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Dynamics of Stars and Black Holes in Dense Stellar Systems:

Lecture IV:

Dynamical processes induced by mass spectrum

0. EFFECTS OF MASS SPECTRUM
1. MASS SEGREGATION
2. EQUIPARTITION of ENERGY
3. SPITZER'S INSTABILITY

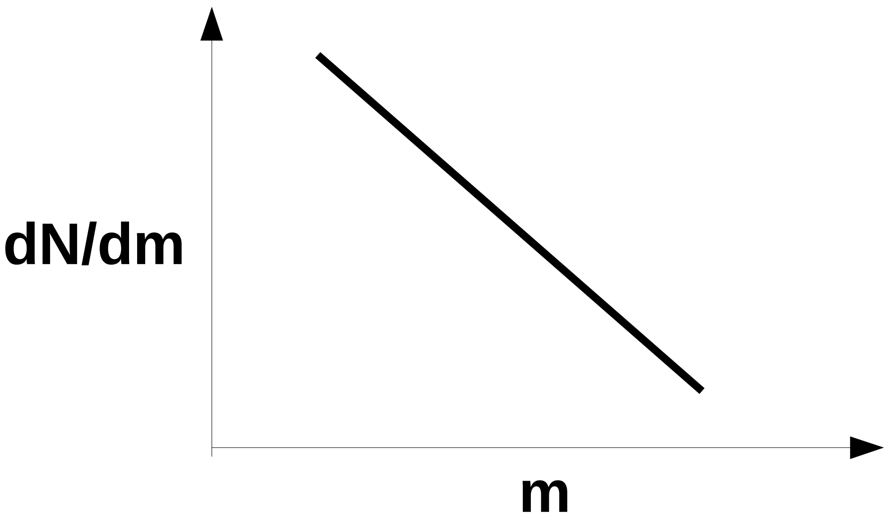
0. EFFECTS OF MASS SPECTRUM

Does core collapse require a mass function (= a system of non-equal mass stars) to occur?

0. EFFECTS OF MASS SPECTRUM

Processes described up to now (two-body relaxation, evaporation, gravothermal instability, core collapse and reversal) OCCUR EVEN IF STARS ARE EQUAL MASS

BUT stars form with a mass function



The most important effects of unequal-mass system are
MASS SEGREGATION and SPITZER'S INSTABILITY

OTHER PROCESSES (eg core collapse) occurring even in equal-mass systems are INFLUENCED by MASS SPECTRUM (eg occur FASTER)

1. MASS SEGREGATION

- what is mass segregation:
  is it a physical process
  or a “state” of a stellar system?

- what drives mass segregation?

1. MASS SEGREGATION

From lecture 1: DYNAMICAL FRICTION

Massive stars feel a DRAG FORCE by light stars, which decelerates them

Same driver as to two-body relaxation:
two-body encounters where the SOLE gravity force is involved

→ same timescale but different relevant mass

$$t_{df} \sim \frac{m}{M} \, t_{rlx}$$

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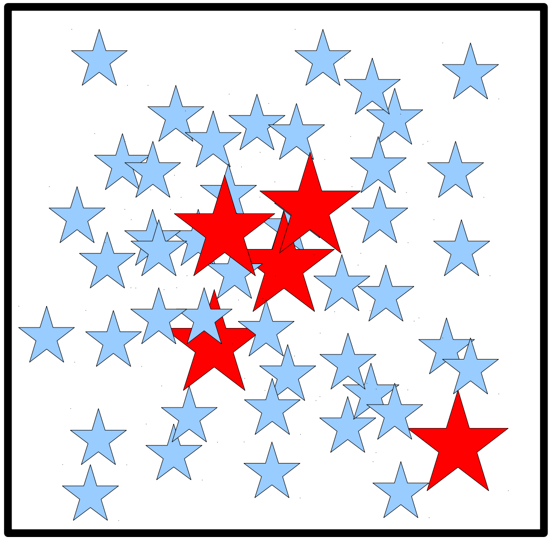
→ Dynamical friction leads MASSIVE STARS to SLOW DOWN wrt light stars

→ MASSIVE STARS SINK to the CENTRE of the CLUSTER (= centre of the potential well)



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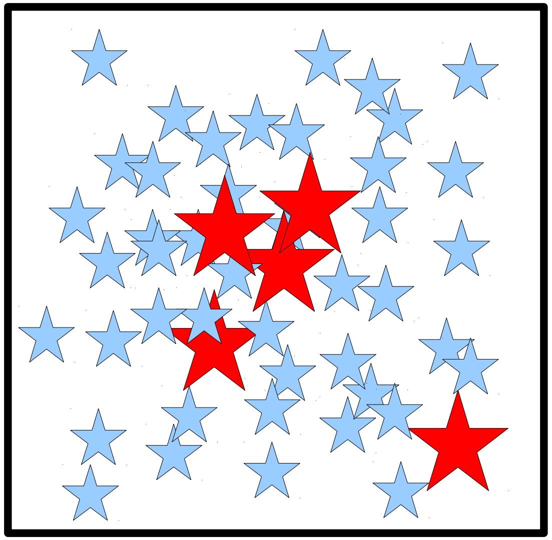


The result is a cluster where the relative frequency of massive stars is higher in the core than in the outskirts:

a MASS STRATIFIED or MASS SEGREGATED cluster

<u>BOTTOM LINE: mass segregation indicates the state of the cluster</u>
<u>The physical process driving mass segregation is dynamical friction</u>

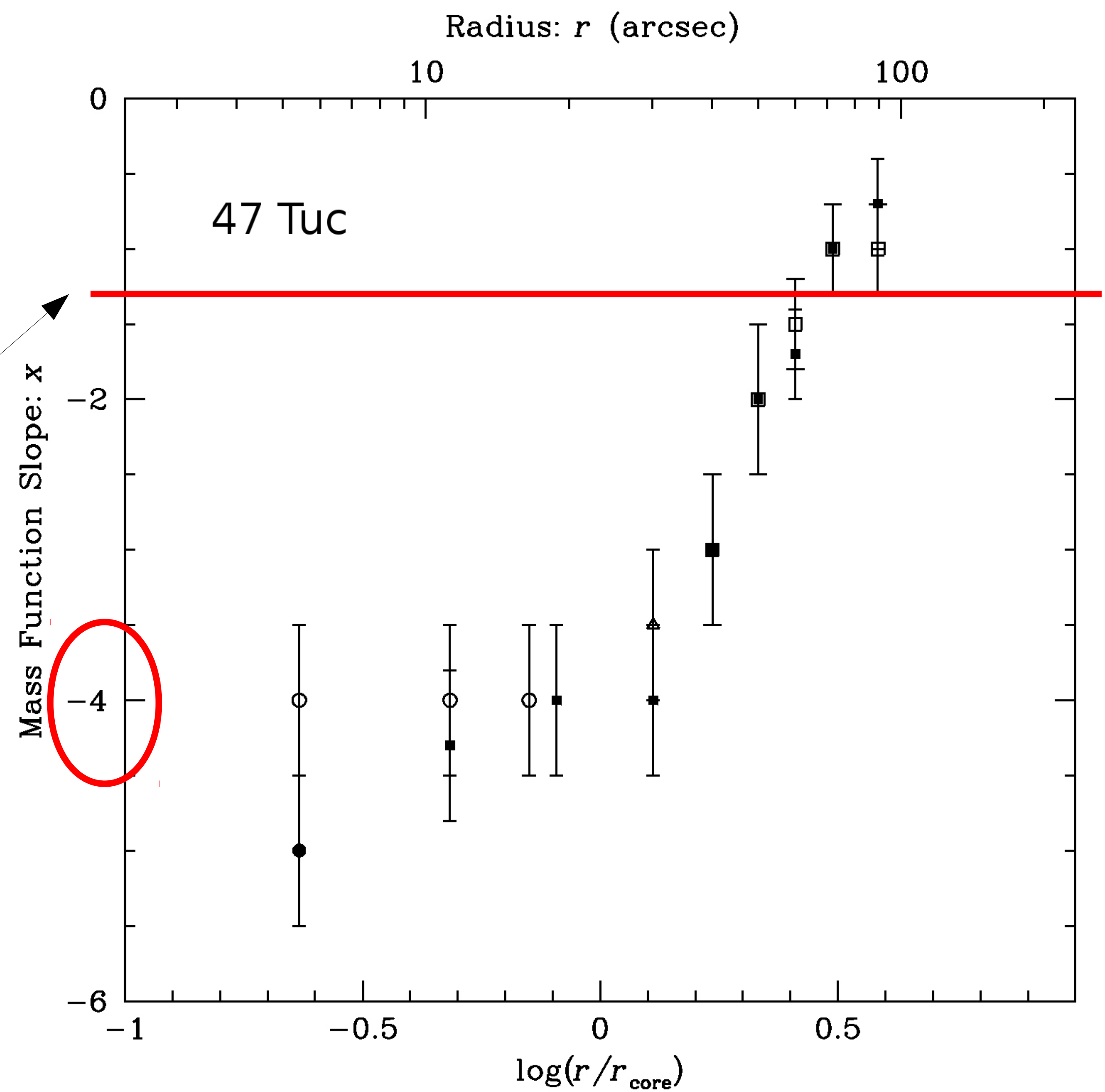
1. MASS SEGREGATION

In mass segregated clusters, LOCAL mass function is different from IMF

e.g. core of 47 Tucanae (Monkman et al. 2006, ApJ, 650, x:= mass function slope

$$dN/dm = m^{-1+x}$$

where Salpeter
$x = -1.35$



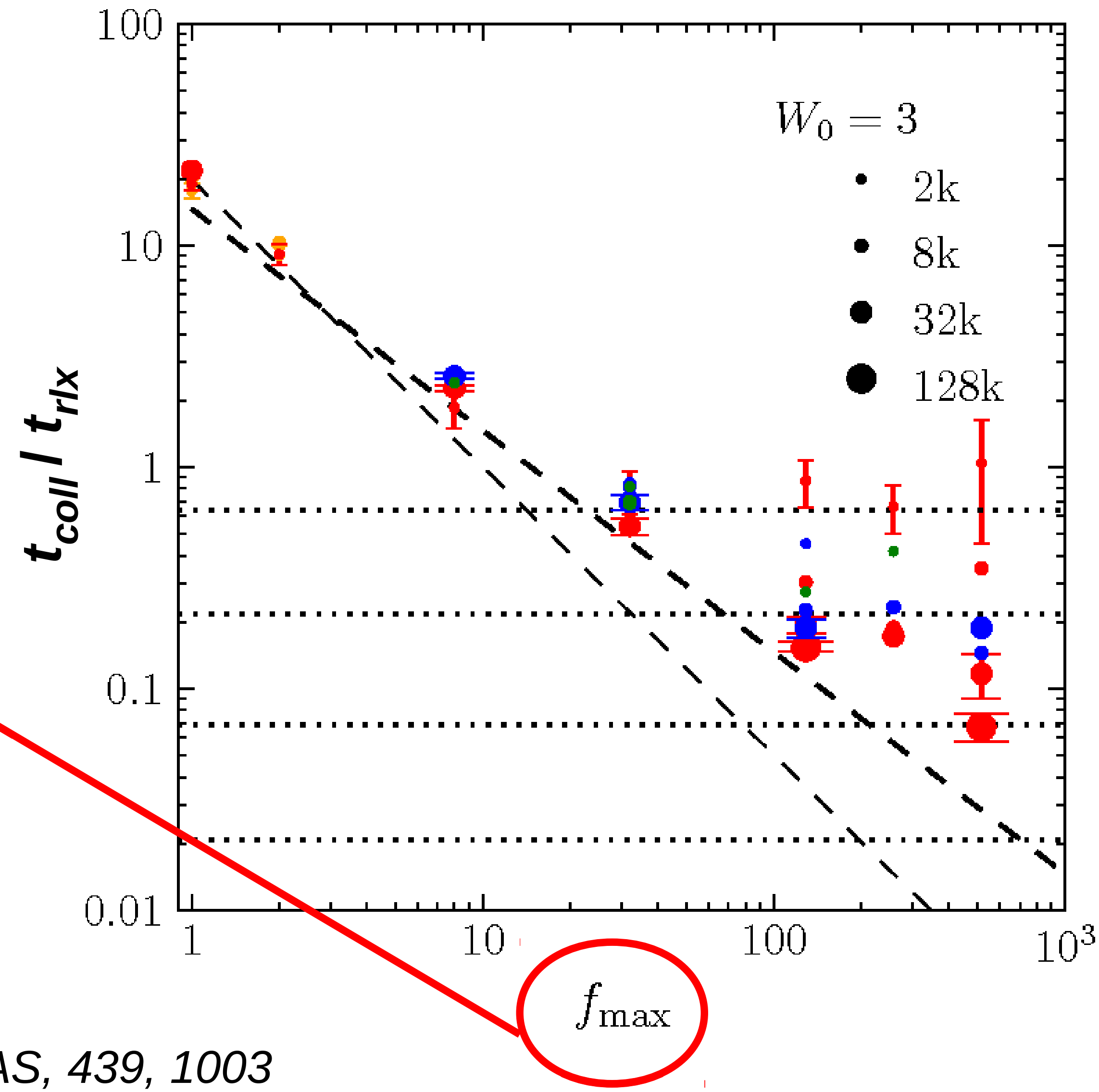
1. MASS SEGREGATION

EFFECT ON CORE COLLAPSE:
MASS SEGREGATION increases the instability of the system and induces a **FASTER COLLAPSE (t_{coll}~0.2 t_{rlx} rather than t_{coll}~15 t_{rlx})**.

