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Dynamics of Stars and Black Holes in Dense Stellar Systems:

Lecture II:

CORE COLLAPSE AND REVERSAL

0. Granularity
1. Evaporation
2. Core collapse
3. Post-core collapse
4. Gravothermal oscillations

0. GRANULARITY of the GRAVITATIONAL FIELD:

Interactions between 2 stars (**two-body encounters**) PRODUCE LOCAL FLUCTUATIONS of ENERGY BALANCE i.e. **CHANGE LOCALLY THE MAGNITUDE of STELLAR VELOCITIES**

On the RELAXATION TIMESCALE, this induces
GLOBAL CHANGES in the CLUSTER EQUILIBRIUM

This happens because a COLLISIONAL system CANNOT be treated as a continuous fluid,

it has DISCRETENESS, GRANULARITY

1. EVAPORATION:

Escape velocity of a star from a cluster: $\frac{1}{2}\,v_e^2 = |\phi|$

where ϕ = potential

as the kinetic energy of the star must overcome its potential energy

MEAN SQUARE escape velocity of a star from a cluster:

$$\langle v_e^2 \rangle = \frac{\int \rho(r)\, v_e^2(r)\, dV}{\int \rho(r)\, dV} = \frac{\int \rho(r)\, 2\, |\phi(r)|\, dV}{M} = -4\,\frac{W}{M}$$

$$W = \frac{1}{2} \int \rho(r)\, \phi(r) dV$$

from virial theorem:

$$\langle v_e^2 \rangle = 4\,\frac{2\,K}{M} = 4\,\langle v^2 \rangle$$

a star can escape if its velocity is higher than 2 times the root mean square velocity

1. EVAPORATION (description from Spitzer 1987):

The concept of evaporation is simple:

if $v > v_e \Rightarrow$ the star escapes, i.e. evaporates from system

What are the global effects of evaporation in a collisional system?

1. EVAPORATION (description from Spitzer 1987):

Mathematical model to understand the evolution of the system induced by evaporation in the case of CONSTANT RATE OF MASS LOSS PER UNIT MASS PER TIME INTERVAL dt / t_{rlx}

This assumption implies self-similarity, as the radial variation of density, potential and other quantities are time-invariant except for TIME DEPENDENT SCALE FACTORS

Example: a contracting uniform sphere which remains uniform (:=density independent of radius) during contraction

MASS LOSS RATE:

$$\frac{dM}{dt} = \frac{-\xi_e\, M(t)}{t_{rlx}(t)} = -\frac{\xi_e\, M(0)}{t_{rlx}(0)} \left[\frac{R(t)}{R(0)}\right]^{-3/2} \left[\frac{M(t)}{M(0)}\right]^{1/2}$$

CONSTANT RATE OF MASS LOSS

where we used the fact that

$$M(t) = \frac{M(0)}{M(0)}\, M(t)$$

$$t_{rlx}(t) = t_{rlx}(0) \left(\frac{R(t)}{R(0)}\right)^{3/2} \left(\frac{M(t)}{M(0)}\right)^{1/2}$$

1. EVAPORATION (description from Spitzer 1987):

Previous equation has two unknowns (*M(t), R(t)*) → we need another equation: Change of total cluster energy, as each escaping star carries away a certain **kinetic energy per unit mass (=ζE_m, where E_m is the mean energy per unit mass of the cluster)**

$$\frac{dE_{TOT}}{dt} = \zeta E_m \frac{dM}{dt} = \frac{\zeta E_{TOT}}{M} \frac{dM}{dt}$$

Since E_{TOT} μ - M^2/R

$$\frac{dE_{TOT}}{dt} = -\frac{d}{dt}\left(\frac{M^2}{R}\right) = -\frac{2M}{R}\frac{dM}{dt} + \frac{M^2}{R^2}\frac{dR}{dt}$$

$$\frac{\zeta E_{TOT}}{M}\frac{dM}{dt} = -\zeta \frac{M}{R}\frac{dM}{dt}$$

$$(2-\zeta)\frac{dM}{M} = \frac{dR}{R}$$

$$\frac{R}{R(0)} = \left[\frac{M}{M(0)}\right]^{2-\zeta}$$

1. EVAPORATION (description from Spitzer 1987):

Inserting

$$\frac{R}{R(0)} = \left[\frac{M}{M(0)}\right]^{2-\zeta} \qquad \text{(@)}$$

into the equation for mass loss rate, i.e.

$$\frac{dM}{dt} = \frac{-\xi_e\, M(t)}{t_{rlx}(t)} = -\frac{\xi_e\, M(0)}{t_{rlx}(0)} \left[\frac{R(t)}{R(0)}\right]^{-3/2} \left[\frac{M(t)}{M(0)}\right]^{1/2}$$

we find:

$$\frac{dM}{dt} = -\frac{\xi_e\, M(0)}{t_{rlx}(0)} \left[\frac{M}{M(0)}\right]^{(-5+\zeta)/2}$$

Integrating the above equation:

$$\frac{M}{M(0)} = \left[1 - \frac{\xi_e\,(7-3\,\zeta)}{2}\,\frac{t}{t_{rlx}(0)}\right]^{2/(7-3\,\zeta)} \equiv \left(1 - \frac{t}{t_{coll}}\right)^{2/(7-3\,\zeta)}$$

t_{coll} := collapse time, time at which *M* and *R* vanish.

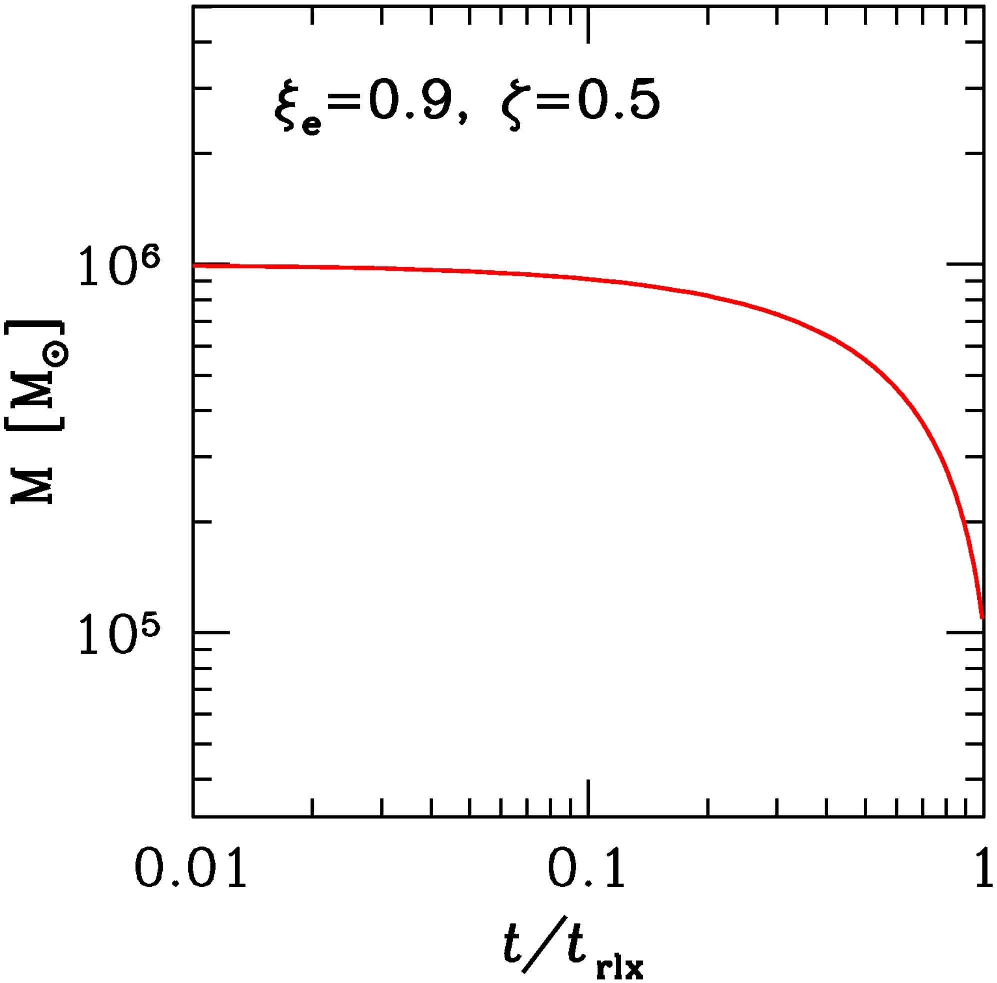
1. EVAPORATION (description from Spitzer 1987):

