = \frac{70\,m}{3}\,\sigma^2 = 70\,k_B\,T \sim 10^2\,k_B\,T$$

↑

$$\frac{1}{2}m\,\sigma^2 = \frac{3}{2}\,k_B\,T$$

E_b is the binding energy exchanged by a hard binary during its life (i.e. before it is ejected).

3. IMBHs: runaway collisions

We calculate now the number of binaries necessary to reverse core collapse (estimated as 10% of the total potential energy of the cluster, Goodman 1987):

$$N_{bin}\,10^2\,k_B\,T \sim 0.1\,|W| = 0.1\,(2\,\frac{3}{2}\,N\,k_B\,T)$$

$$\longrightarrow N_{bin} \sim 10^{-3}\,N$$

Hard binary formation rate:

$$R_{bin} \sim 10^{-3}\,\frac{N}{t_{rlx}}$$

Assuming that ~ each hard binary undergoes <=1 collision, we estimate the collision rate

$$R_{coll} \sim 10^{-3}\,f_{coll}\,\frac{N}{t_{rlx}}$$

3. IMBHs: runaway collisions

(ii) ESTIMATE of δm_{coll}

From dynamical friction timescale $$t_{df} \sim \frac{\langle m \rangle}{m_{df}} t_{rlx}$$

where <*m*>= average star mass, *M*=total cluster mass, *N*= number of stars

We estimate the minimum mass of star that can sink to the centre in a time *t*

$$\delta m_{coll} \geq m_{df} \sim \langle m \rangle \frac{t_{rlx}}{t_{\rm df}}$$

the mass that can be acquired after a collision (!!!)

3. IMBHs: runaway collisions

$$\frac{dM_{raway}}{dt} \sim 10^{-3}\, f_{coll}\, \frac{N}{t}\, \langle m \rangle$$

M_{raway}~$10^{2\text{-}3}$ $M_\odot$ for a dense young cluster with N>10^5

1st CONDITIO SINE QUA NON:

core collapse time << massive star evolution time

→ t_{coll}<3 – 25 Myr

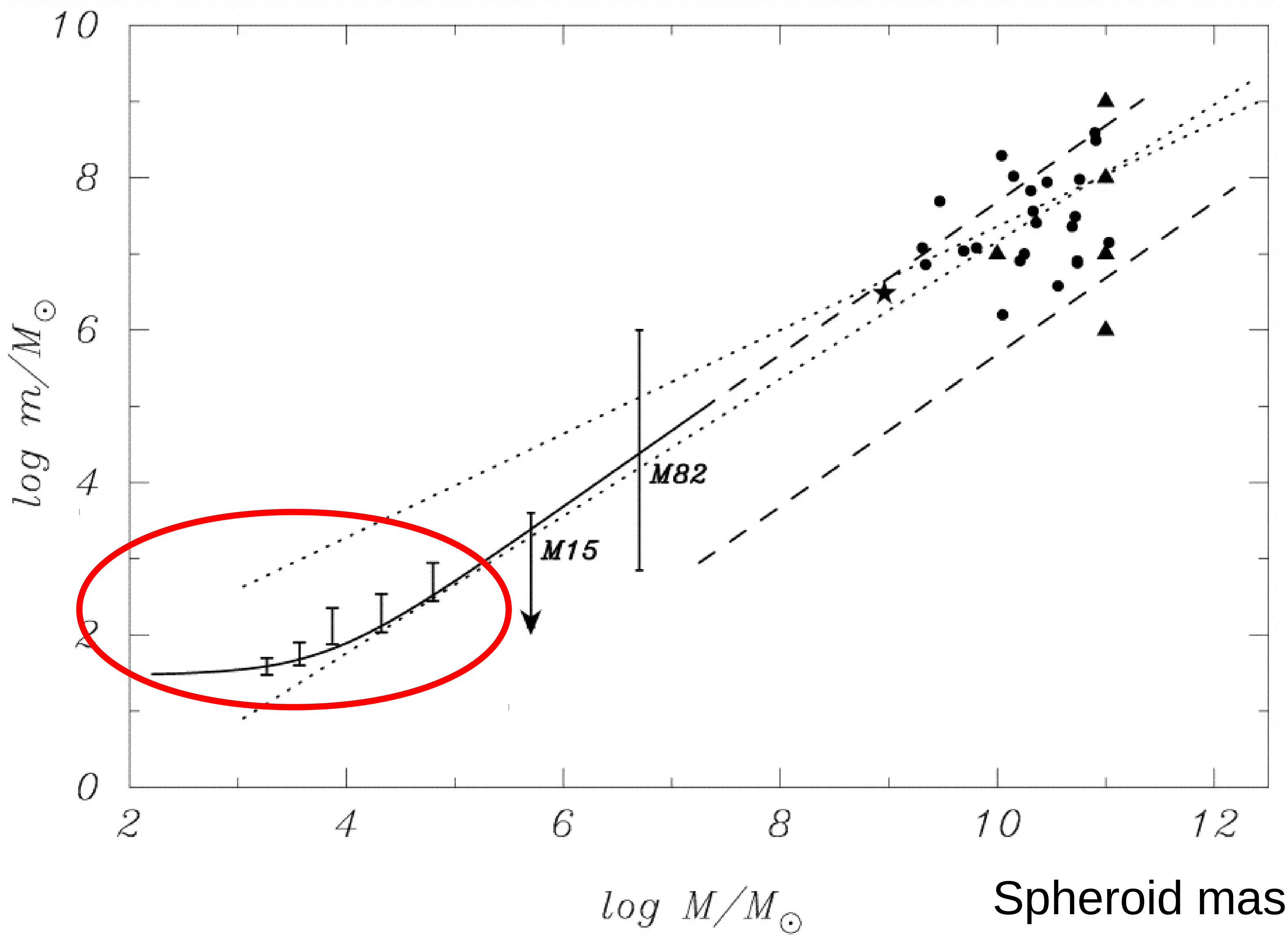
2nd CONDITIO SINE QUA NON:

STAR CLUSTER SUFFICIENTLY MASSIVE AND CONCENTRATED

3. IMBHs: runaway collisions

“Analytic” *formalism by Portegies Zwart & McMillan 2002, ApJ, 576,899* confirmed by their simulations

BH mass



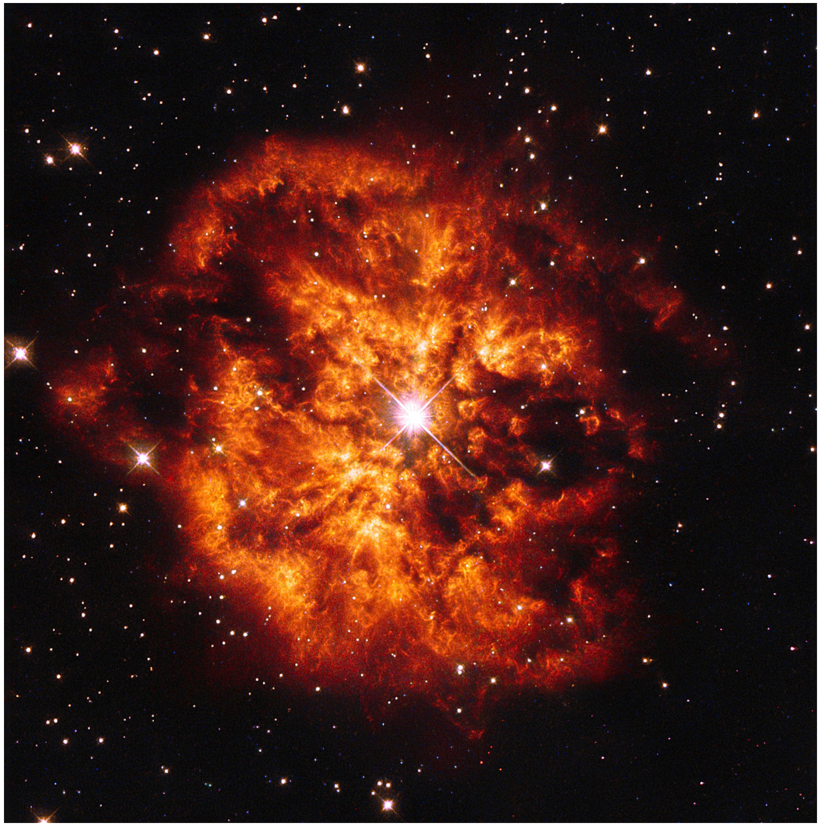
Spheroid mass