Measured with N-body simulations

If m_{max} is the maximum mass of a star in system and <m> is average mass

$$f_{\rm max} \equiv \frac{m_{\rm max}}{\langle m \rangle}$$



Fujii & Portegies Zwart et al. 2014, MNRAS, 439, 1003

2. EQUIPARTITION

Theorem of statistical mechanics (Boltzmann 1876):

If a system of ideal gas particles is in thermal equilibrium, energy is shared equally by all particles

Stellar systems can be considered the same as gas particles if temperature is defined as

$$\frac{1}{2}\, m \left\langle v^2 \right\rangle = \frac{3}{2}\, \kappa_B \, T$$

EQUIPARTITION for stellar systems: if a collisional system (i.e. where two-body relaxation is efficient) is in thermal equilibrium,
PARTICLES TEND TO HAVE THE SAME KINETIC ENERGY

Thus, equipartition occurs EVEN if stars are equal mass.

If stars are equal mass → equipartition implies that have the same **VELOCITY LOCALLY**

$$k_i = \frac{1}{2}\, m \, v_i^2$$

2. EQUIPARTITION

If particles have different masses:

$$m_i \langle v_i^2 \rangle = m_j \langle v_j^2 \rangle$$

$$\text{if} \quad m_i > m_j \Rightarrow \langle v_i^2 \rangle < \langle v_j^2 \rangle$$

During two-body encounters, massive stars transfer kinetic energy to light stars.
Massive stars slow down, light stars move to higher velocities.

SEEMS equivalent to mass segregation: dynamical friction slows down massive stars and they sink to centre

IS EQUIPARTITION THE SAME AS MASS SEGREGATION?

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SHORT ANSWER: NO

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MASS SEGREGATION: Massive stars tend to slow down by dynamical friction and sink to the core leading the system to be mass segregated
VS
EQUIPARTITION: if the system is in thermal equilibrium the relation

$$m_i \left\langle v_i^2 \right\rangle = m_j \left\langle v_j^2 \right\rangle$$ MUST BE TRUE

Also equipartition is reached (if it is reached) by dynamical friction but is a more stringent condition than mass segregation

→ **a system can be mass segregated WITHOUT being in equipartition**

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→ **a system can be mass segregated WITHOUT being in equipartition**

ADDITIONAL CAVEAT: equipartition is a LOCAL condition

PROBLEM: equipartition for star systems is based on analogy with statistical mechanics of gas systems MIGHT BE MISLEADING!!!!

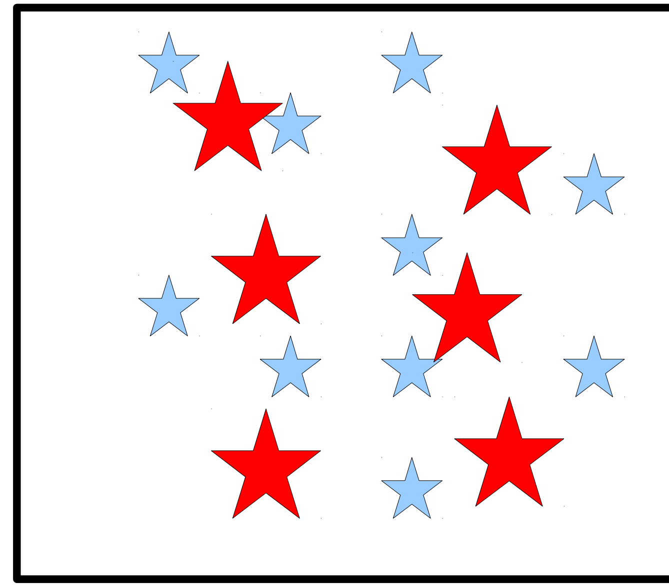
2. EQUIPARTITION

IS EQUIPARTITION ALWAYS REACHED IN A TWO-BODY RELAXED SYSTEM?

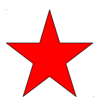
3. SPITZER'S INSTABILITY (or MASS STRATIFICATION instability):

It is not always possible to reach equipartition in a multi-mass system.

Let us suppose that there are two populations with two different masses:



$m_2 > m_1$



HEAVY POPULATION
m_2 (total mass M_2)

LIGHT POPULATION
m_1 (total mass M_1)

$M_2 \sim M_1$

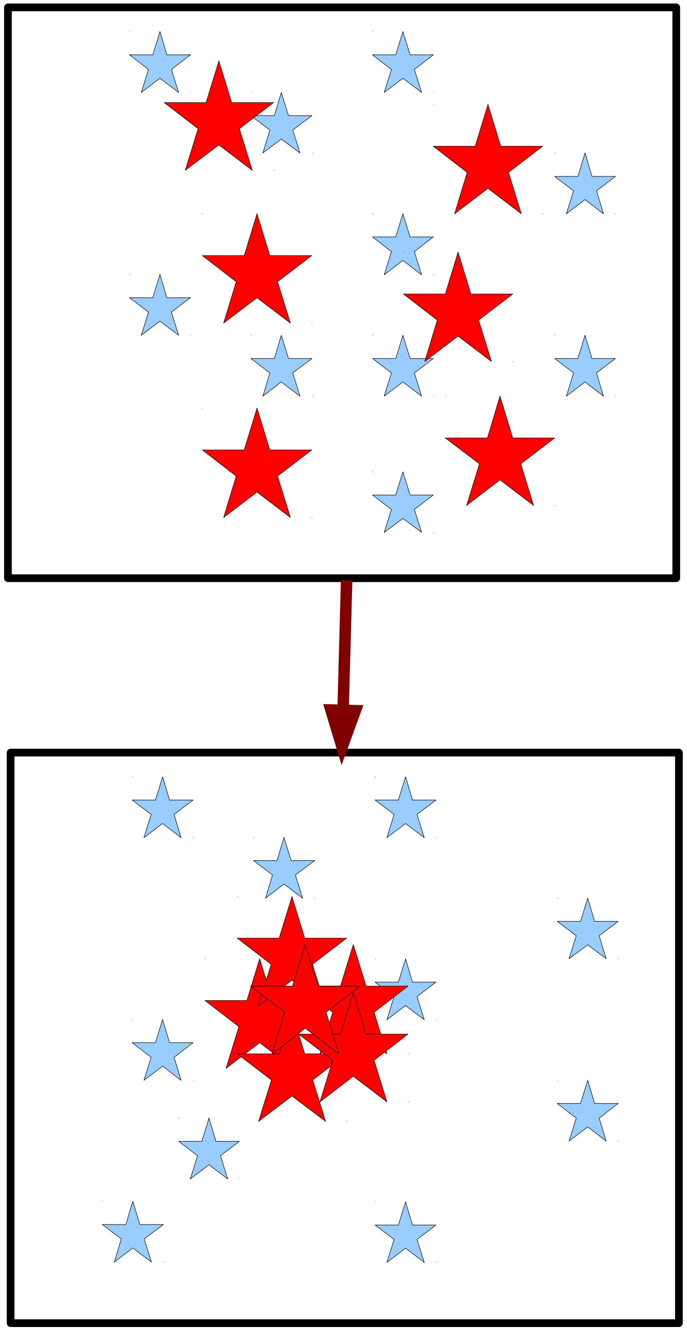
If the total mass of the heavy population is similar to the total mass of the light population, equipartition is not possible:

$M_2 \langle v_2^2 \rangle >> M_1 \langle v_1^2 \rangle$

THE LIGHT POPULATION CANNOT ABSORB ALL THE KINETIC ENERGY THAT MUST BE TRANSFERRED FROM THE HEAVY POPULATION TO REACH EQUIPARTITION

3. SPITZER'S INSTABILITY

The heavy population forms a
CLUSTER WITHIN THE CLUSTER
(sub-cluster at the centre of the cluster),
DYNAMICALLY DECOUPLED
from the rest of the cluster.



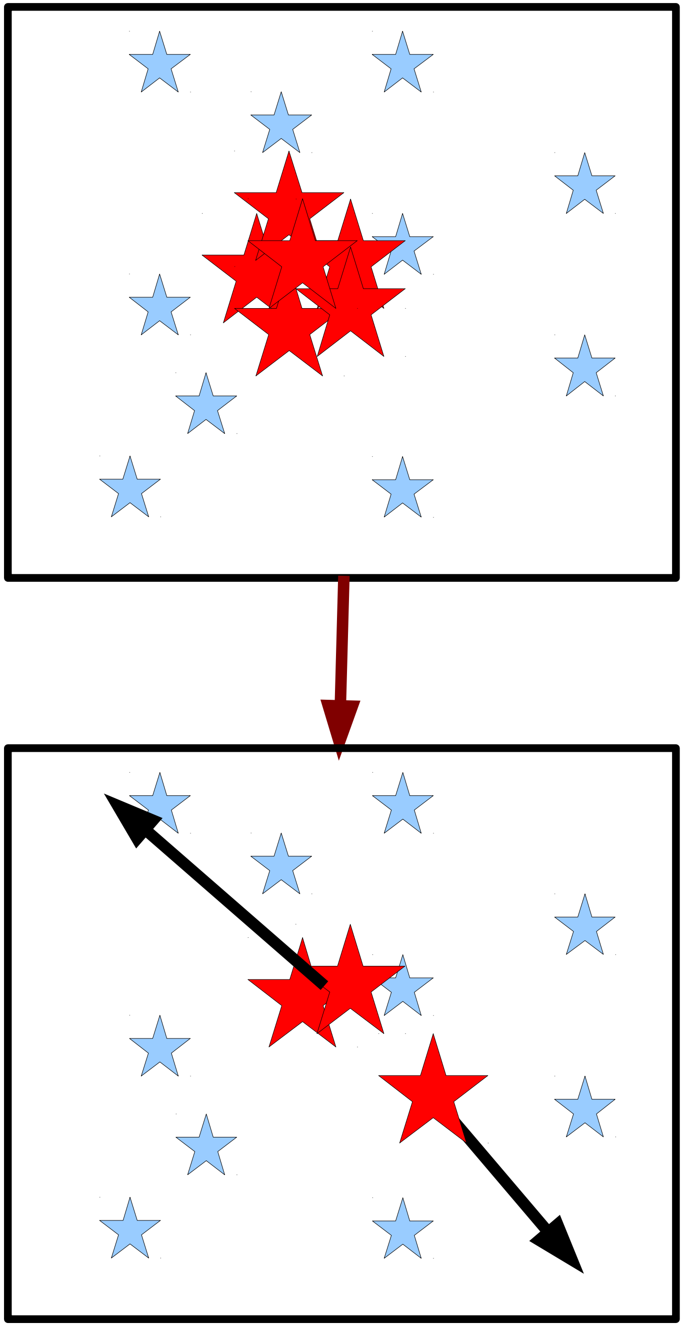
The massive stars in the sub-cluster keep transferring kinetic energy to the lighter stars but cannot reach equipartition:
the core of massive stars continues to
CONTRACT TILL INFINITE DENSITY!

3. SPITZER'S INSTABILITY

The contraction stops

– when most of the massive stars eject each-other from the SC by 3-body encounters

SPITZER INSTABILITY ENHANCES THE EJECTION OF MASSIVE OBJECTS (E.G. BLACK HOLES) FROM SCs !!!!