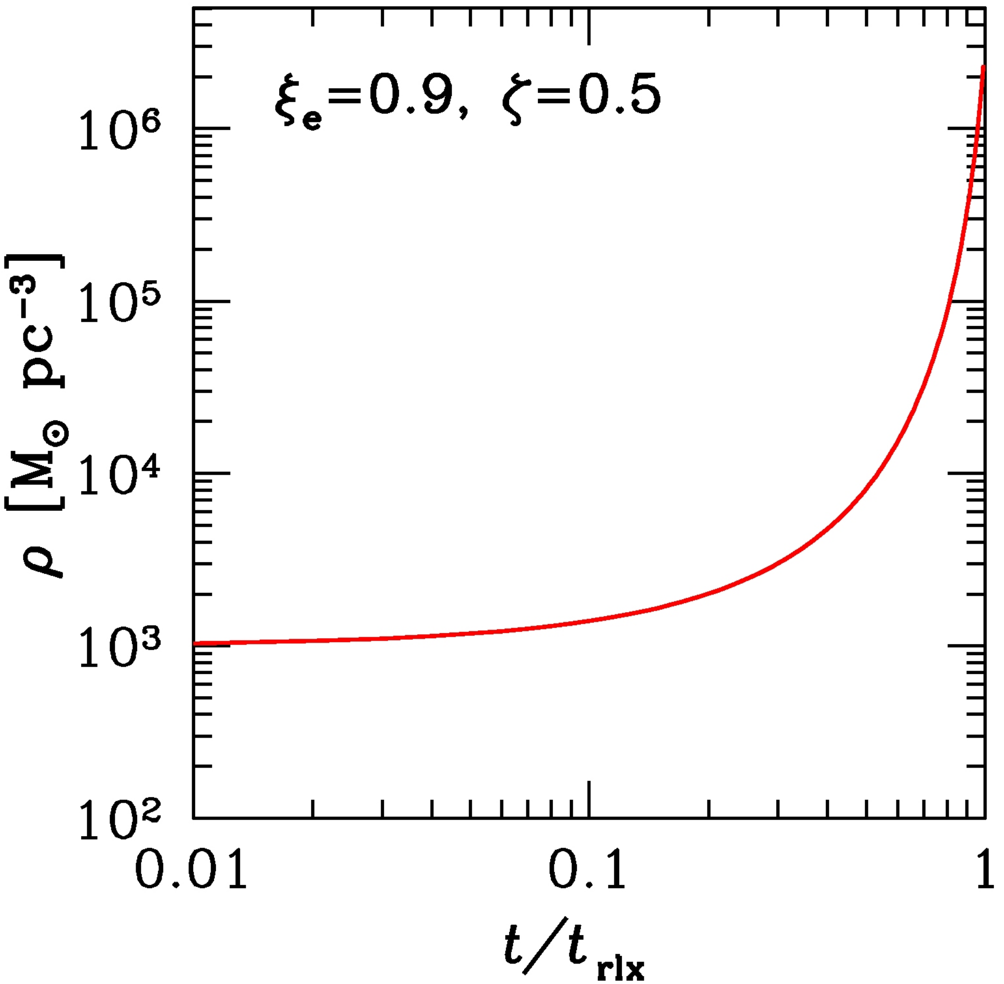
Note: from (@), using the fact that $\rho = 3\,M\,/\,(\,4\,\pi\,R^3\,)$

$$\frac{\rho}{\rho(0)} \propto \left[\frac{M}{M(0)}\right]^{-(5-3\,\zeta)}$$

Since $\zeta < 1$ (for realistic clusters), when M decreases for evaporation, ρ increases:

Collapse! Evaporation may induce collapse!!





2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

Instability which occurs in a small core
confined in outer ISOTHERMAL halo

The core contracts to a zero radius and infinite density
in a runaway sense

NOTE: Gravothermal instability occurs
even if **STARS ARE EQUAL MASS!!!!!!**

1. IDEAL-GAS APPROACH

based on the analogy with ideal gas

2. PHYSICAL APPROACH

based on the virial theorem

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

1. IDEAL-GAS APPROACH:

analogy with IDEAL GAS

We define the temperature T of a self-gravitating system $\frac{1}{2}\, m\, \langle v^2 \rangle = \frac{3}{2}\, \kappa_B\, T$

Total kinetic energy of a system

$$K = \sum \frac{1}{2}\, m\, \langle v^2 \rangle = \frac{3}{2}\, \kappa_B\, N\, \frac{\int_V \rho(\mathbf{x})\, T(\mathbf{x})\, d\mathbf{x}}{\int_V \rho(\mathbf{x})\, d\mathbf{x}}$$

Virial theorem: $$E_{\rm TOT} = -K = -\frac{3}{2}\, N\, \kappa_B\, \langle T \rangle$$

Definition of heat capacity: $$C \equiv \frac{dE}{d\langle T \rangle} = -\frac{3}{2}\, N\, \kappa_B$$

MEANS THAT by LOSING ENERGY THE SYSTEM BECOMES HOTTER

→ **system contracts more and becomes hotter in runaway sense**

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

How is it possible that losing energy the system becomes hotter?

If we put a negative heat capacity system in a bath and heat is transferred to the bath

$$\mathrm{d}Q = dE > 0$$

→ temperature of the system changes by $T - \mathrm{d}Q / C = \mathrm{T} - \mathrm{d}E/C > T$

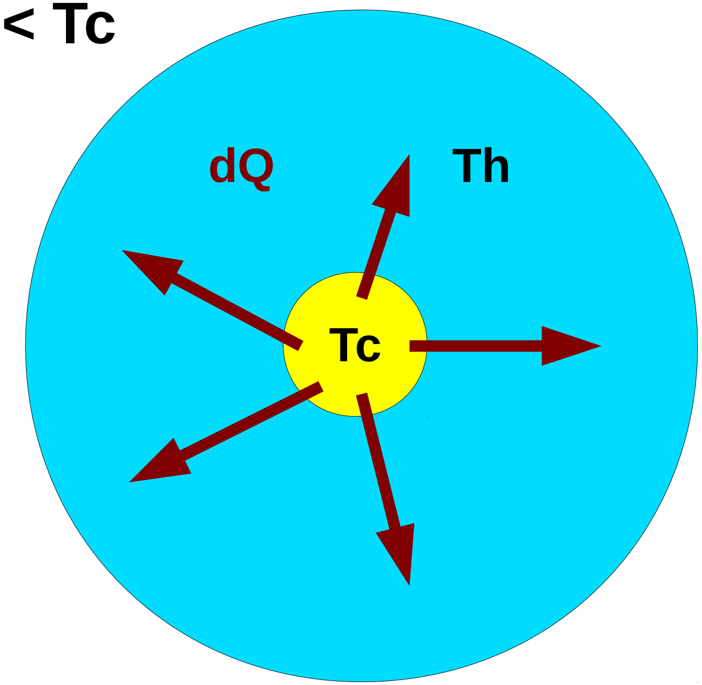
because $dQ/C = dE/C = dE / (-3/2\ N\ Kb) < 0$ **always negative!**

→ system becomes hotter and heat keeps flowing from system to bath: *T* rises without limits!!

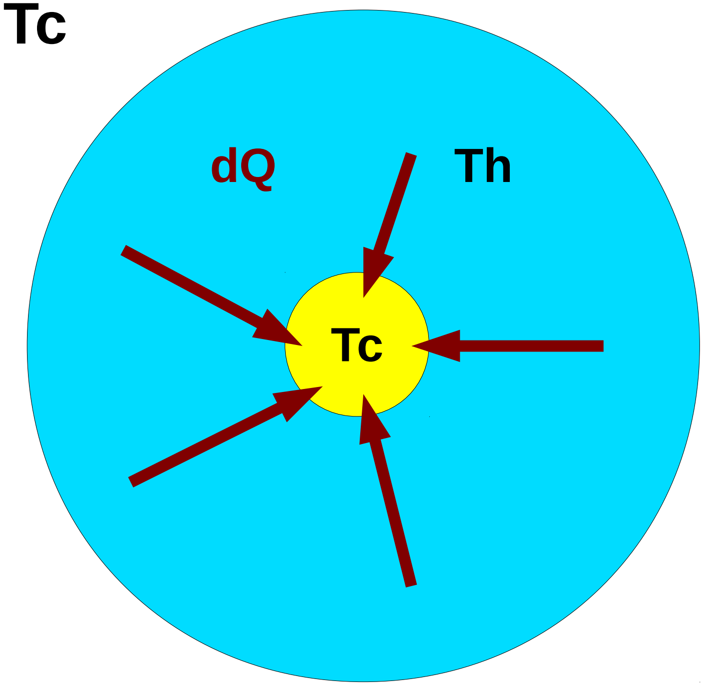
Note: Any bound finite system in which dominant force is gravity exhibits *C*<0

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

If Th < Tc



If Th > Tc



Note: CONDITION that HALO is LARGE with respect to the core is crucial!
so that K continuously injected into the halo does not imply the heating of the halo (perfect thermal bath)

Otherwise, if the *K* of the halo overcomes the *K* of the core,
The energy injected into the halo FLOWS BACK to the core
and stops contraction!!!