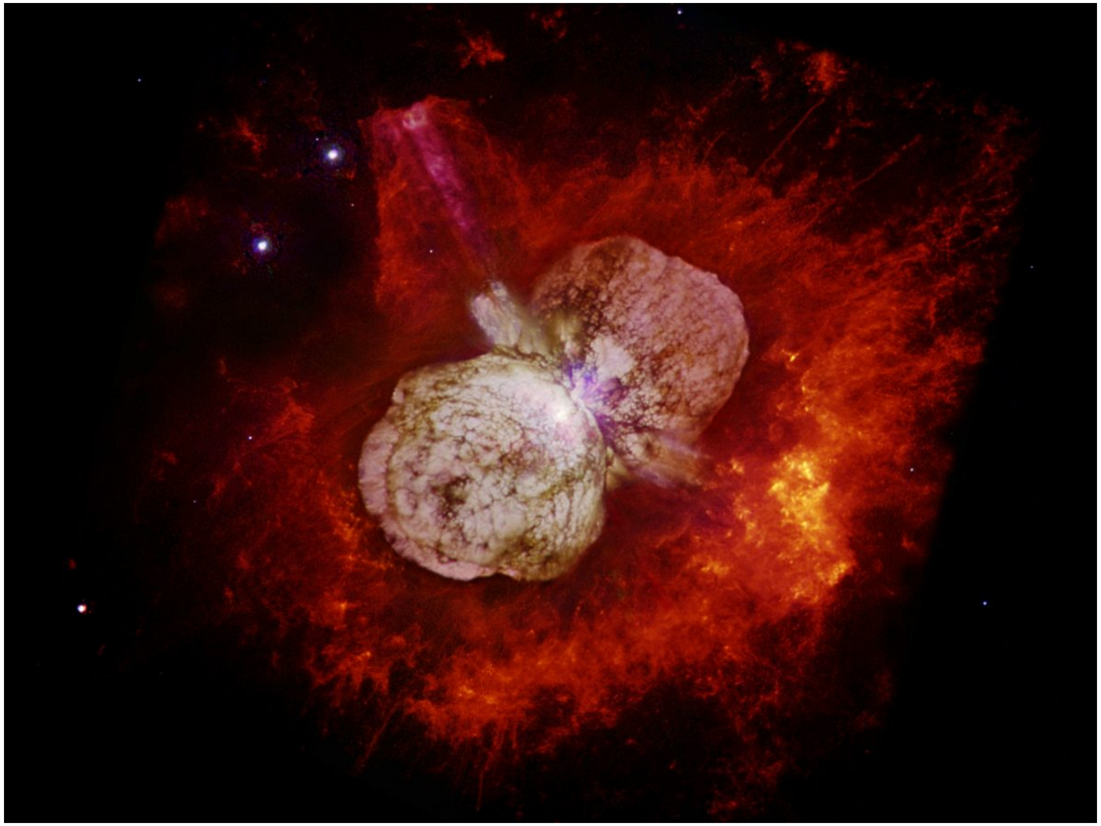
3. IMBHs: runaway collisions

MAIN ISSUE: MASS LOSS!!!

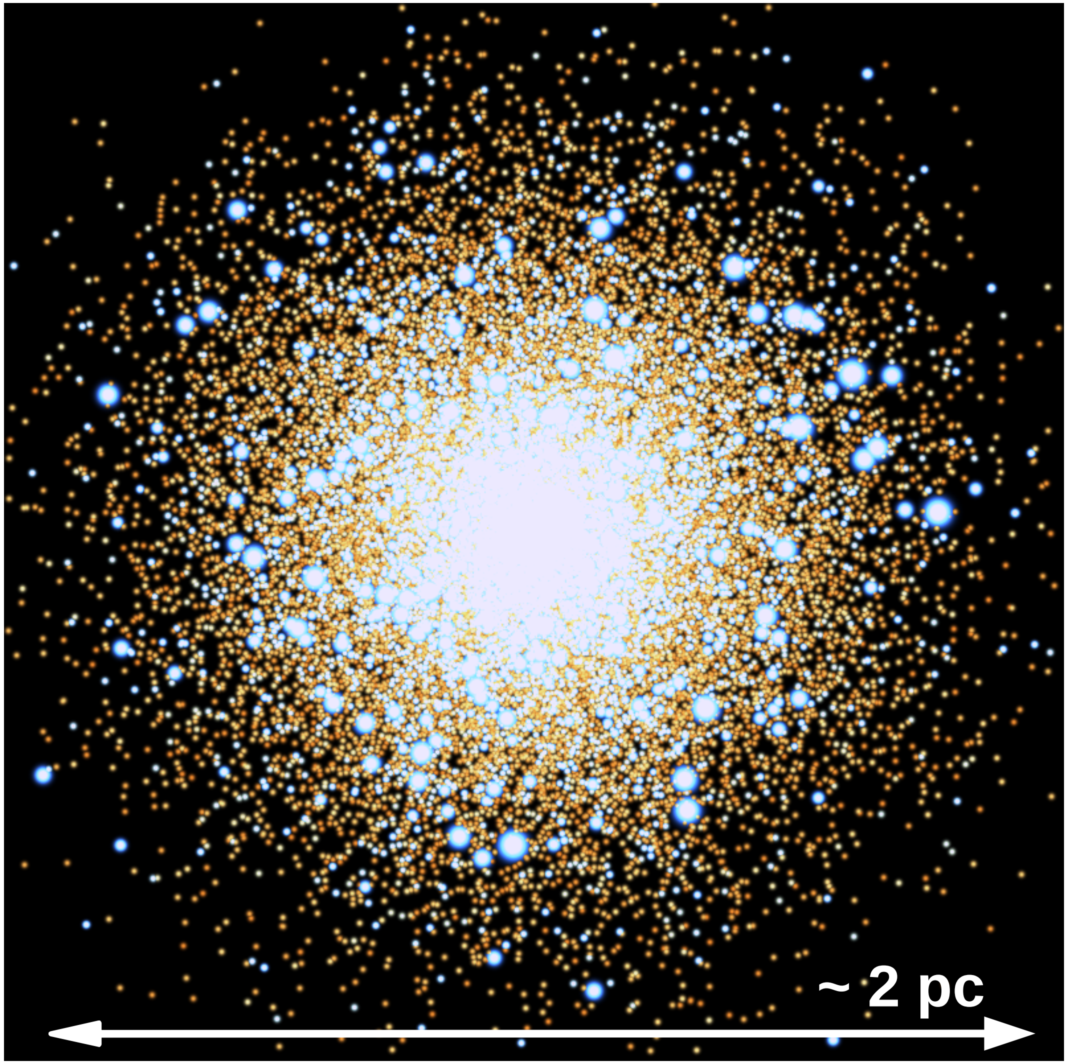
(1) during merger
simulations show mass loss
up to 25% of total mass
(Gaburov et al. 2010, MNRAS, 402, 105)