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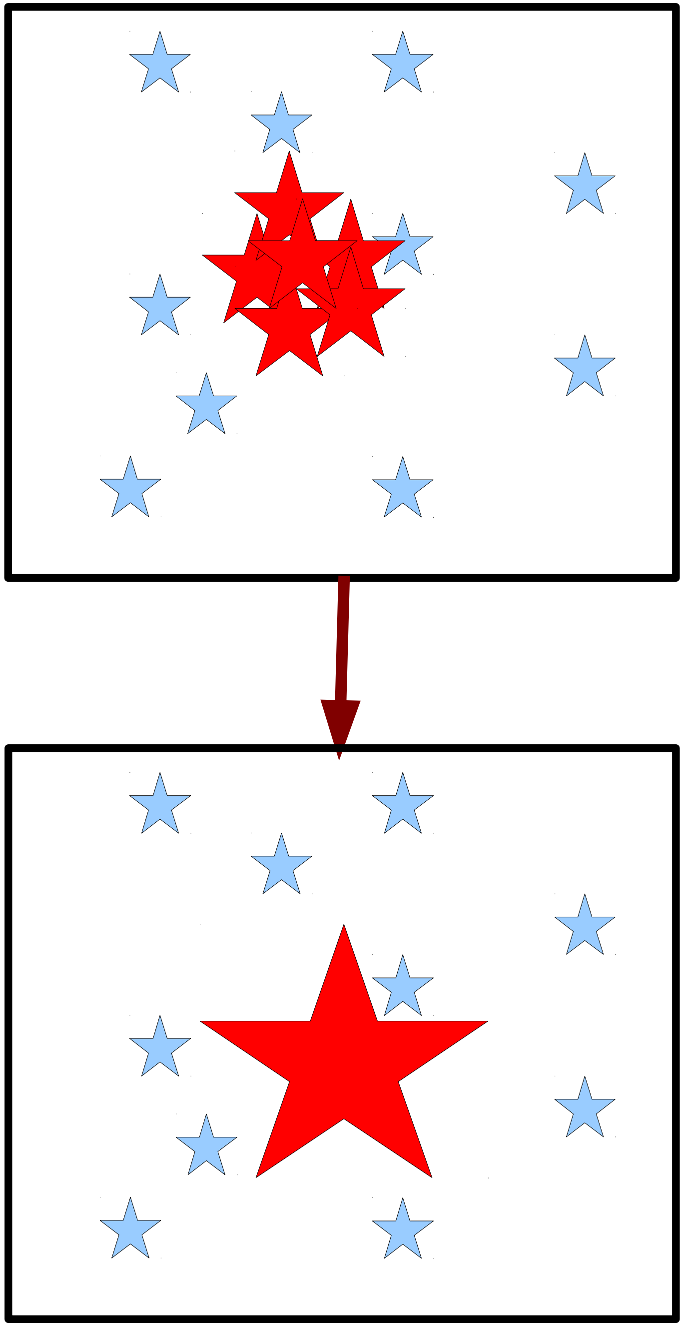
– or when most of the massive stars collapse into a single object



3. SPITZER'S INSTABILITY

MATH DEMONSTRATION FROM Spitzer (1969)*:*

Let us suppose that there are two populations with two different masses: m_1 (total mass M_1) and m_2 (total mass M_2), with $m_1<m_2$.

We explore 2 limit cases where equipartition is impossible.

1) $M_2>>M_1 \Rightarrow$ potential is dominated by massive stars

$\Rightarrow \langle v^2\rangle$ of the massive stars is ~ ¼ $\langle v_{esc}^2\rangle$

$\Rightarrow$ if $m_2/m_1 > 4$, the $\langle v^2\rangle$ of light stars is higher than $\langle v_{esc}^2\rangle$

$\Rightarrow$ ***ALL LIGHT STARS EVAPORATE FROM THE CLUSTER!!!***

Not very important in practice because IMF is not sufficiently top-heavy

3. SPITZER'S INSTABILITY

2) M_2~M_1 (the case of the so called Spitzer's instability)

If the total mass of the heavy population is similar to the total mass of the light population, equipartition is not possible:
the heavy population forms a cluster within the cluster,
i.e. a sub-cluster at the centre of the cluster,
dynamically decoupled from the rest of the cluster.
The sub-cluster of the heavy population tends to contract.

DEMONSTRATION:
(Note that I did not put numerical coefficients & simplified!)
(a) Assume that there are two populations (1 and 2) with $m_2 >> m_1$
(b) assume total mass $M_2 < M_1$
(c) assume $M_1(r) \sim \rho_{01}\, r^3$ (ρ_{01}:= initial density of population 1)

$\rho_{m2} \sim M_2/r_2^3$ *(ρ_{m2}:= average density of population 2, r_2:=half mass radius of population 2)*

$\rho_{m1} \sim M_1/r_1^3$ *(ρ_{m1}:= average density of population 1, r_1:=half mass radius of population 1)*

3. SPITZER'S INSTABILITY

From equipartition: $m_i \langle v_i^2 \rangle = m_j \langle v_j^2 \rangle$ (1)

From virial theorem: $\langle v_2^2 \rangle \sim \frac{G\,M_2}{r_2} + \frac{G}{M_2} \int_0^\infty \frac{\rho_2\,M_1(\vec{r})}{\vec{r}} dV$ (2)

$$\langle v_1^2 \rangle \sim \frac{G\,M_1}{r_1} + \frac{G}{M_1} \int_0^\infty \frac{\rho_1\,M_2(\vec{r})}{\vec{r}} dV \quad (3)$$

$M_1 \gg M_2(\vec{r})$ → 0

Substituting (2) and (3) into (1) and using the assumptions a, b and c:

$$m_2 \left(\frac{G\,M_2}{r_2} + \frac{G\,\rho_{m2}}{M_2} \int \rho_{01} \frac{\vec{r}^3}{\vec{r}} dV \right) = m_1 \frac{G\,M_1}{r_1}$$

$$m_2 \left[\frac{M_2}{\left(\frac{M_2}{\rho_{m2}}\right)^{1/3}} + \frac{\rho_{m2}}{M_2} \int \rho_{01}\,\vec{r}^2\,dV \right] = m_1 \frac{M_1}{\left(\frac{M_1}{\rho_{m1}}\right)^{1/3}}$$

3. SPITZER'S INSTABILITY

$$m_2\, M_2^{2/3}\, \rho_{m2}^{1/3}\, \left(1 + \frac{\rho_{m2}\, \int \rho_{01}\, \vec{r}^2\, dV}{M_2^{5/3}\, \rho_{m2}^{1/3}}\right) = m_1\, M_1^{2/3}\, \rho_{m1}^{\;1/3}$$

$$\frac{m_2}{m_1}\, \left(\frac{M_2}{M_1}\right)^{2/3} = \frac{\left(\rho_{m1}/\rho_{m2}\right)^{1/3}}{\left[1 + \frac{\rho_{m2}\, \int \rho_{01}\, \vec{r}^2\, dV}{\left(\rho_{m2}\, r_2^3\right)^{5/3}\, \rho_{m2}^{1/3}}\right]}$$

$$\frac{m_2}{m_1}\, \left(\frac{M_2}{M_1}\right)^{2/3} = \frac{\left(\rho_{m1}/\rho_{m2}\right)^{1/3}}{\left(1 + \frac{\rho_{m2}\, \int \rho_{01}\, \vec{r}^2\, dV}{\rho_{m2}^2\, r_2^5}\right)}$$

$$\frac{m_2}{m_1}\, \left(\frac{M_2}{M_1}\right)^{2/3} = \frac{\left(\rho_{m1}/\rho_{m2}\right)^{1/3}}{\left[1 + \frac{\rho_{01}}{\rho_{m2}}\, \left(\frac{r_2}{r_2}\right)^5\right]}$$

3. SPITZER'S INSTABILITY

$$\left(\frac{m_2}{m_1}\right)^{3/2} \left(\frac{M_2}{M_1}\right) = \frac{\left(\rho_{m1}/\rho_{m2}\right)^{1/2}}{\left[1+\frac{\rho_{01}}{\rho_{m2}}\right]^{3/2}}$$

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$$\left(\frac{m_2}{m_1}\right)^{3/2} \left(\frac{M_2}{M_1}\right) = \frac{\left(\rho_{m1}/\rho_{m2}\right)^{1/2}}{\left[1+\frac{\rho_{01}}{\rho_{m1}}\left(\frac{\rho_{m1}}{\rho_{m2}}\right)\right]^{3/2}}$$

3. SPITZER'S INSTABILITY

$$\left(\frac{m_2}{m_1}\right)^{3/2} \left(\frac{M_2}{M_1}\right) = \frac{(\rho_{m1}/\rho_{m2})^{1/2}}{\left(1+\alpha\,\frac{\rho_{m1}}{\rho_{m2}}\right)^{3/2}}$$

Our simplified α :

$$\alpha \equiv \frac{\rho_{01}}{\rho_{m1}}$$

Spitzer's α :

$$\alpha \equiv \frac{5\,\rho_{01}}{4\,\rho_{m1}} \left(\frac{r_{2s}}{r_2}\right)^2 \sim 5.6$$

where r_{2s}^2 is the mean value of r^2 for the population 2

Maximum possible value for the right-hand term ~ 0.16

$$\Rightarrow \frac{M_2}{M_1} < 0.16 \left(\frac{m_2}{m_1}\right)^{3/2}$$

OTHERWISE EQUIPARTITION CANNOT BE REACHED!

3. SPITZER'S INSTABILITY

SPITZER'S INSTABILITY:
It is not possible to reach equipartition if $M_2/M_1 < 0.16\ (m_2/m_1)^{3/2}$.

If the total mass of the heavy population is similar to the total mass of the light population, equipartition is not possible:

the heavy population forms a cluster within the cluster,
i.e. a sub-cluster at the centre of the cluster,
dynamically decoupled from the rest of the cluster.

The massive stars in the sub-cluster keep transferring kinetic energy to the lighter stars but cannot reach equipartition: the core of massive stars continues to contract till infinite density!

The contraction stops when most of the massive stars eject each-other from thecluster by 3-body encounters or when most of the massive stars collapse into a single object (see last lecture).

3. SPITZER'S INSTABILITY

How common is Spitzer's instability?

No analytic calculation that generalizes Spitzer calculation (2 mass system) for a generic mass function (Vishniac 1978 has dangerous assumptions – see Merritt 1981, also Miocchi 2006)

But several numerical tests:

Trenti & van der Marel 2013, MNRAS, 435, 3272
Spera, MM & Jeffries 2016, MNRAS, 460,317
Parker et al. 2016, MNRAS, 459, L119
Bianchini et al. 2016, MNRAS, 458, 3644