2. PHYSICAL APPROACH:

REQUIREMENTS:

* SMALL HIGH-DENSITY CORE in a very LARGE ISOTHERMAL HALO (the bath)

* MAXWELLIAN VELOCITY DISTRIBUTION (or, in general, velocity distribution where stars can evaporate)

$$f(v) \propto v^2\, e^{-v^2} \qquad f(v) > 0 \quad \text{if } v \to \infty$$

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

2. PHYSICAL APPROACH:

IF velocity distribution allows stars with $v > v_{escape}$

⇒ high velocity stars ESCAPE from the core into halo (EVAPORATION)

⇒ $$K = \sum_i \frac{1}{2} m_i \, v_i^2$$ ↓ because high velocity stars escape

$$W = -\sum_{i,j,i \neq j} \frac{G \, m_i \, m_j}{r_{ij}}$$ ↑ because the mass of escaping stars is lost

BUT DECREASE in *K* is more important than increase in *W* since the FASTER STARS LEAVE the cluster!

$$2\,K_f + W_f < 2K_i + W_i$$

⇒ GRAVITY is NO longer supported by *K*, by random motions

⇒ SYSTEM CONTRACTS (*)

⇒ TO REACH NEW VIRIAL EQUILIBRIUM AVERAGE VELOCITY MUST INCREASE

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

Or, to say it in a different way (more physical?)

DECREASE in *K* is more important than increase in *W* since the FASTER STARS LEAVE the cluster!

$$2\,K_f + W_f < 2K_i + W_i$$

⇒ GRAVITY is NO longer supported by *K*, by random motions

⇒ SYSTEM CONTRACTS (*)

(*) IF the system contracts, it becomes DENSER

⇒ higher density implies MORE two-body encounters (higher two-body encounter rate)

⇒ stars exchange more energy and become dynamically hotter

⇒ faster stars tend to EVAPORATE even more than before

⇒ *K* decreases faster than *W* increases

⇒ system contracts even more

⇒ CATASTROPHE!!!

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

BUT WE DO NOT SEE STAR CLUSTERS WITH INFINITE DENSITY IN NATURE!!

WHAT DOES REVERSE THE CORE COLLAPSE??

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

REVERSE OF CORE COLLAPSE ONLY
BY SWITCHING ON A NEW SOURCE OF $K = K_{ext}$

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

REVERSE OF CORE COLLAPSE ONLY
BY SWITCHING ON A NEW SOURCE OF K = K_{ext}

THIS SOURCE CAN OPERATE IN TWO WAYS

(1) $$2\,K_f + W_f = 2\,(K_{ext} + K_i) + W_i > 2\,K_i + W_i$$

⇒ Kinetic energy increases not by gravitational contraction
but by an **EXTERNAL** SOURCE

Energy injection breaks virial equilibrium and negative heat capacity

→ **CORE EXPANDS** (lasts only till energy source is on)

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

REVERSE OF CORE COLLAPSE ONLY
BY SWITCHING ON A NEW SOURCE OF K = K_{ext}

(2) THE NEW KINETIC ENERGY TRANSFERRED TO CORE STARS
INDUCES THE EJECTION OF STARS
that were not necessarily the faster stars before
receiving the new kinetic energy:

$$K = \sum_i \frac{1}{2} m_i\, v_i^2$$

↓ because stars which received external kinetic energy escape

$$W = -\sum_{i,j,i\neq j} \frac{G\, m_i\, m_j}{r_{ij}}$$

↑ because the mass of escaping stars is lost

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

NET RESULT:

INCREASE in *W* (DECREASE OF |*W*|) is more important than decrease in *K* because

(I) STARS that LEAVE the cluster were not the faster before receiving the kick and

(II) K_f is the sum of K_i and K_{ext}

$$2\,K_f + W_f > 2K_i + W_i$$

⇒ **POTENTIAL WELL BECOMES PERMANENTLY SHALLOWER**

⇒ **AND SYSTEM EXPANDS** (*)

⇒ TO REACH NEW VIRIAL EQUILIBRIUM AVERAGE VELOCITY MUST DECREASE

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

Or, to say it in a different way (more physical?)

$\Rightarrow$ **POTENTIAL WELL BECOMES PERMANENTLY SHALLOWER**

$\Rightarrow$ **AND SYSTEM EXPANDS** (*)

(*) IF the system expands, it becomes LESS DENSE

$\Rightarrow$ lower density implies LESS two-body encounters

$\Rightarrow$ stars exchange less energy and become dynamically cooler

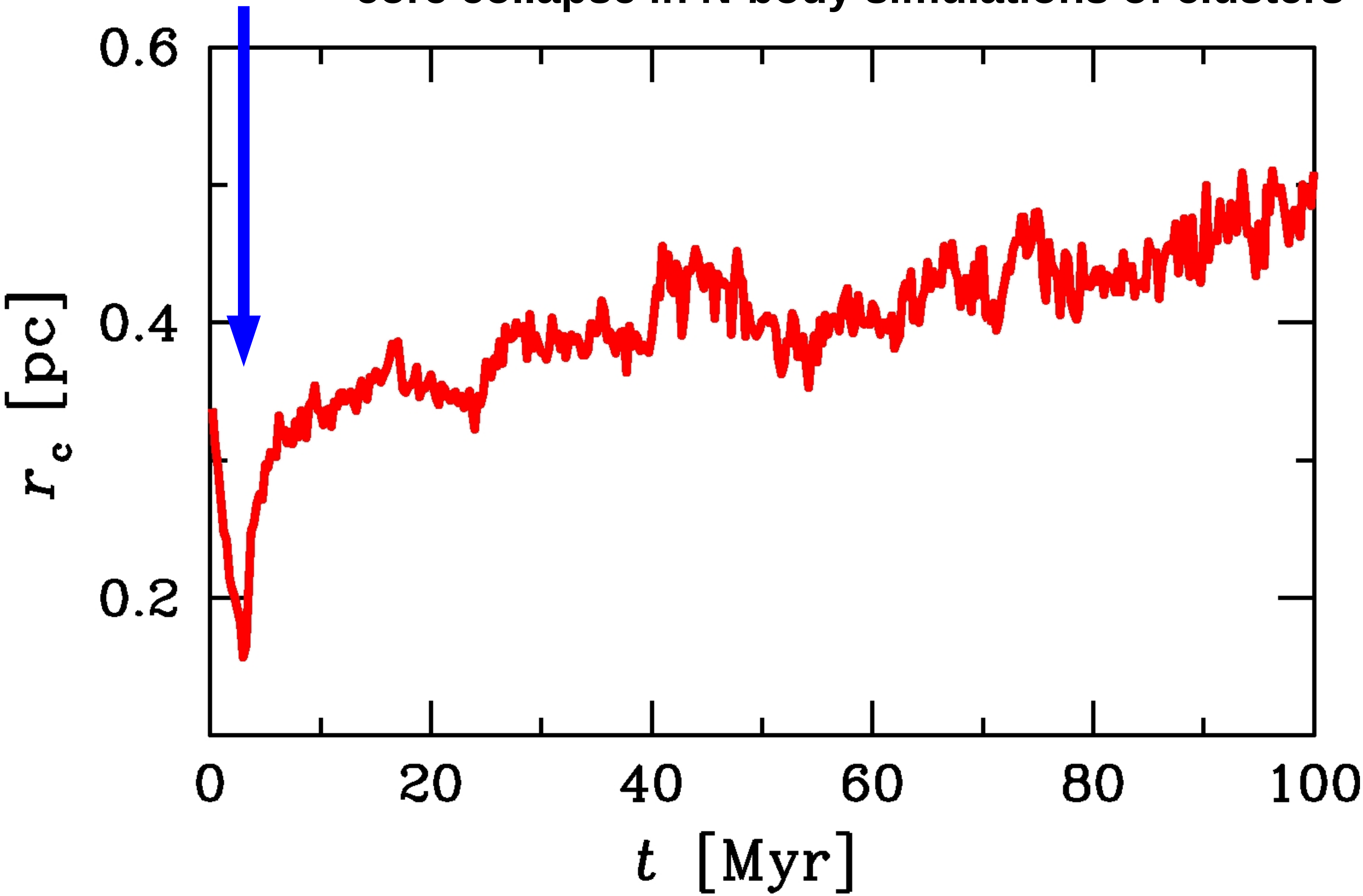
$\Rightarrow$ **gravothermal CATASTROPHE is reversed !!!**

Even if sources of heating (partially) switch off, the ejection of stars and the lowering of potential well ensures reversal of catastrophe