(2) after merger, by stellar winds
the super-massive star
will be very unstable
(radiation pressure dominated)
e.g. MM 2016, MNRAS, 459, 3432





3. IMBHs: runaway collisions



N-body simulations
of collisional systems
(direct summation N-body)

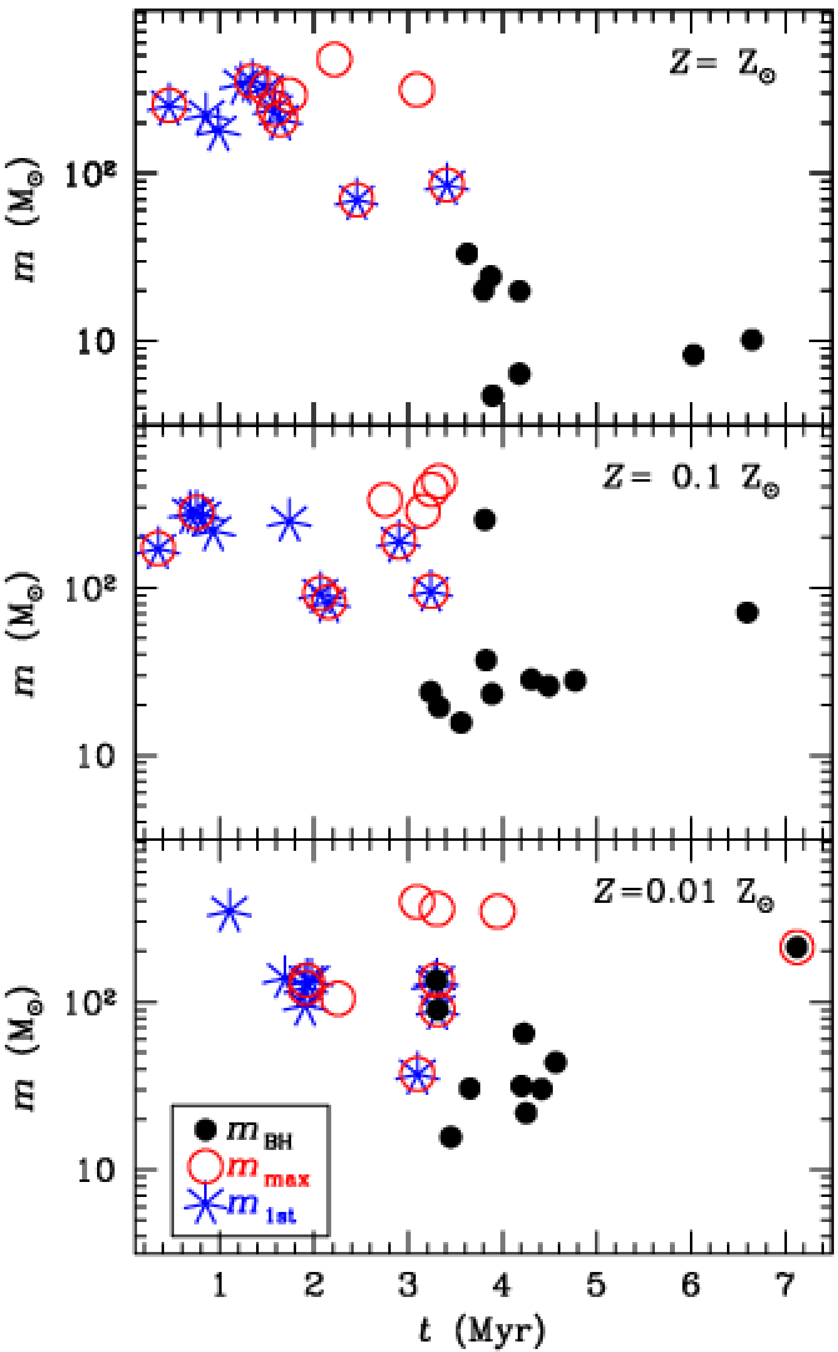
+

stellar and binary evolution
(population synthesis)
embedded in N-body

=

can be used to study
IMBH formation accounting
for mass loss