SUGGEST SPITZER'S INSTABILITY IS COMMON

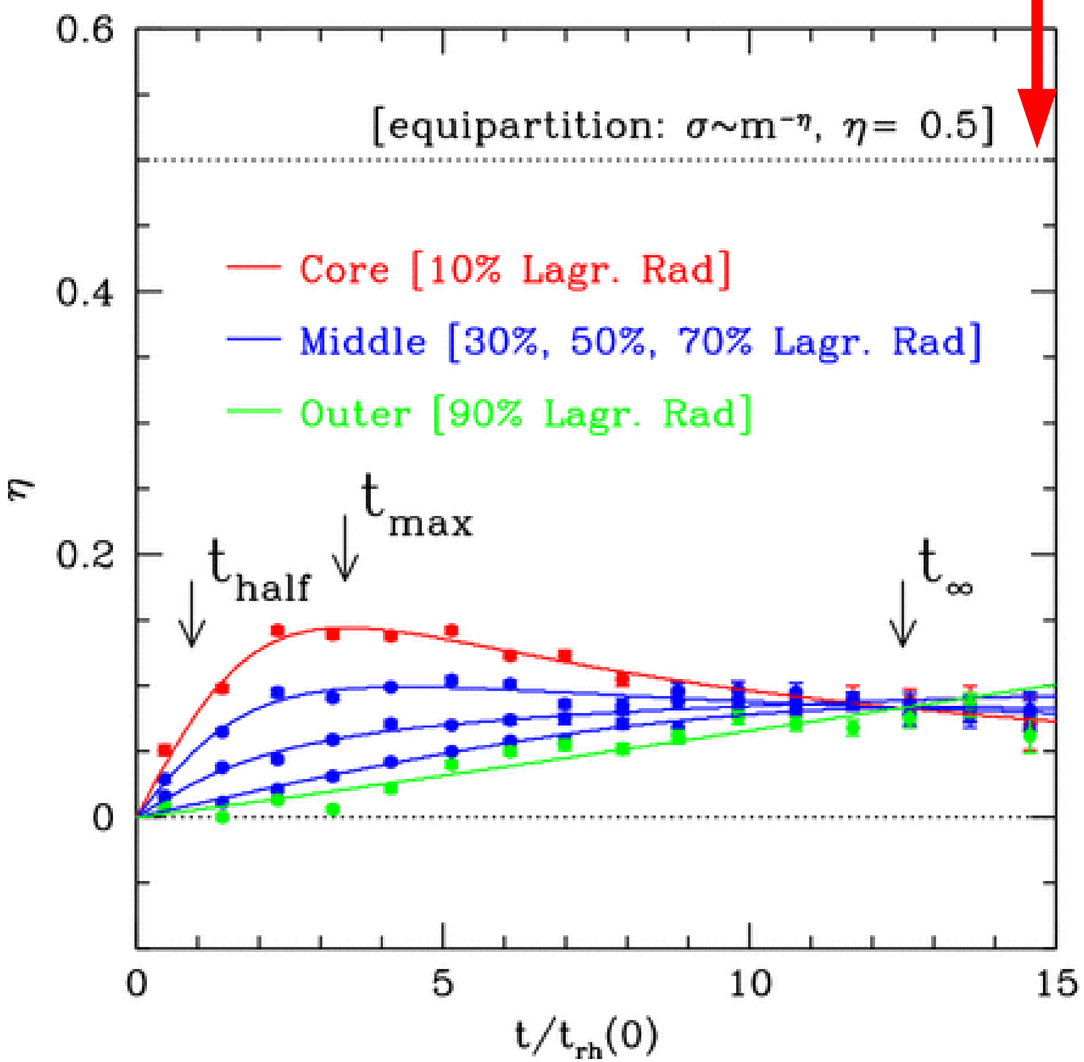
EQUIPARTITION MAY BE IMPOSSIBLE TO REACH

3. SPITZER'S INSTABILITY

If equipartition in a non-rotationally supported system (velocity dispersion σ first non-zero term of velocity)

$$m_i \sigma_i^2 \sim m_j \sigma_j^2 \quad \rightarrow \sigma(m)\ \mu\ m^{-0.5}$$

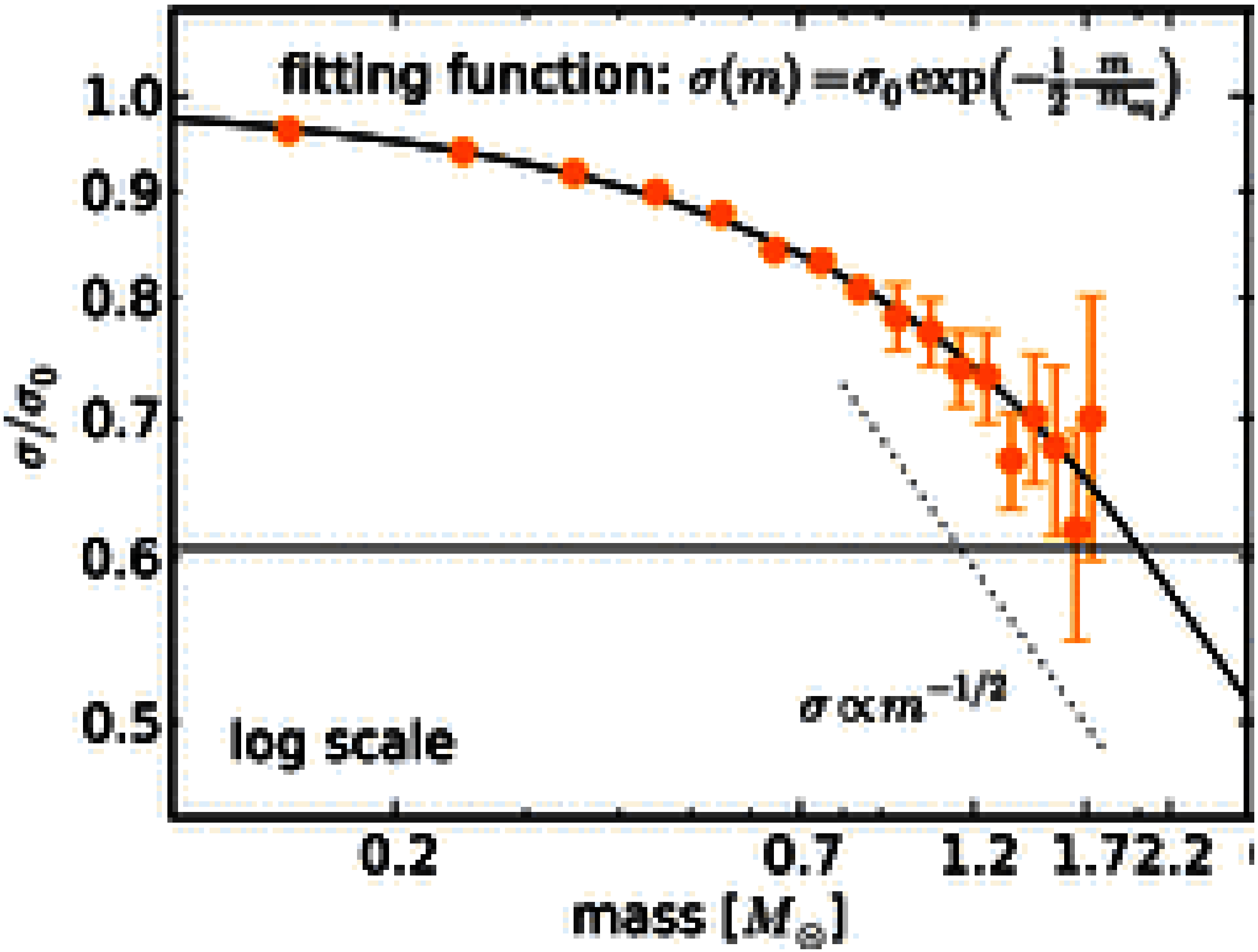
$$\sigma(m) \propto m^{-\eta}$$



- **N-body simulations of globular clusters**
- **initial mass function (Salpeter, Miller-Scalo)**
- **only King models**

Trenti & van der Marel 2013

3. SPITZER'S INSTABILITY



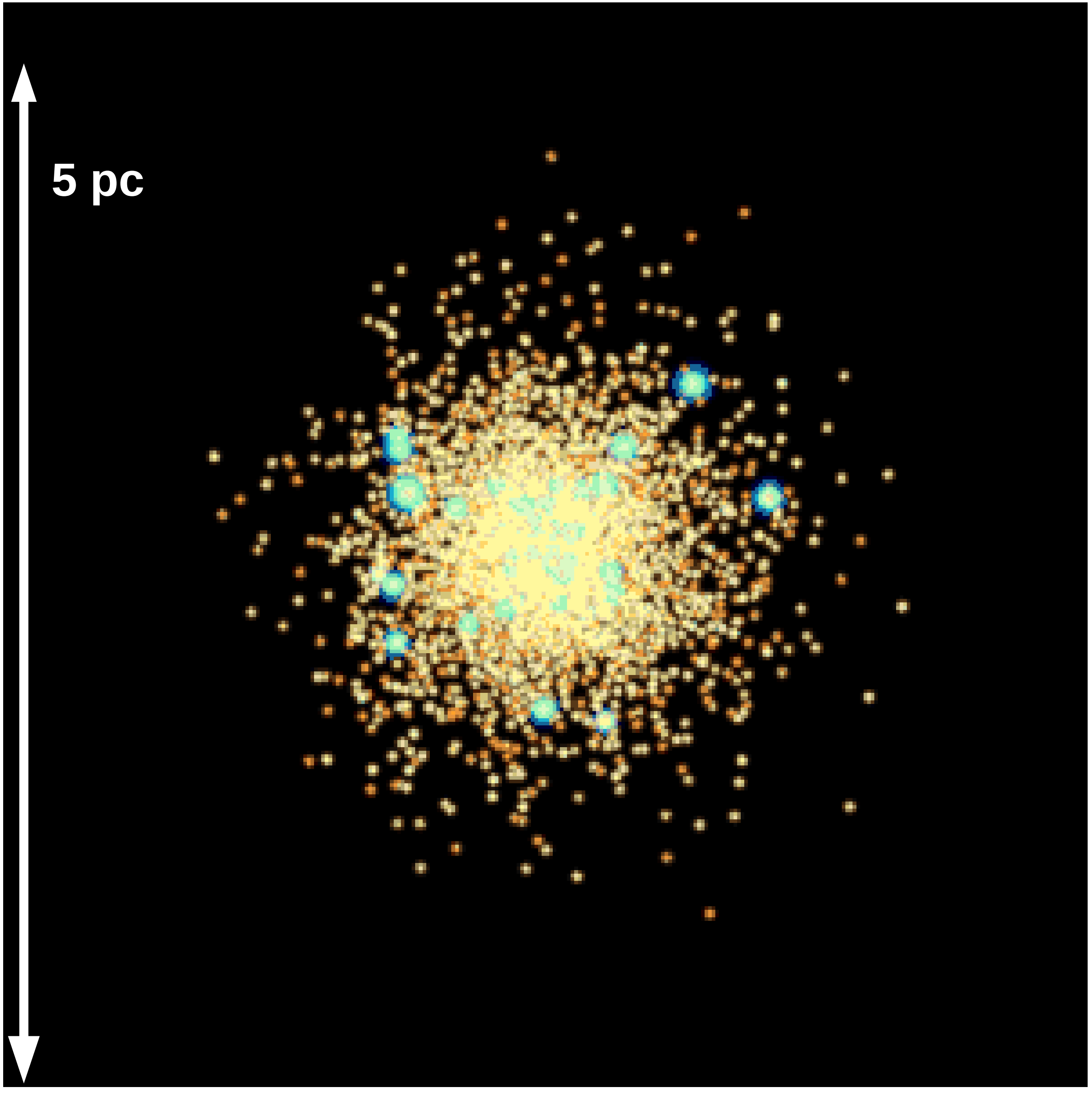
$$\sigma(m) \propto m^{-0.5}$$

- Monte-Carlo simulations of globular clusters
- Initial realistic mass function (Kroupa 2001)
- only Plummer model