(but see gravothermal oscillations at end of lecture)

2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

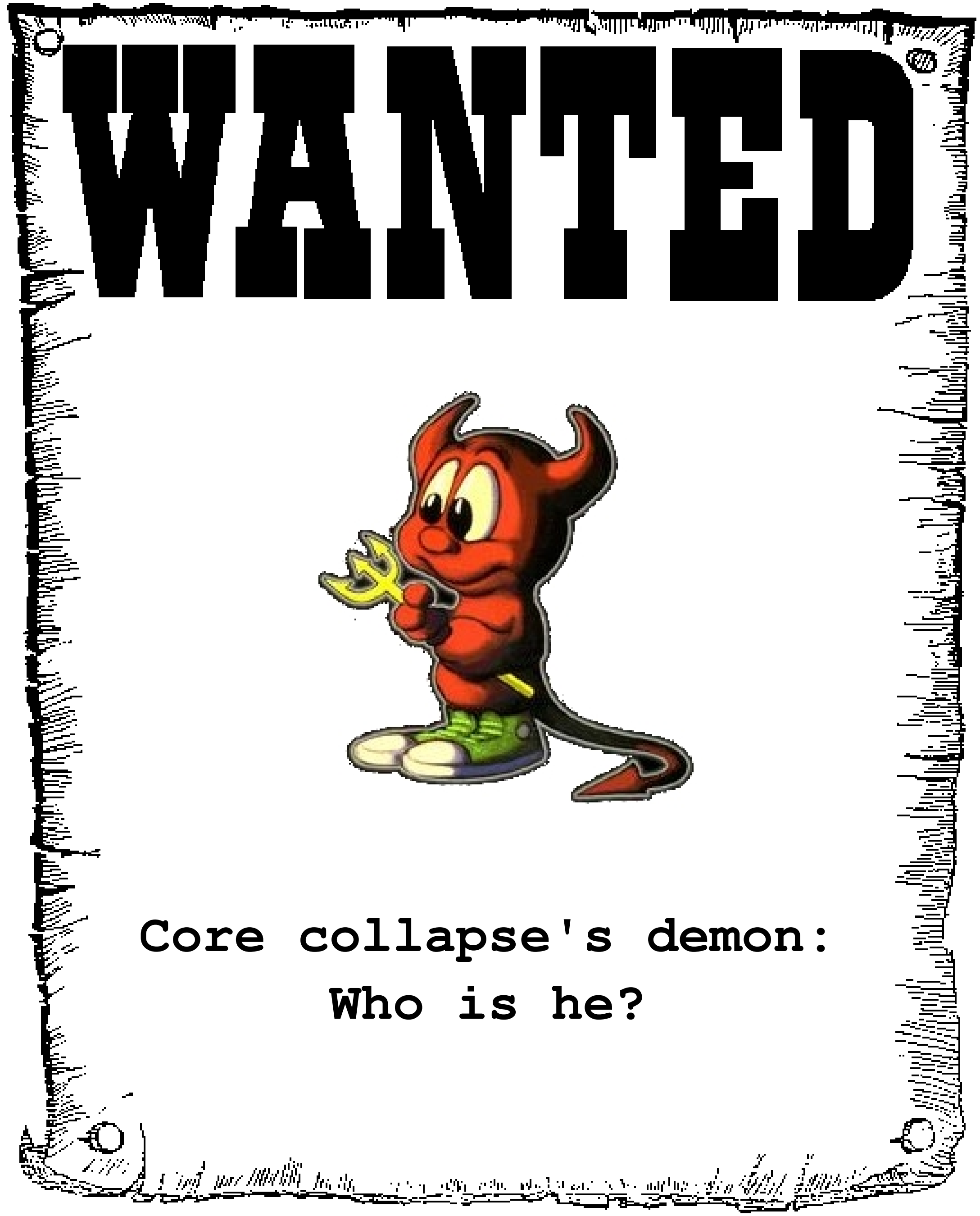
core collapse in N-body simulations of clusters



2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

We did not make any assumption about the source of kinetic energy that reverses core collapse

WHAT ARE THE ENERGY SOURCES IN REAL-LIFE STAR CLUSTERS?



2. GRAVOTHERMAL INSTABILITY or CORE COLLAPSE:

WHAT IS THE NEW SOURCE OF K ENERGY WHICH SWITCHES ON?

(1) MASS LOSS by STELLAR WINDS and SUPERNOVAE which remove mass without changing *K* of other stars

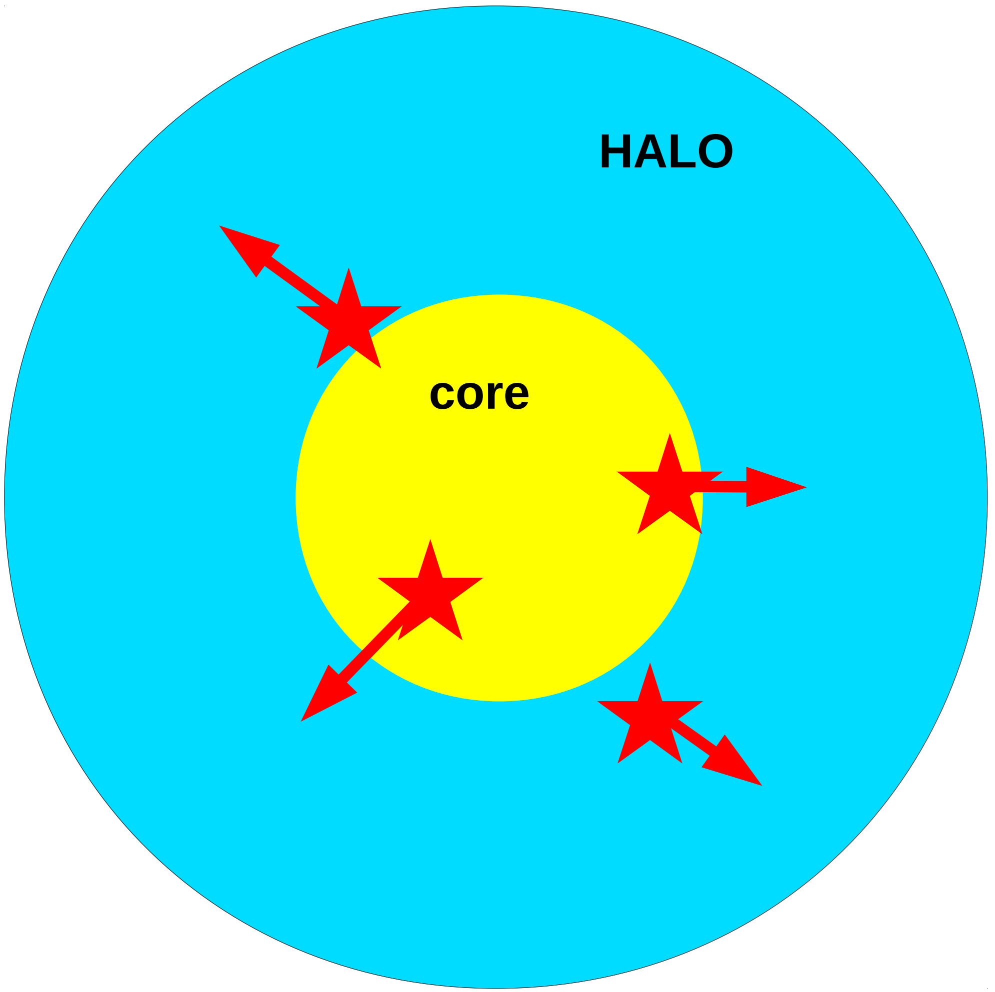
$$2\,K_i + W_f > 2\,K_i + W_i$$

IMPORTANT only if massive star evolution lifetime is similar to core collapse timescale (see last lecture)

(2) BINARIES as ENERGY RESERVOIR (see next lecture)