3. IMBHs: runaway collisions



Mass loss by stellar winds prevents formation of IMBHs from runaway collisions UNLESS METALLICITY < 0.1 Zsun

e.g. MM 2016, MNRAS, 459, 3432

* maximum star mass up to 500 Msun

* 1/10 BH in the IMBH regime (>100 Msun) at Z = 0.01 – 0.1 Zsun

* CAVEAT 1: uncertainties in the evolution of very massive stars

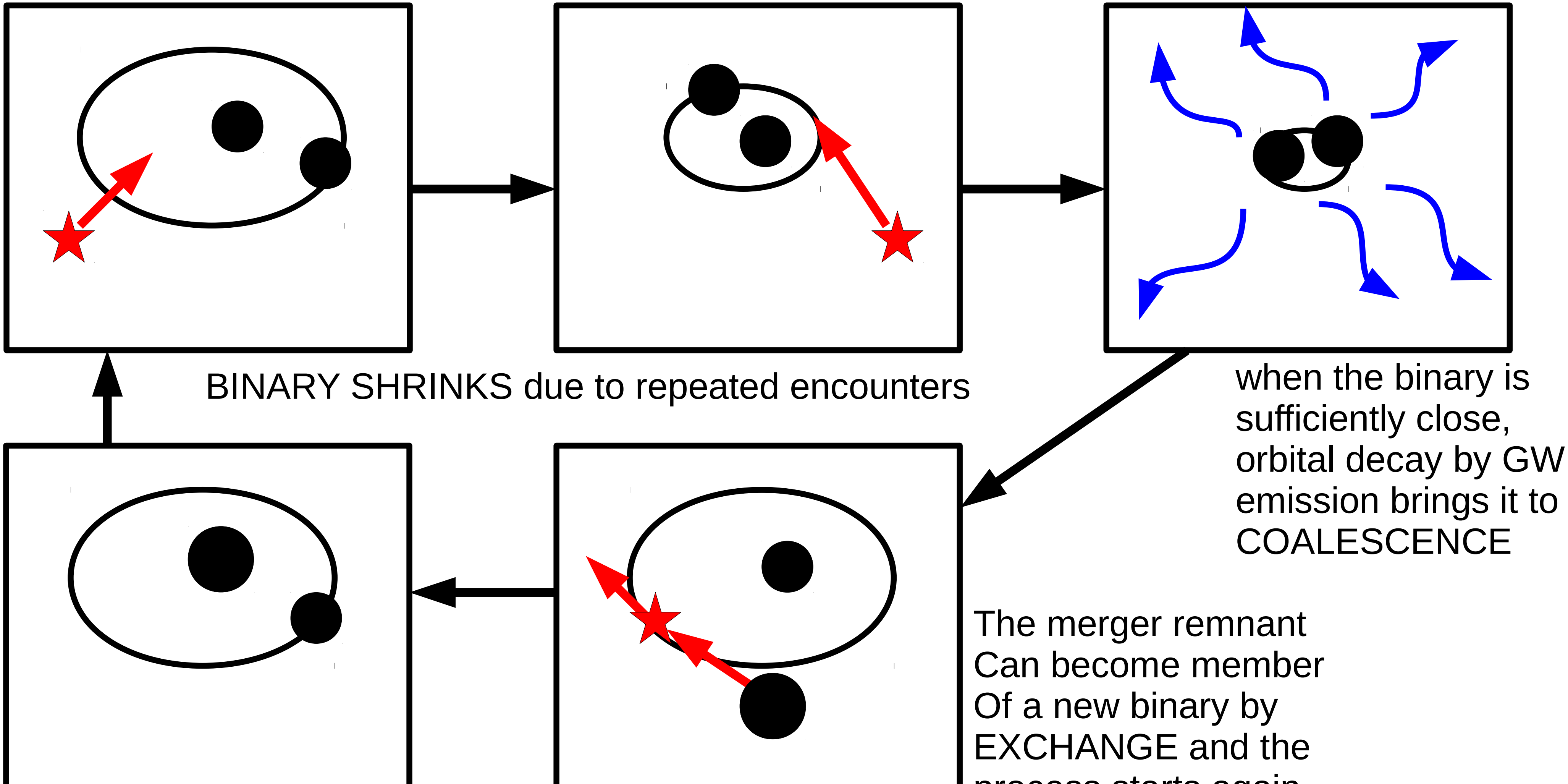
* CAVEAT 2: uncertainties in mass-loss during/after collisions