$$\sigma^2(m) \propto \exp(-m)$$

Bianchini et al. 2016

3. SPITZER'S INSTABILITY



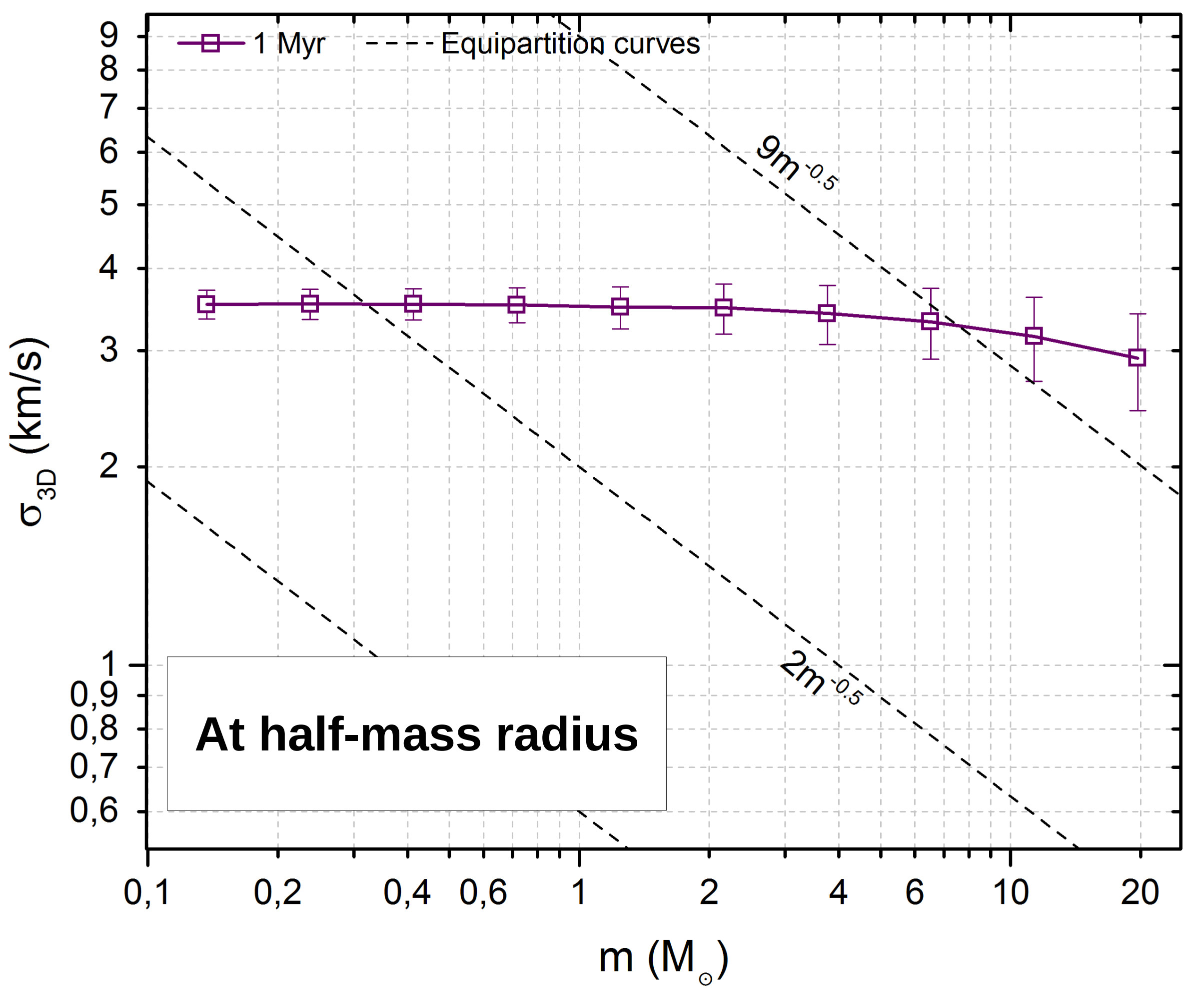
- N-body simulations of open clusters
N ~ 6000
M ~ 3900 M⊙
R_{vir} ~ 1 pc

- Initial realistic mass function, stellar evolution and Milky Way tidal field

-King models

Spera, MM & Jeffries 2016

3. SPITZER'S INSTABILITY

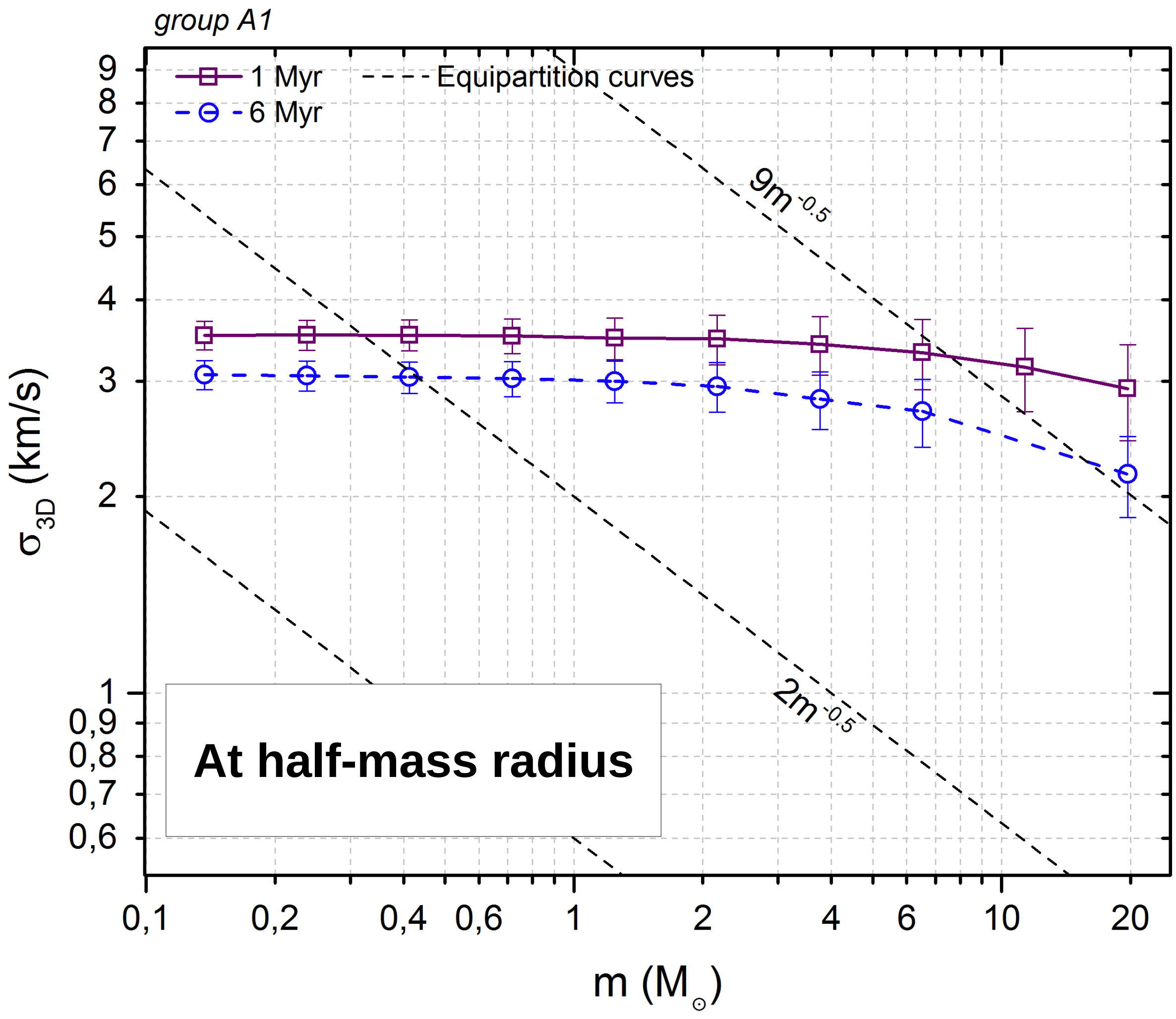


Star clusters try to reach equipartition but never attain it in steady state:

- initially flat sigma profile

Spera, MM & Jeffries 2016

3. SPITZER'S INSTABILITY



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- high mass stars tend to equipartition

Spera, MM & Jeffries 2016

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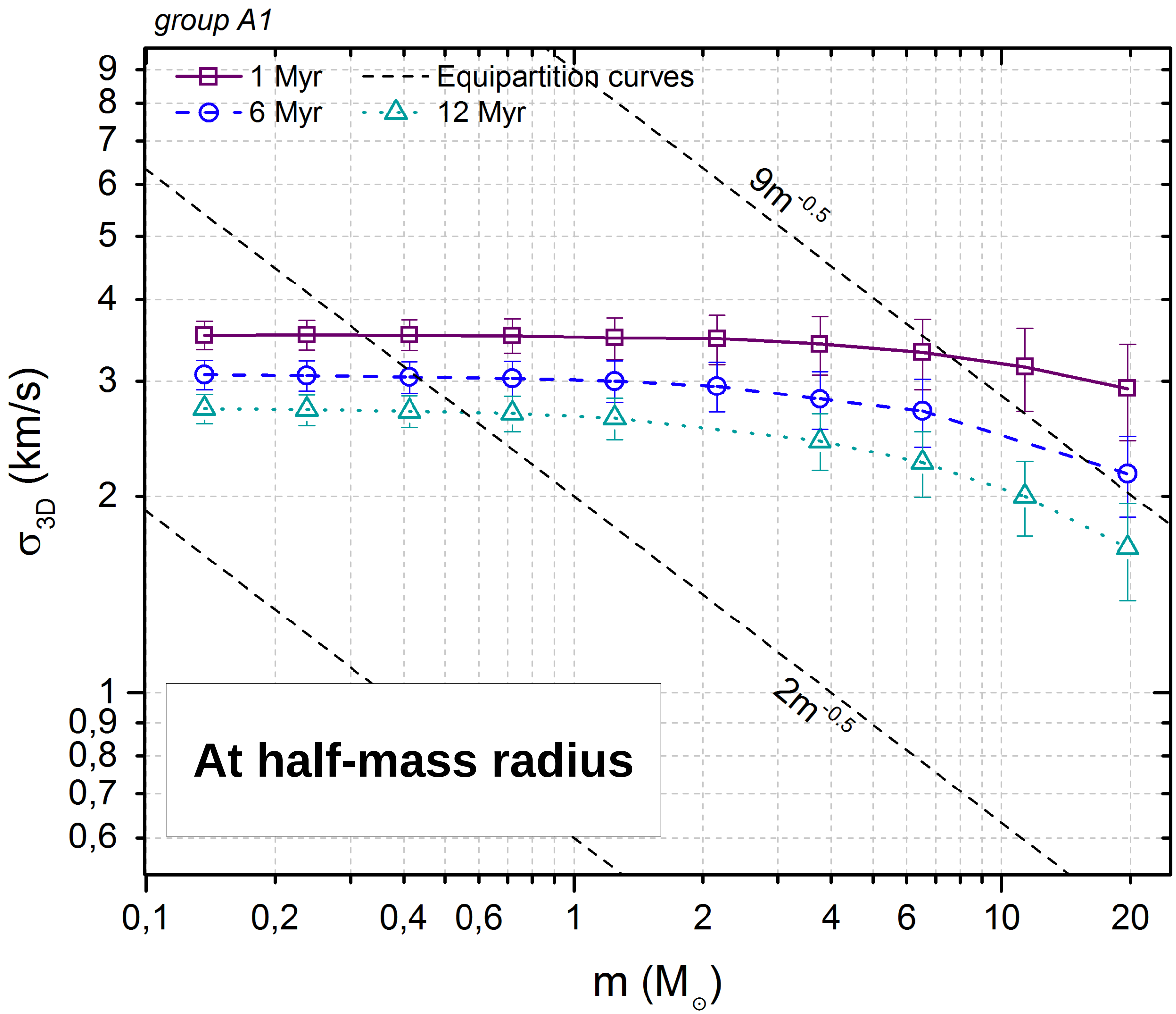

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Spera, MM & Jeffries 2016

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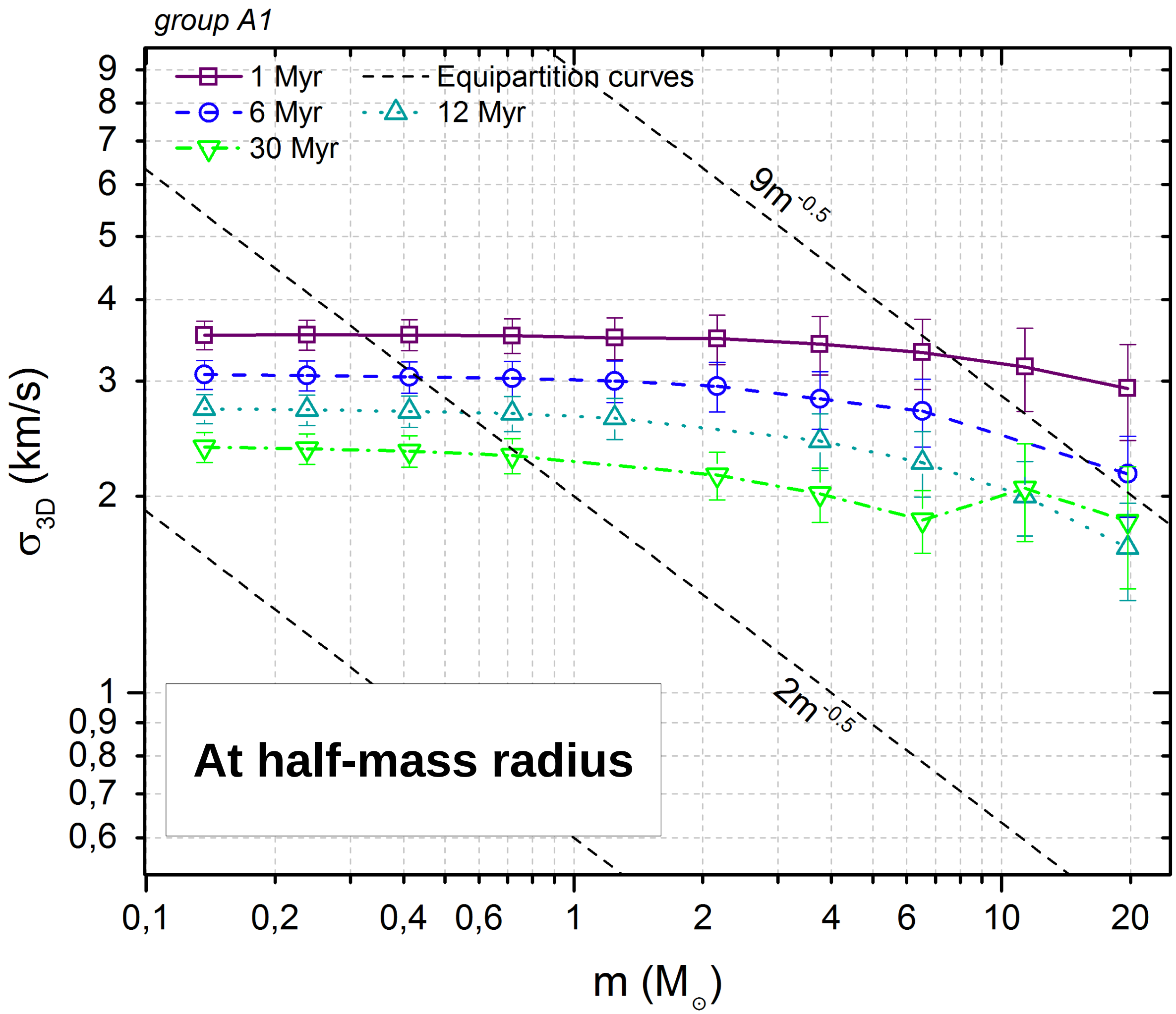