2. CORE COLLAPSE with CARTOONS:

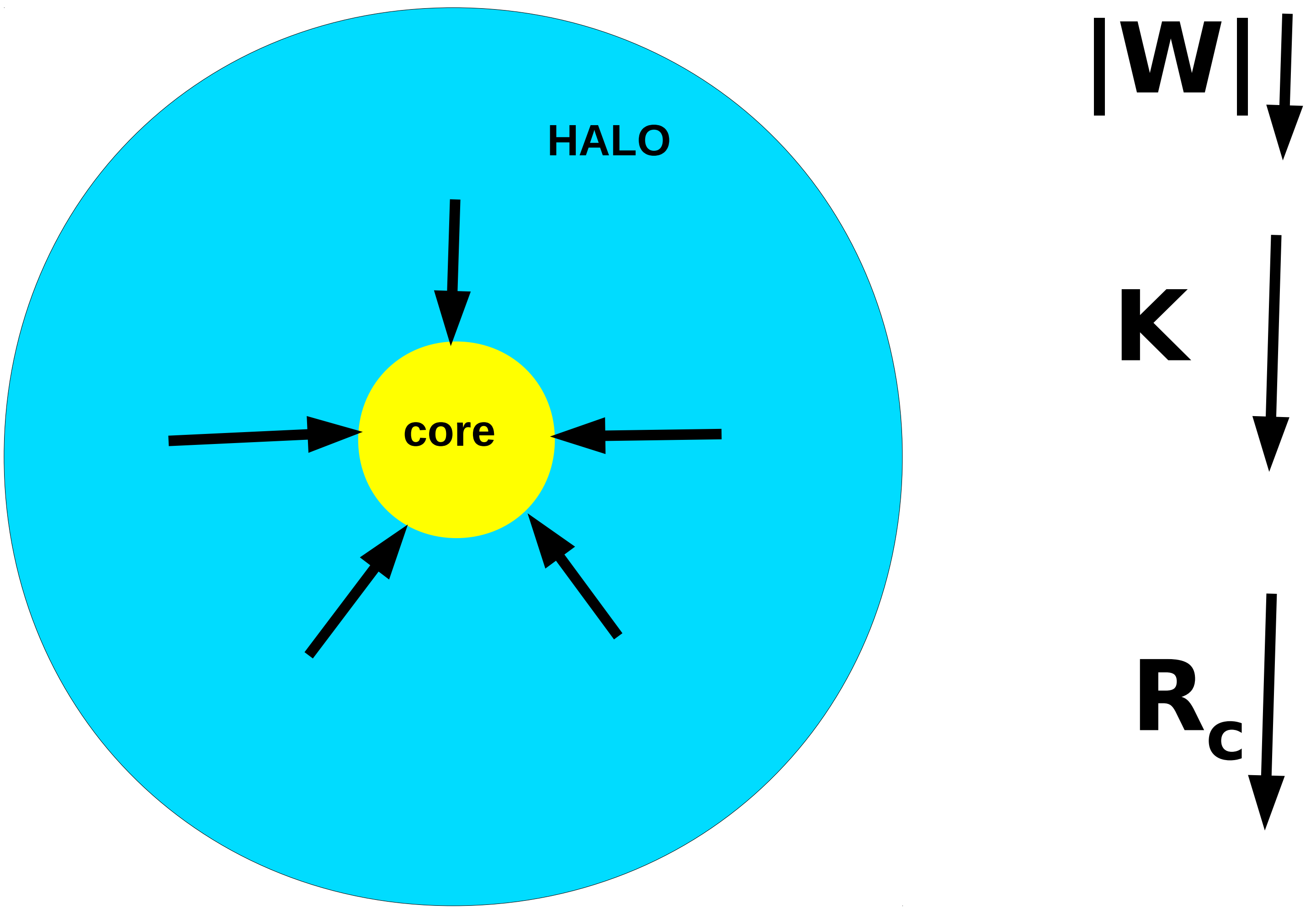
- two-body encounters are efficient
  → leads to evaporation of the fastest stars from core



2. CORE COLLAPSE with CARTOONS:

- leads to decrease of |W| and K
- since 'fastest' stars are lost, the decrease in K is stronger than in |W|

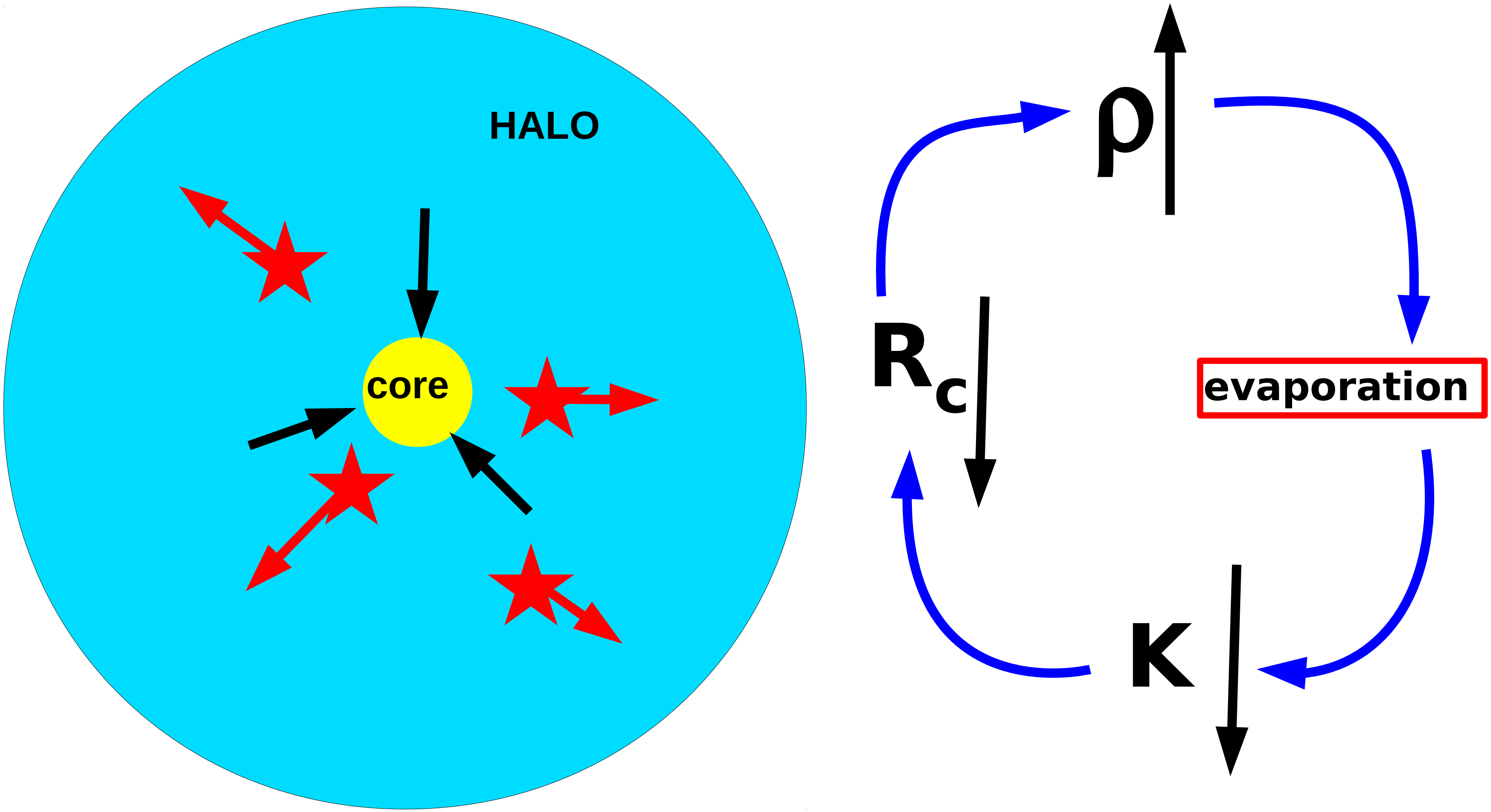
→ core contracts because |W| no longer balanced by K