MM 2016, MNRAS, 459, 3432

3. IMBHs: repeated mergers

Formalism by Miller & Hamilton (2002)

In a old cluster stellar BHs can grow in mass because of repeated mergers with the companion triggered by 3-body encounters



3. IMBHs: repeated mergers

MAIN PROBLEM: seed BH must avoid ejection before merger

$$v_{rec} = \frac{m_3}{m_T}\sqrt{\frac{m_3\,(m_1+m_2)}{m_e\,(m_a+m_b)}\,v_\infty^2 + \frac{2\,m_T}{m_e\,(m_a+m_b)}\,\Delta E_b} \sim \frac{m_3}{m_T}\sqrt{\frac{2\,m_T}{m_3\,(m_1+m_2)}\,\Delta E_b}$$

$$\boxed{m_1 + m_2 \gg m_3}$$

$$\sim \frac{m_3}{m_T}\sqrt{\frac{2\,m_T}{m_3\,(m_1+m_2)}\,\frac{\xi\,m_3}{(m_1+m_2)}\,E_b} \sim \frac{m_3}{m_1+m_2}\sqrt{\frac{2\,\xi}{m_T}\,E_b}$$

Find the minimum binding energy for EJECTION ($E_{b,min}$) by imposing $v_{rec} = v_{esc}$

$$\Rightarrow E_{b,\,min} \sim \frac{(m_1+m_2)^3}{2\,\xi\,m_3^2}\,v_{esc}^2$$

where we assumed $m_1+m_2 \sim m_T$

$$E_{b,\,min} \sim 2\times 10^{50}\ \mathrm{erg}\left(\frac{m_1}{50\,M_\odot}\right)^3\left(\frac{m_3}{10\,M_\odot}\right)^{-2}\left(\frac{\xi}{0.2}\right)^{-1}\left(\frac{v_{esc}}{50\,\mathrm{km\ s^{-1}}}\right)^2$$

3. IMBHs: repeated mergers

Orbital separation in gravitational wave merger regime:

$$a_{GW} \sim 3 \times 10^{11}\ \mathrm{cm} \left(\frac{t_{GW}}{10^6\ \mathrm{Myr}}\right)^{1/4} \left(\frac{m_1}{50\ M_\odot}\right)^{1/2} \left(\frac{m_2}{10\ M_\odot}\right)^{1/4}$$

Binding energy in merger regime:

$$E_{b,\,merg} = \frac{G\,m_1\,m_2}{2\,a_{GW}} \sim 2 \times 10^{50}\ \mathrm{erg} \left(\frac{t_{GW}}{10^6\ \mathrm{Myr}}\right)^{-1/4} \left(\frac{m_1}{50\ M_\odot}\right)^{1/2} \left(\frac{m_2}{10\ M_\odot}\right)^{3/4}$$

COMPARING $E_{b,min}$ with $E_{b,merg}$:

$$x = \frac{E_{b,\,min}}{E_{b\,merg}} \sim \left(\frac{m_1}{50\ M_\odot}\right)^{5/2} \left(\frac{m_2}{10\ M_\odot}\right)^{-11/4} \left(\frac{t_{GW}}{10^6\ \mathrm{Myr}}\right)^{1/4}$$

If $x>1$ BINARY MERGES BEFORE EJECTION

If $x<1$ BINARY IS EJECTED BEFORE MERGER

3. IMBHs: repeated mergers

ADDITIONAL PROBLEM: INEFFICIENT!