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- high mass stars become hotter

Spera, MM & Jeffries 2016

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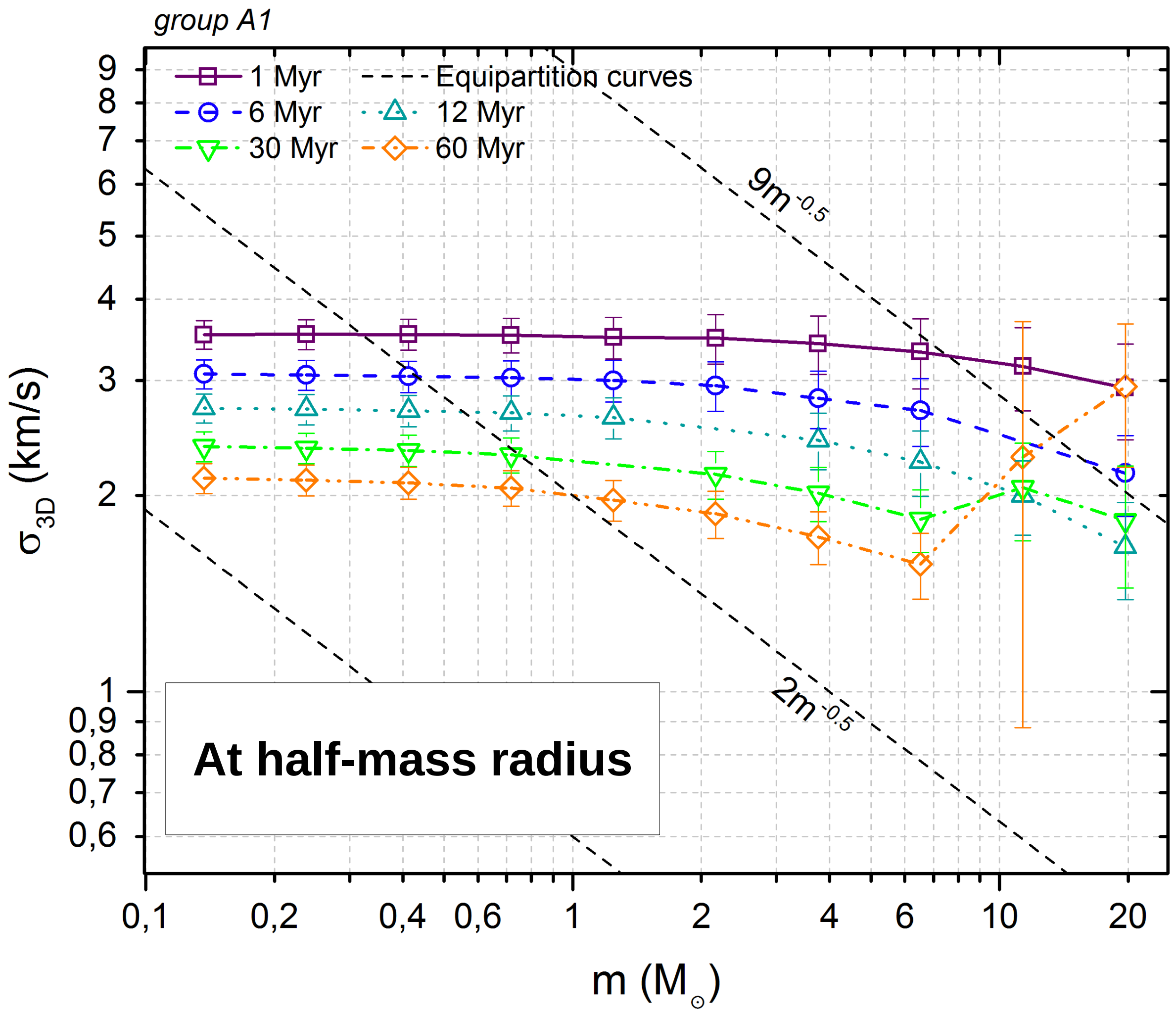

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Spera, MM & Jeffries 2016

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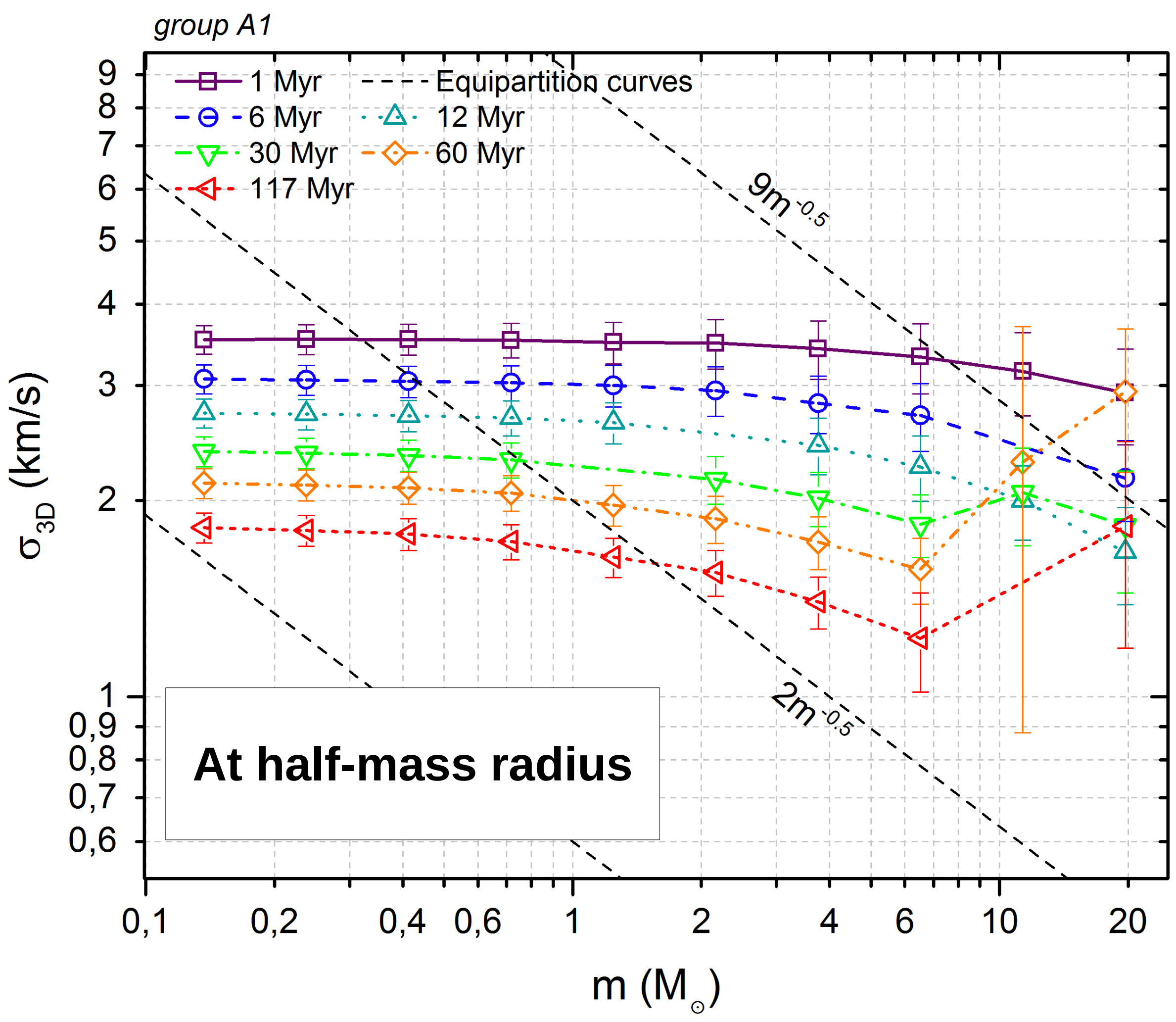

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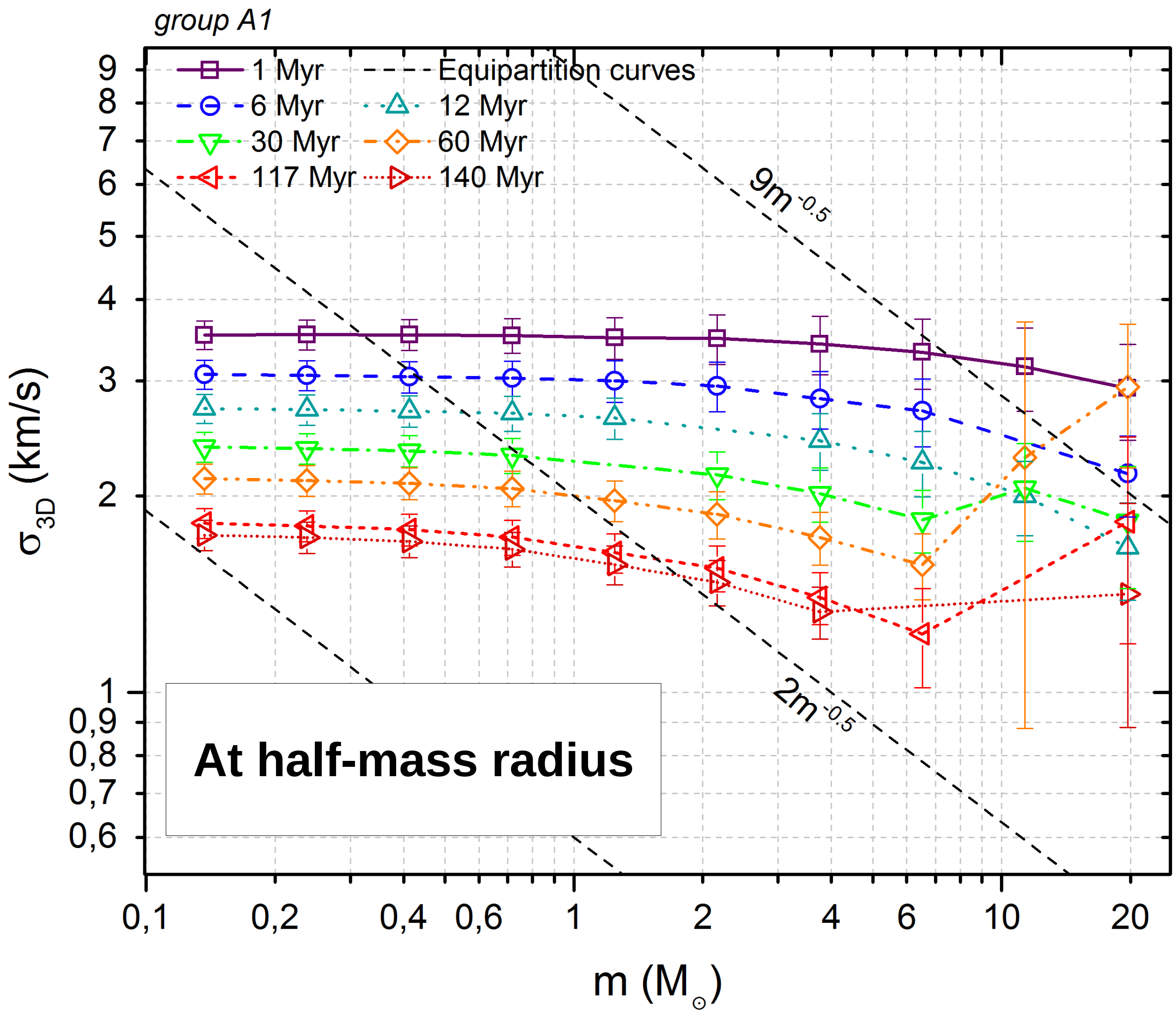