2. CORE COLLAPSE with CARTOONS:

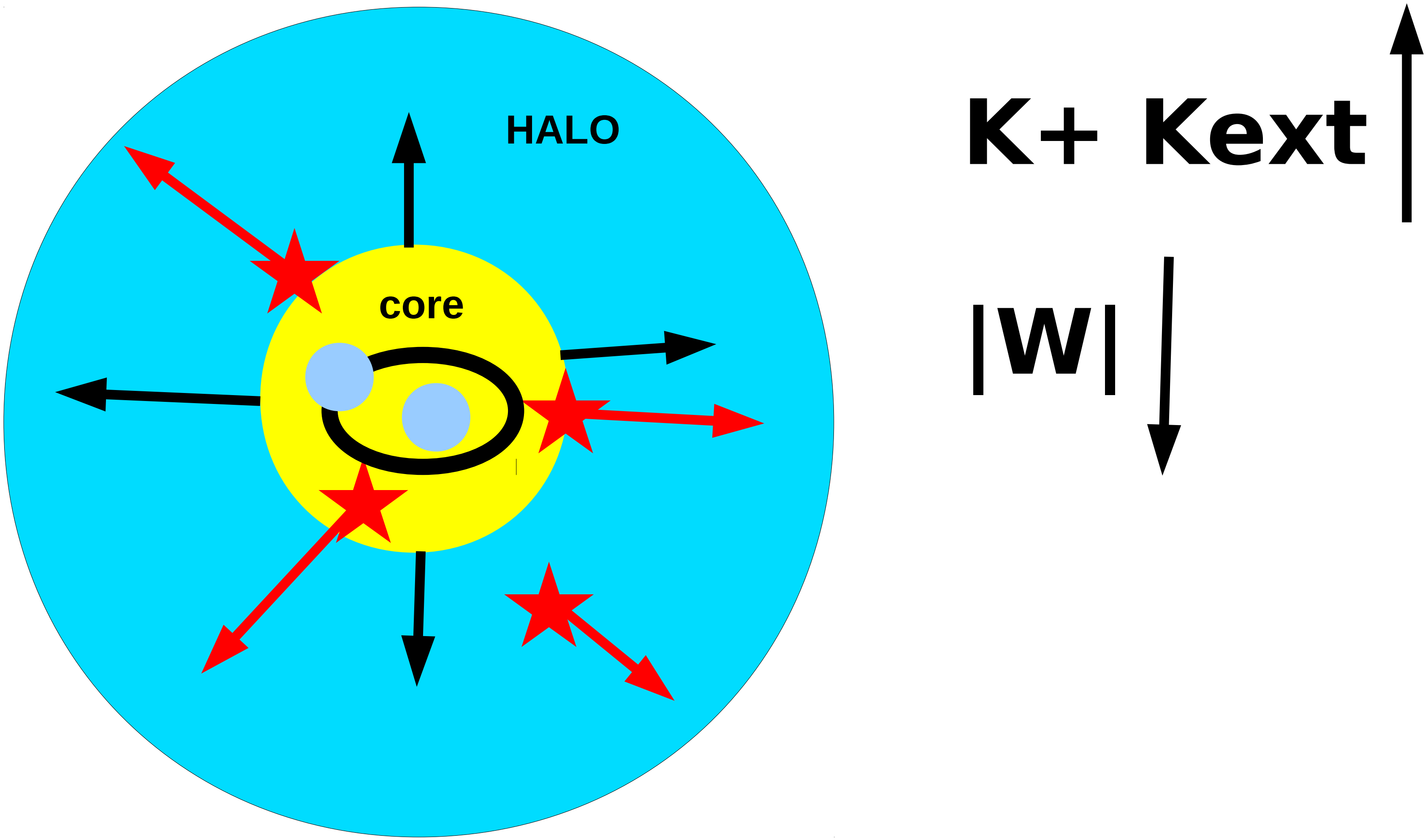
- density increases and 2body encounter rate increases
→ more fast stars evaporate, K decreases further, radius contracts more

RUNAWAY MECHANISM : core collapse!!!



2. CORE COLLAPSE with CARTOONS:

- NEEDS AN EXTERNAL SOURCE (Kext) TO BREAK IT:
    - * 3body encounters: E extracted from binaries decreases |W| and increases K
    - * Mass loss by stellar winds decrease |W|



2. CORE COLLAPSE properties:

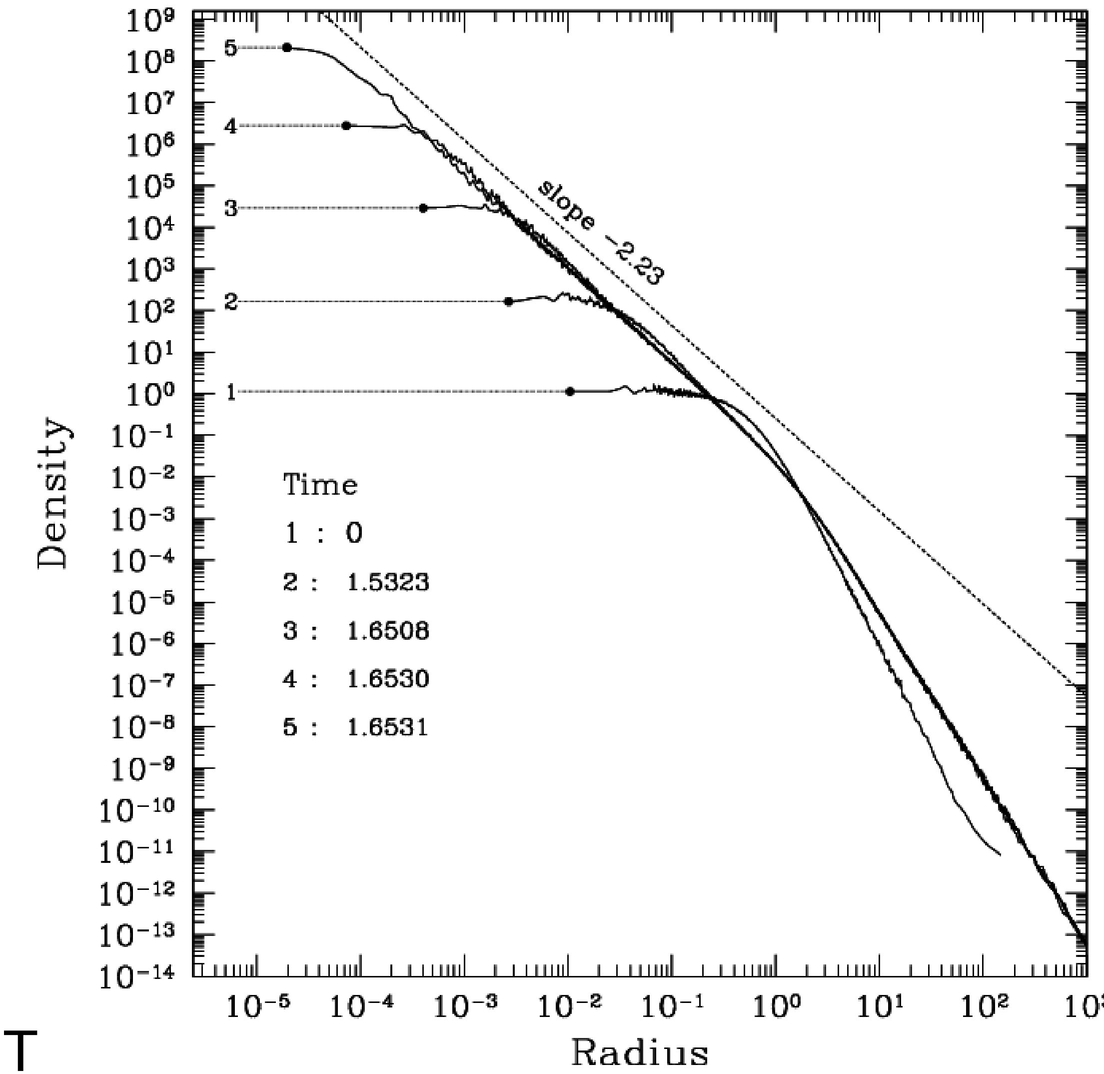
CORE COLLAPSE is SELF-SIMILAR (cfr. model of evaporation)