Number of 3-body encounters for a BH to merge with its companion (from lecture 3):

$$N_{merg} = \frac{1}{\xi}\,\frac{m_T}{\langle m\rangle}\,\ln\left(\frac{a_0}{a_{GW}}\right)$$

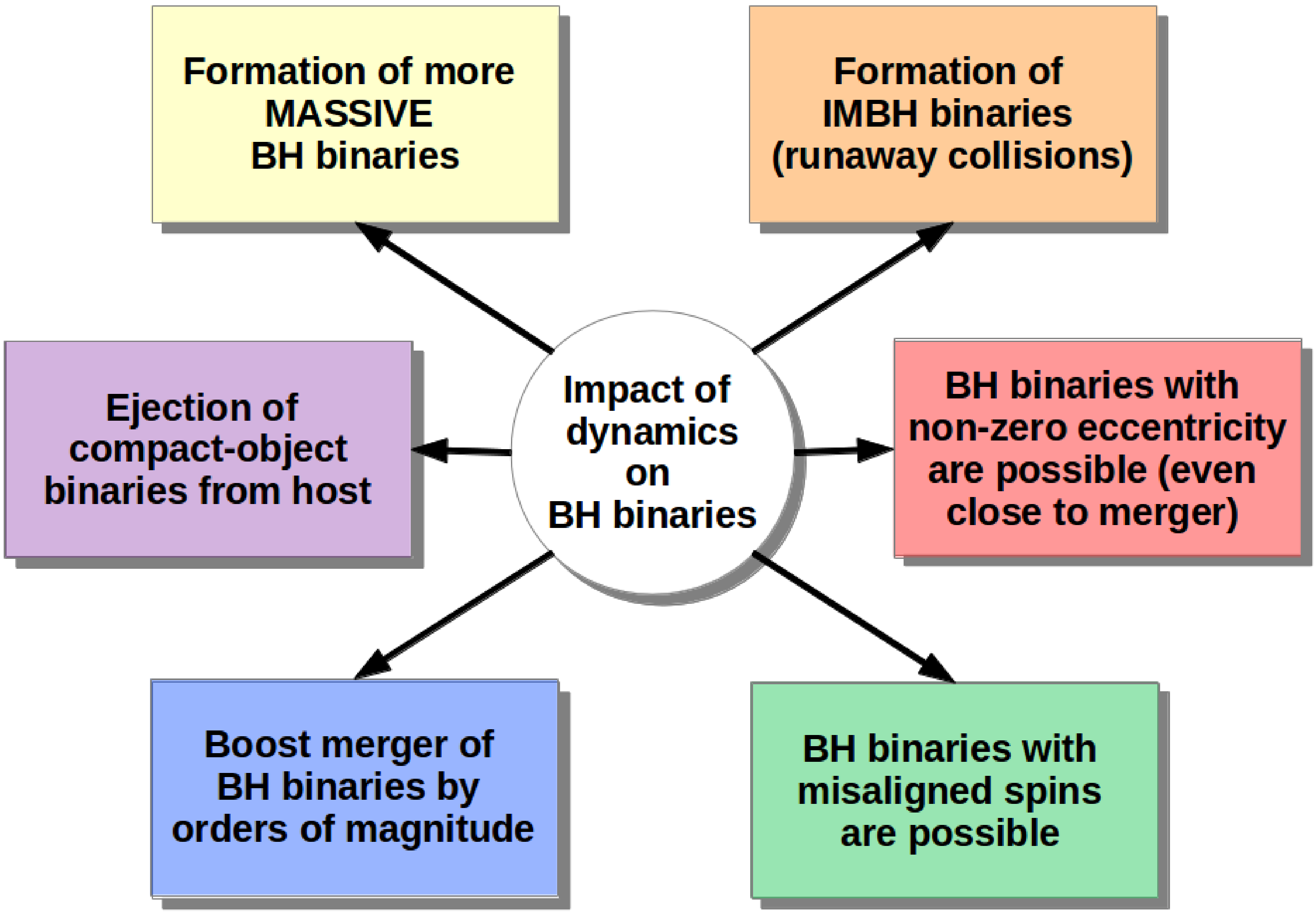
Time required for 1 merger:

$$dt = -\frac{\sigma}{2\,\pi\,G\,\xi\,\rho}\,\frac{da}{a^2} \quad \longrightarrow \quad \int_0^{t_{GW}} dt = -\frac{\sigma}{2\,\pi\,G\,\xi\,\rho}\int_{a_0}^{a_{GW}}\frac{da}{a^2}$$