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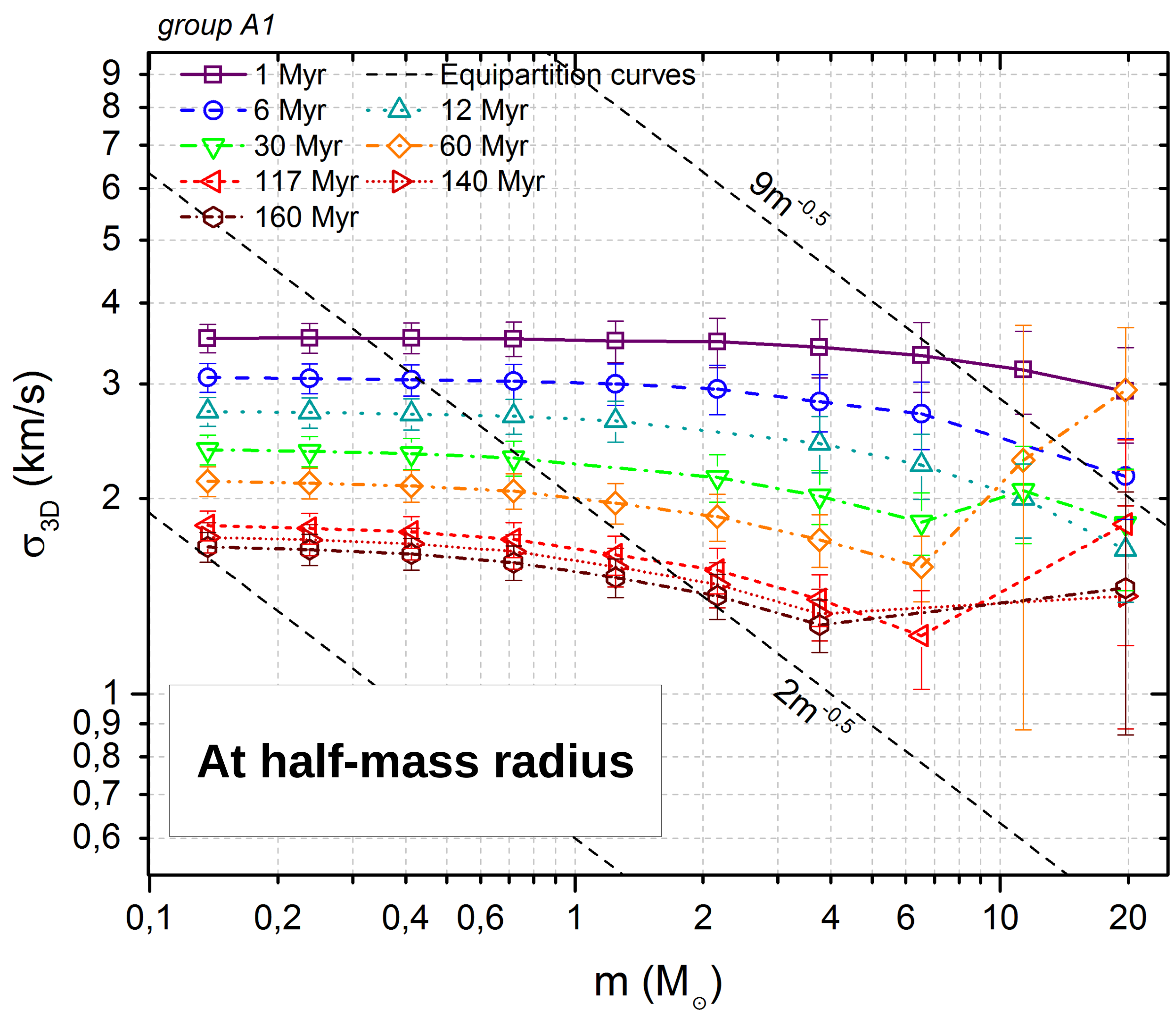

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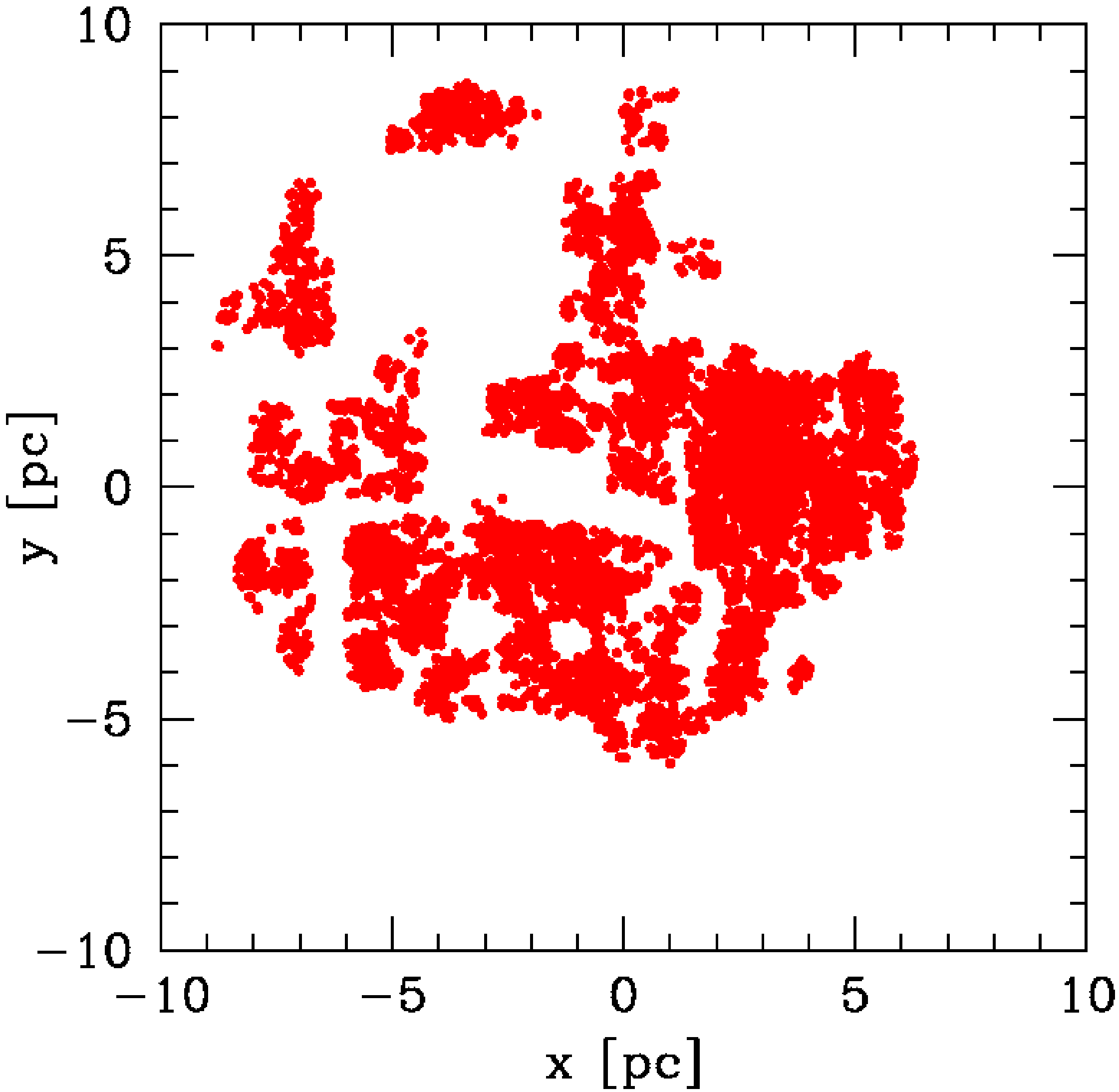
Spera, MM & Jeffries 2016

3. SPITZER'S INSTABILITY

Star clusters with King model + virial equilibrium do not seem to reach equipartition

What about non-virial and non-King models?



- **N-body simulations of open clusters**

- **Initial realistic mass function (Kroupa 2001)**

- **FRACTAL (sub-virial) initial conditions**

Parker+ 2016

3. SPITZER'S INSTABILITY

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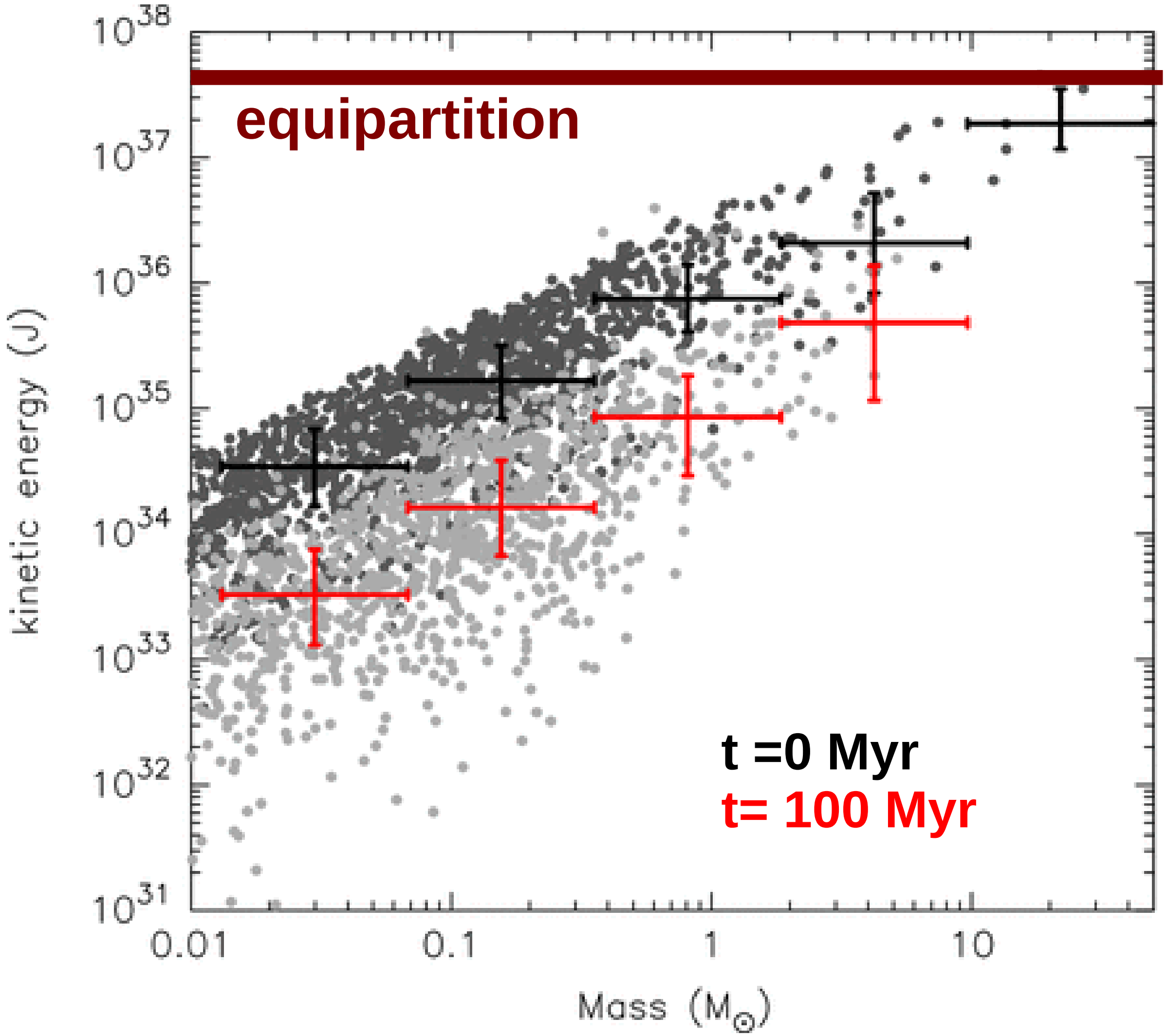