$$\frac{d\rho_c}{dt} \rightarrow \mathrm{const}\ \frac{\rho_c}{t_{cr}}$$

central relaxation time

const ~ 3.6x10^-3 – 6x10^-3 from N-body simulations

* DURING CORE COLLAPSE
  HALF-MASS RADIUS ~CONSTANT



Freitag & Benz 2001, A&A, 375, 711

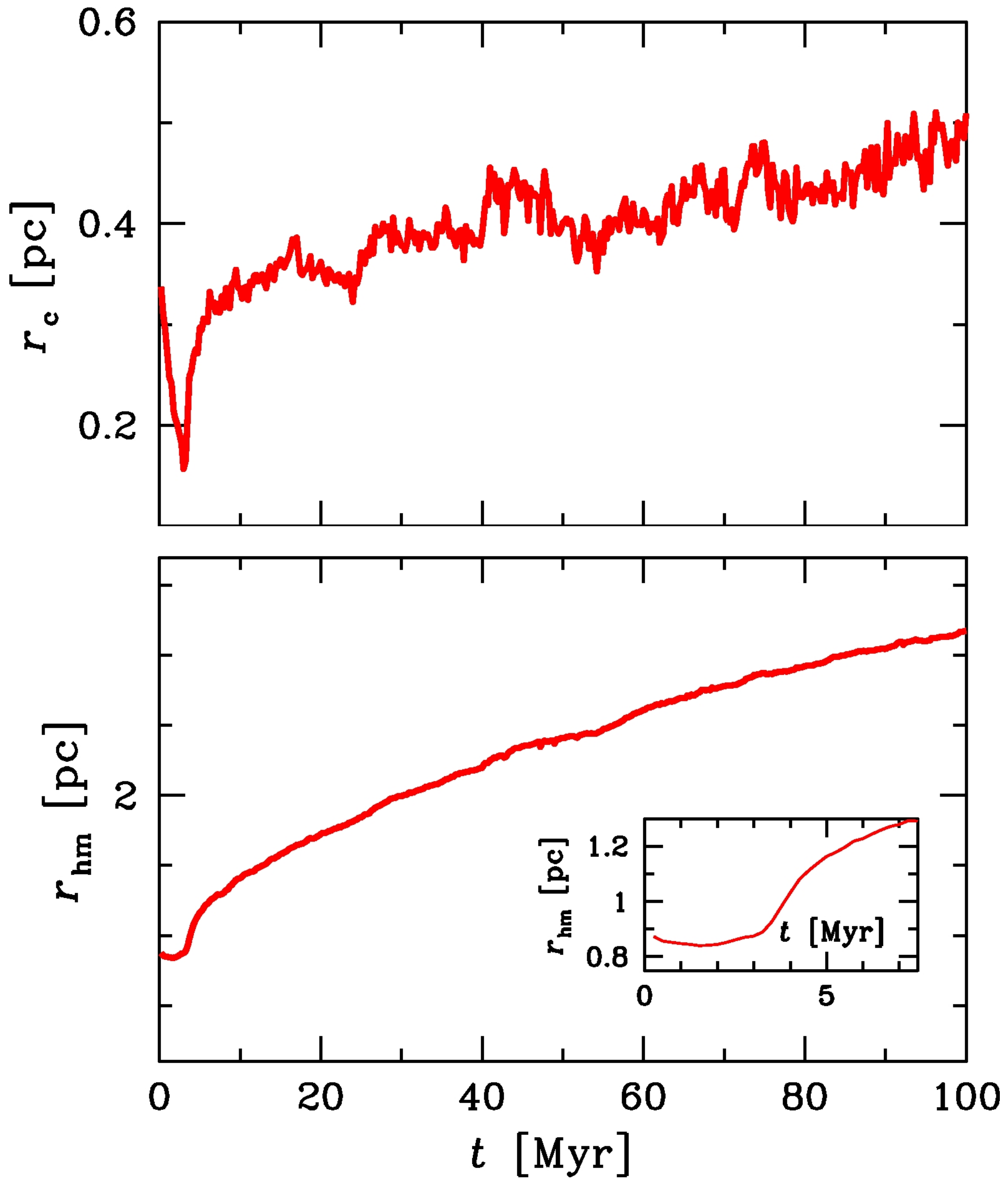
3. POST CORE COLLAPSE PHASE:

CORE EXPANDS
→**INJECTS ENERGY IN THE HALO IN THE FORM OF HIGH VELOCITY STARS**

HALO is a good bath but not an ideal (i.e. perfect) bath:

HALO EXPANDS due to energy injection and also half-mass radius expands

(Note: when speaking of half-mass radius, we refer mostly to the halo as core generally is << 1/10 of total mass)



3. POST CORE COLLAPSE PHASE:

HOW does halo expand?

(1) core collapse is self-similar
half-mass relaxation time $t_{hm} \propto t$

(2) from 1st lecture $t_{hm} \propto \frac{N}{\ln N} t_{cross} \sim N\, t_{cross}$

(3) VIRIAL theorem $\frac{1}{2} M \langle v^2 \rangle = \frac{1}{2} \frac{G M^2}{r_{hm}}$

(4) $t_{cross} = \frac{r_{hm}}{\langle v \rangle} \Rightarrow r_{hm}^3 \sim G\, M\, t_{cross}^2$

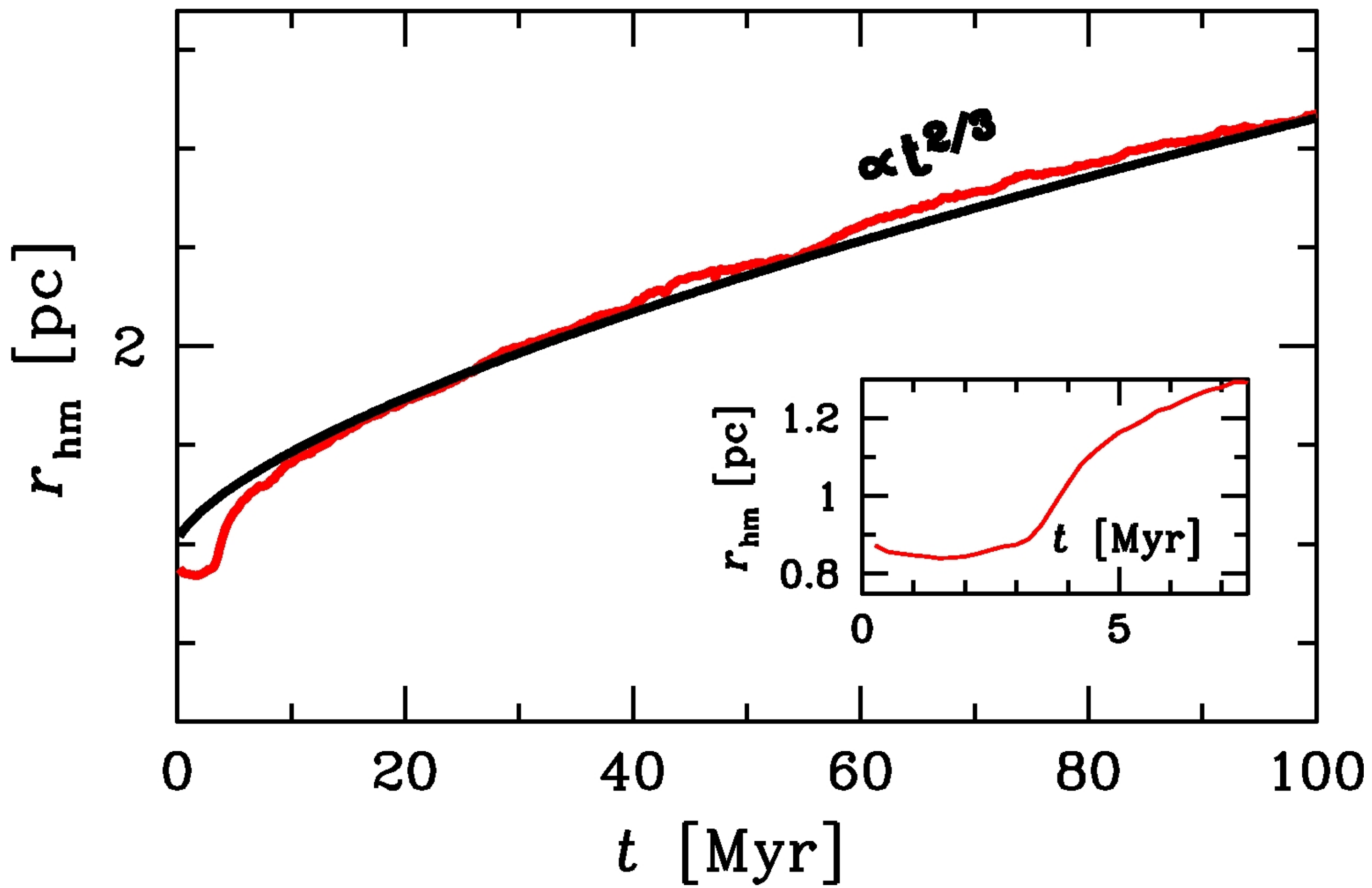
$$\Rightarrow t_{cross} \propto r_{hm}^{3/2} \quad \text{assuming } M \sim const$$

$$\Rightarrow r_{hm} \propto t_{cross}^{2/3} \propto t_{hm}^{2/3} \propto t^{2/3}$$