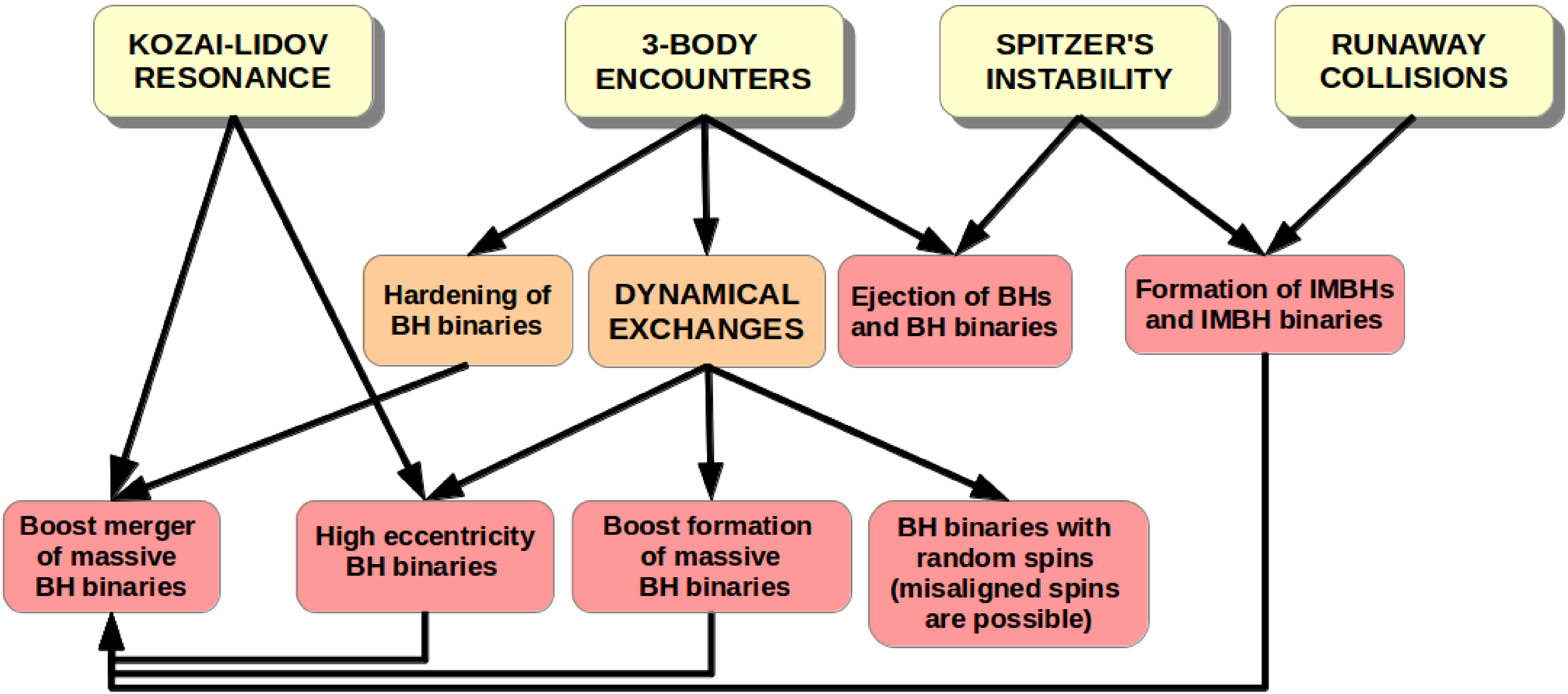
$$t_{GW} = \frac{\sigma}{2\,\pi\,G\,\xi\,\rho}\left(\frac{1}{a_{GW}} - \frac{1}{a_0}\right)$$

$$t_{GW} \sim 3\times 10^8\ \mathrm{yr}\left(\frac{\sigma}{10\ \mathrm{km\ s^{-1}}}\right)\left(\frac{\xi}{0.2}\right)^{-1}\left(\frac{\rho}{10^6\ M_\odot\ \mathrm{pc^{-3}}}\right)^{-1}\left(\frac{a_{GW}}{1\ \mathrm{AU}}\right)^{-1}$$

SUMMARY of EFFECTs of DYNAMICS on BH binaries:



SUMMARY of EFFECTs of DYNAMICS on BH binaries:



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