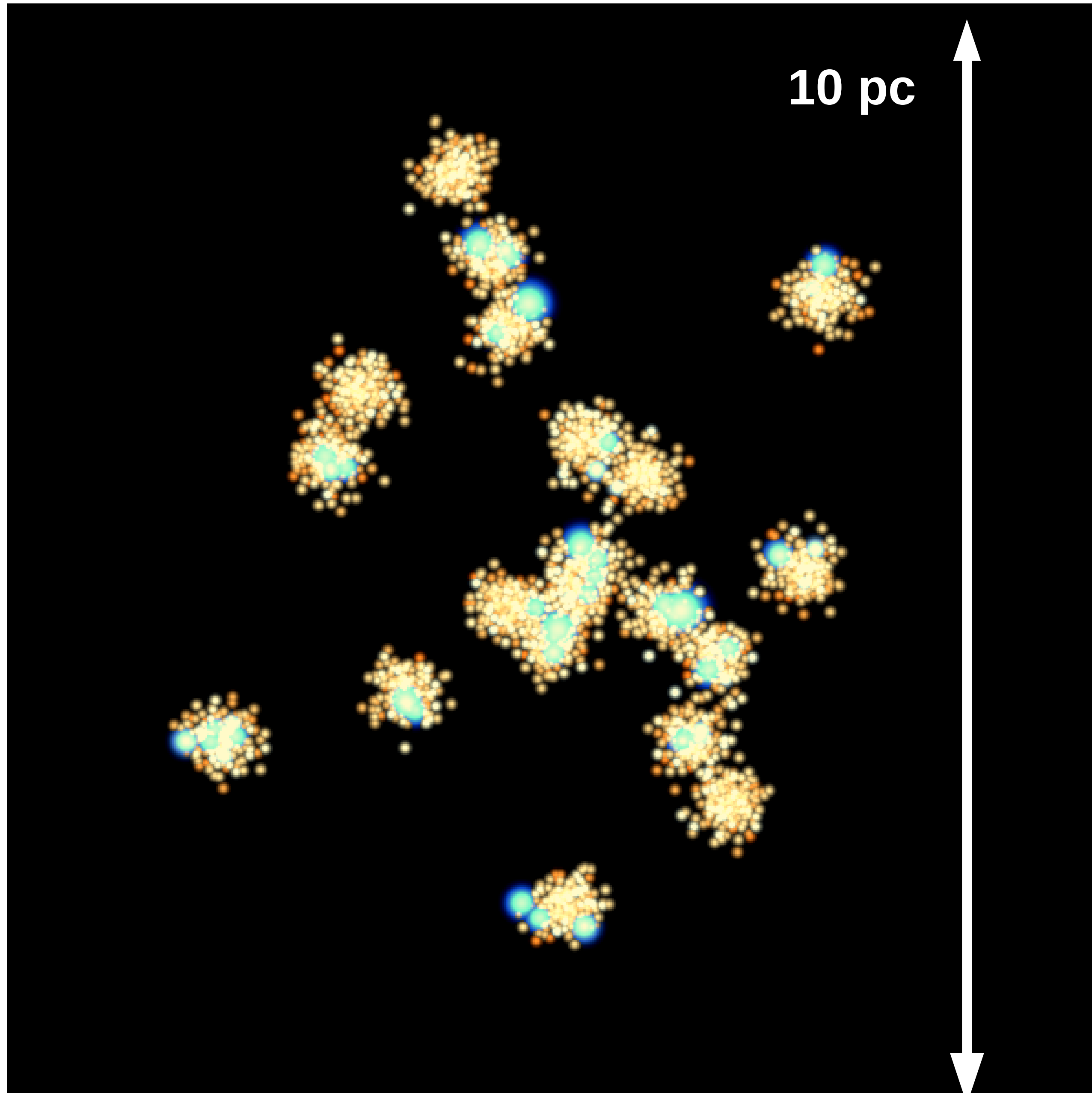
- N-body simulations of open clusters
- Initial realistic mass function (Kroupa 2001)
- FRACTAL (sub-virial) initial conditions
- again: σ(m) does not depend on m

Parker+ 2016

3. SPITZER'S INSTABILITY

What about non-virial and non-King models?

Merger of clumps (20 clumps at rest) equivalent to SUB-VIRIAL initial conditions

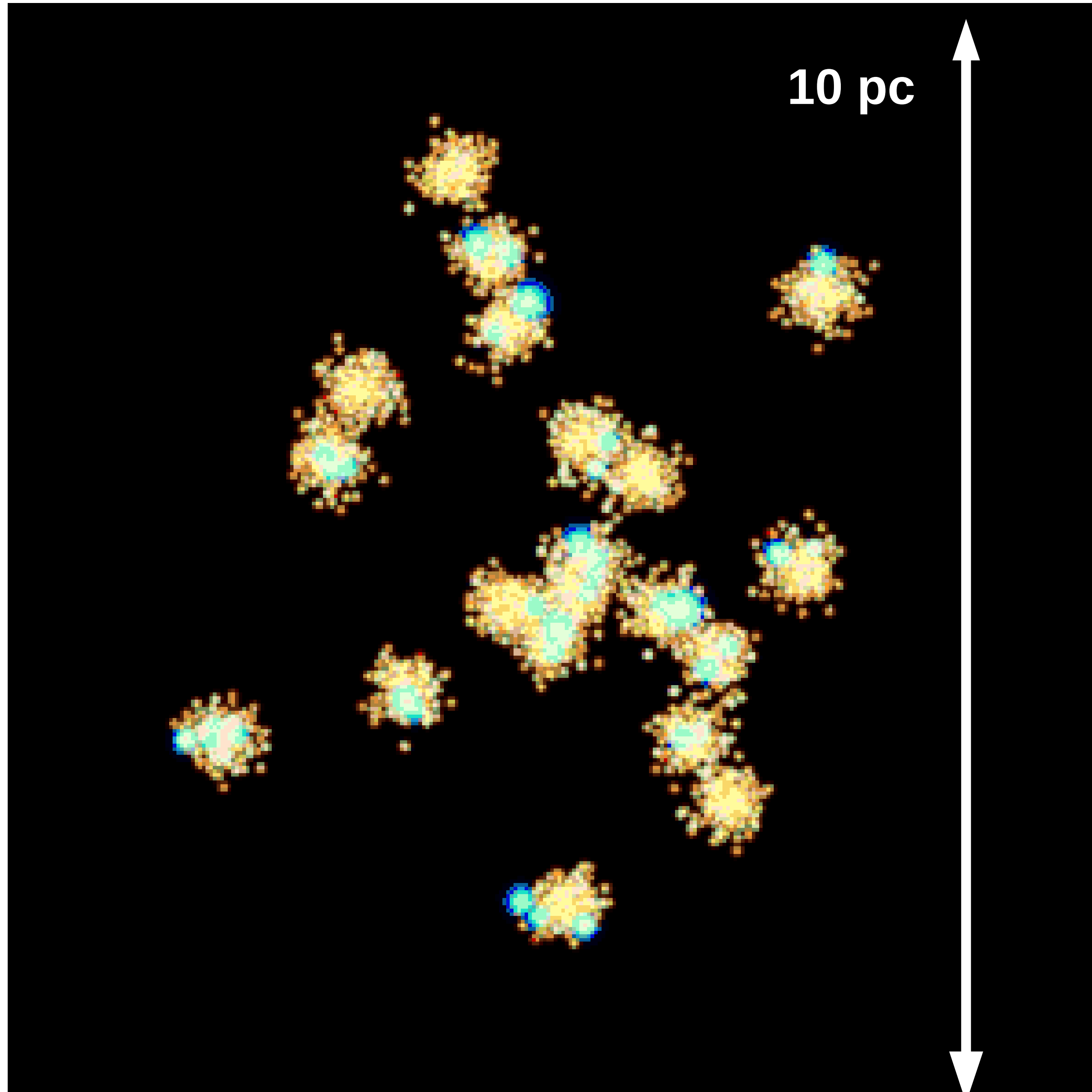


Spera, MM & Jeffries 2016

3. SPITZER'S INSTABILITY

What about non-virial and non-King models?

Merger of clumps (20 clumps at rest) equivalent to SUB-VIRIAL initial conditions



Spera, MM & Jeffries 2016

3. SPITZER'S INSTABILITY

What about non-virial and non-King models?

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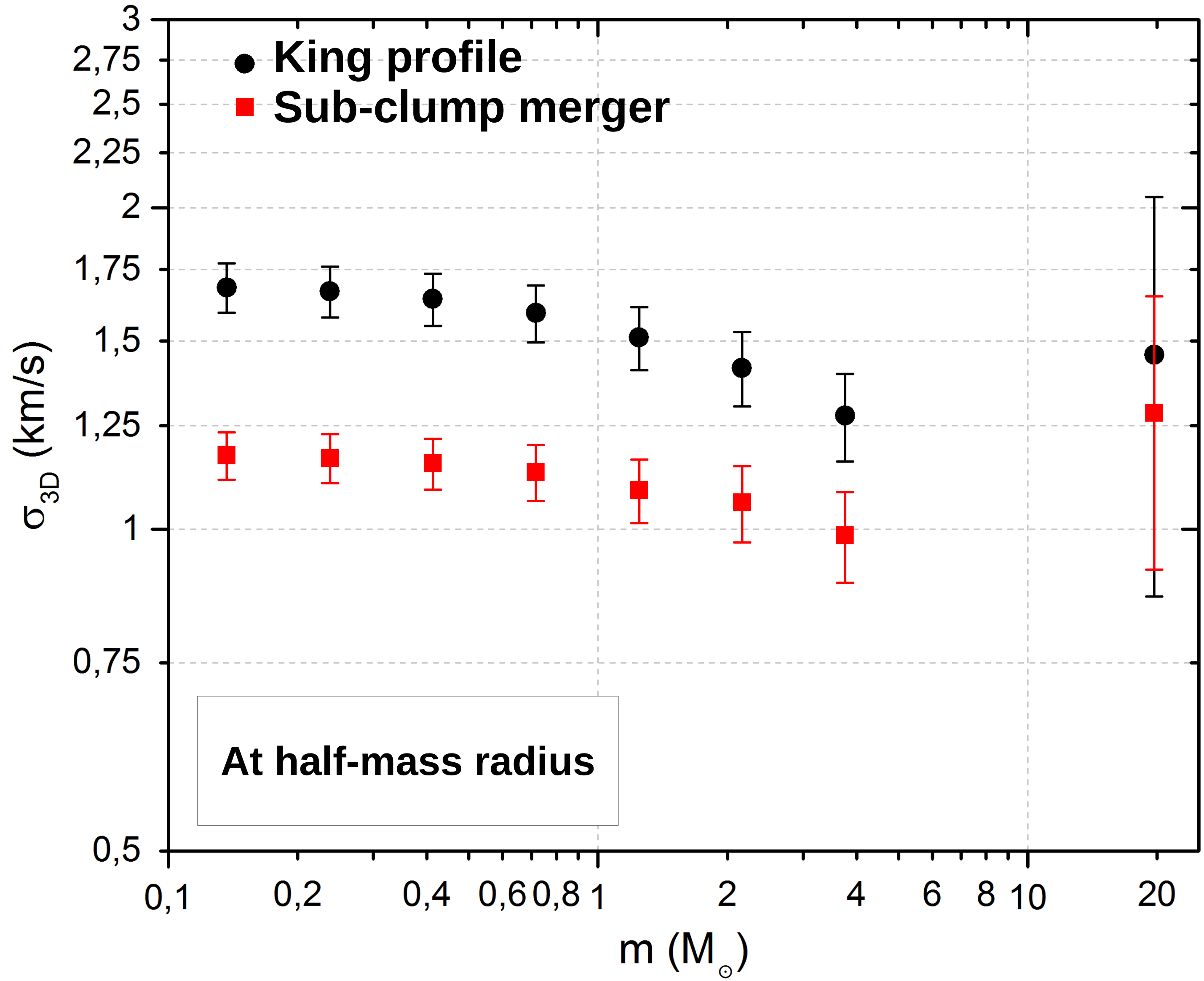
SAME TREND AS VIRIAL KING MODEL



Spera, MM & Jeffries 2016

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