3. POST CORE COLLAPSE PHASE:

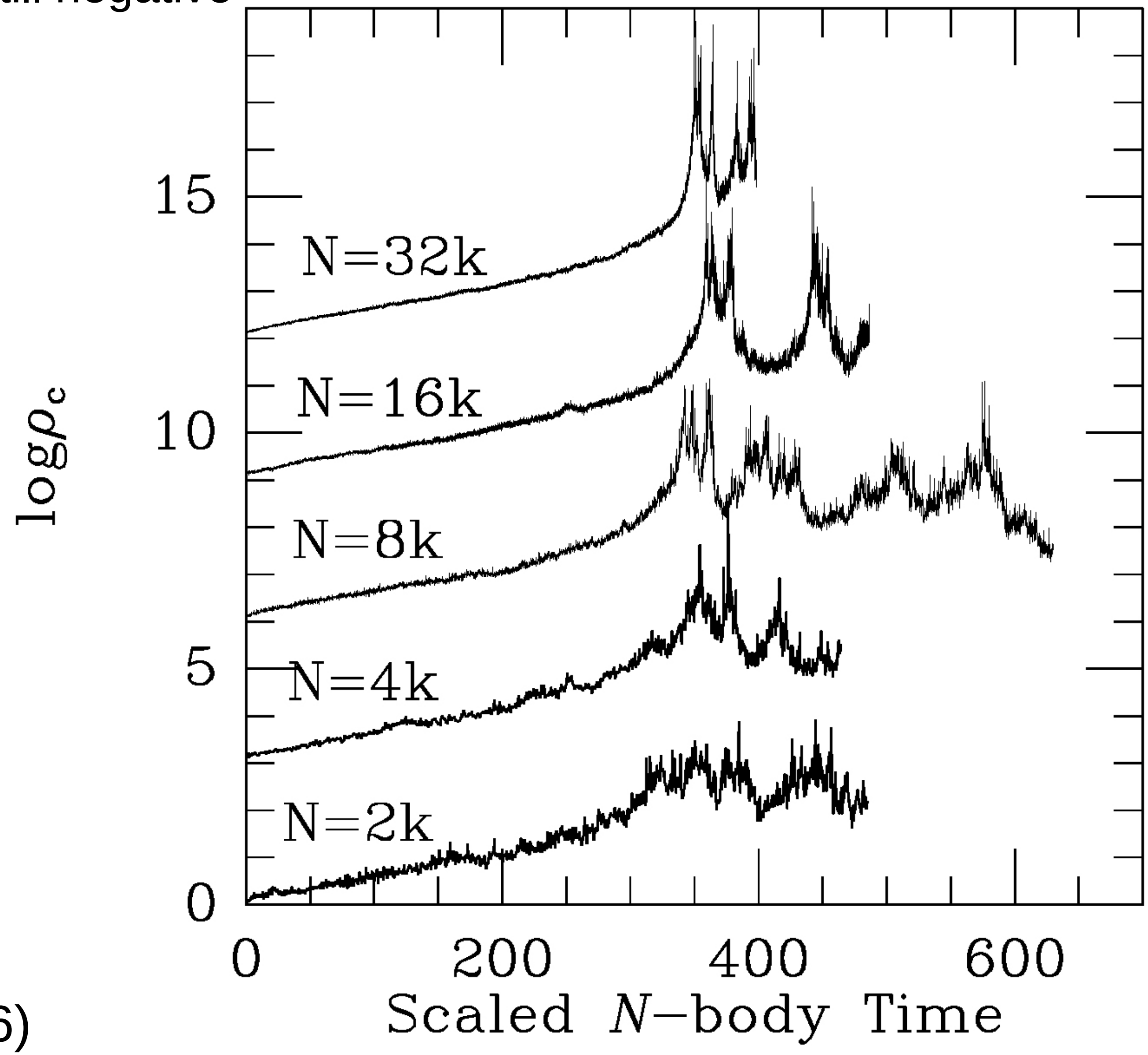
HOW does halo expand?

$$\Rightarrow r_{hm} \propto t_{cross}^{2/3} \propto t_{hm}^{2/3} \propto t^{2/3}$$



4. GRAVOTHERMAL OSCILLATIONS:

After first core collapse there may be a series of core contractions/re-expansions because HEAT CAPACITY still negative



Hut 1997 (astro-ph/9704286)

4. GRAVOTHERMAL OSCILLATIONS:

There has been a lot of discussion whether

- secondary **collapses** are gravothermal

  i.e. are induced by negative heat capacity

  MOST LIKELY ANSWER: yes

- secondary **reverses** of collapse are gravothermal

  i.e. are induced by negative heat capacity

  reverse of the heating flow: when the bath becomes hotter than the core →

  heat keeps flowing from bath to core

  Core becomes cooler and cooler, and expands

  MOST LIKELY ANSWER: NO

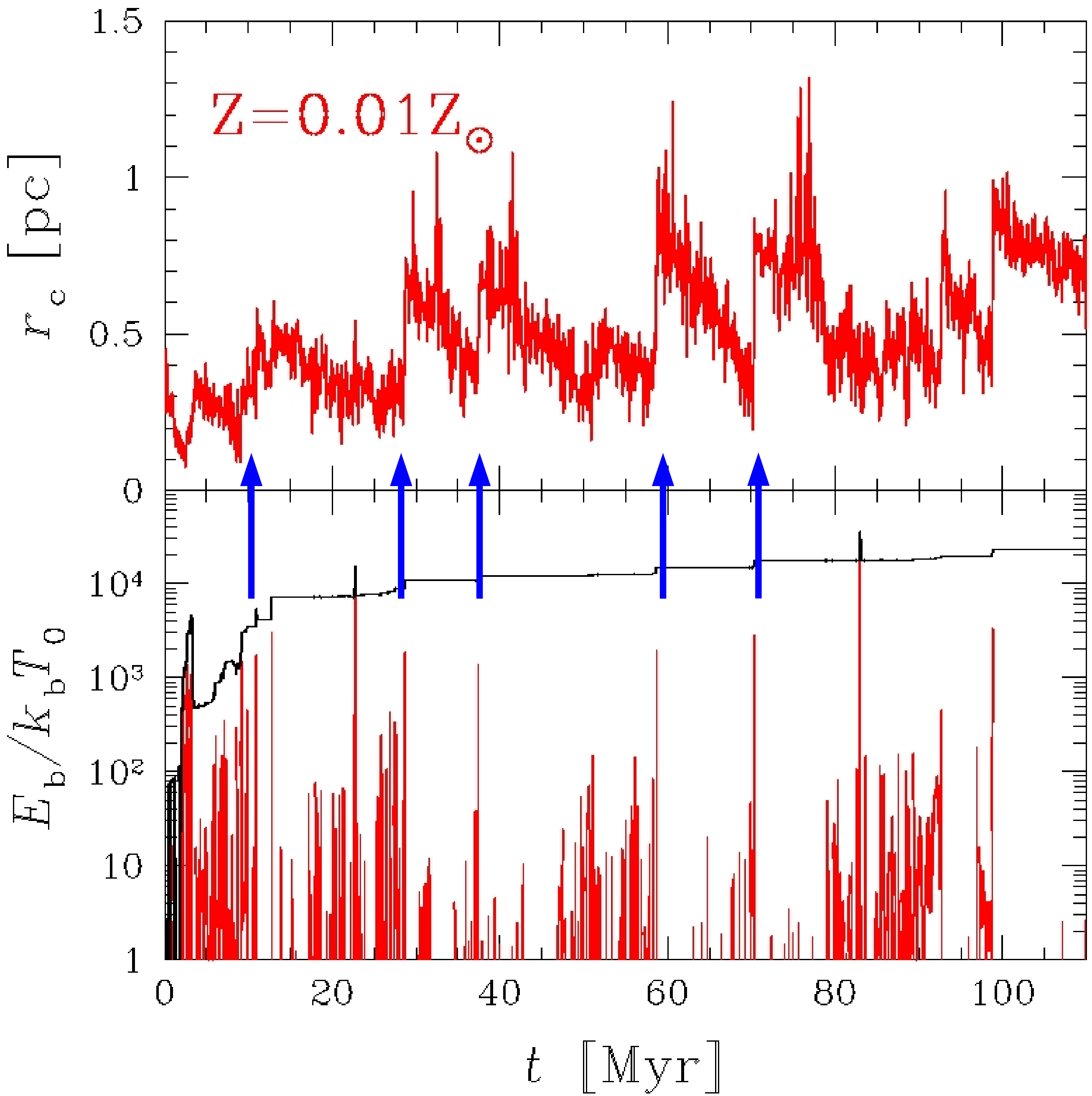
  All reverses are due to 3-body encounters

WHY?

4. GRAVOTHERMAL OSCILLATIONS:

Core radius

Binary binding energy



Reverse of oscillations is not gravothermal because each reverse corresponds to a jump in binary binding energy

→ three-body encounters by binaries play the main role

Trani, MM & Bressan 2014

NOTE: TIMESCALES FOR RELAXATION in different SCs

Table 1.3. Time scales

Time scale	symbol	bulge	globular	YoDeC	Open cluster
Star	t_{ms}	10Gyr	10Gyr	10Myr	10Myr
size	R	100pc	10pc	$\lesssim$ 1pc	10pc
mass	M	$10^9 M_\odot$	$10^6 M_\odot$	$10^5 M_\odot$	$1000 M_\odot$
velocity	$\langle v \rangle$	100km s^{-1}	10km s^{-1}	10km s^{-1}	1km s^{-1}
relaxation	t_{rt}	10^{15}yr	3 Gyr	50Myr	100Myr
crossing	t_{hm}	100Myr	10 Myr	100Kyr	1Myr
t_{rt}/t_{ms}		10^5	3	5	10
t_{hm}/t_{ms}		0.01	1	10^{-4}	0.1

From Portegies Zwart 2004, astro-ph/0406550

Note: t_{coll}~0.2 t_{rlx}

Young dense star clusters (YoDeC) are the only clusters with relaxation and core collapse time of the same order of magnitude as massive star evolution

References:

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