Michela Mapelli

LECTURES on COLLISIONAL DYNAMICS:

4. HOT TOPICS on COLLISIONAL DYNAMICS

Part 2

1) IMBHs: runaway collapse, repeated mergers, ...

2) BHs eject each other?

3) Effects of 3-body on X-ray binaries (formation and escape)

3b) Gravitational waves

4) Effect of metallicity on cluster evolution

5) Formation of blue straggler stars

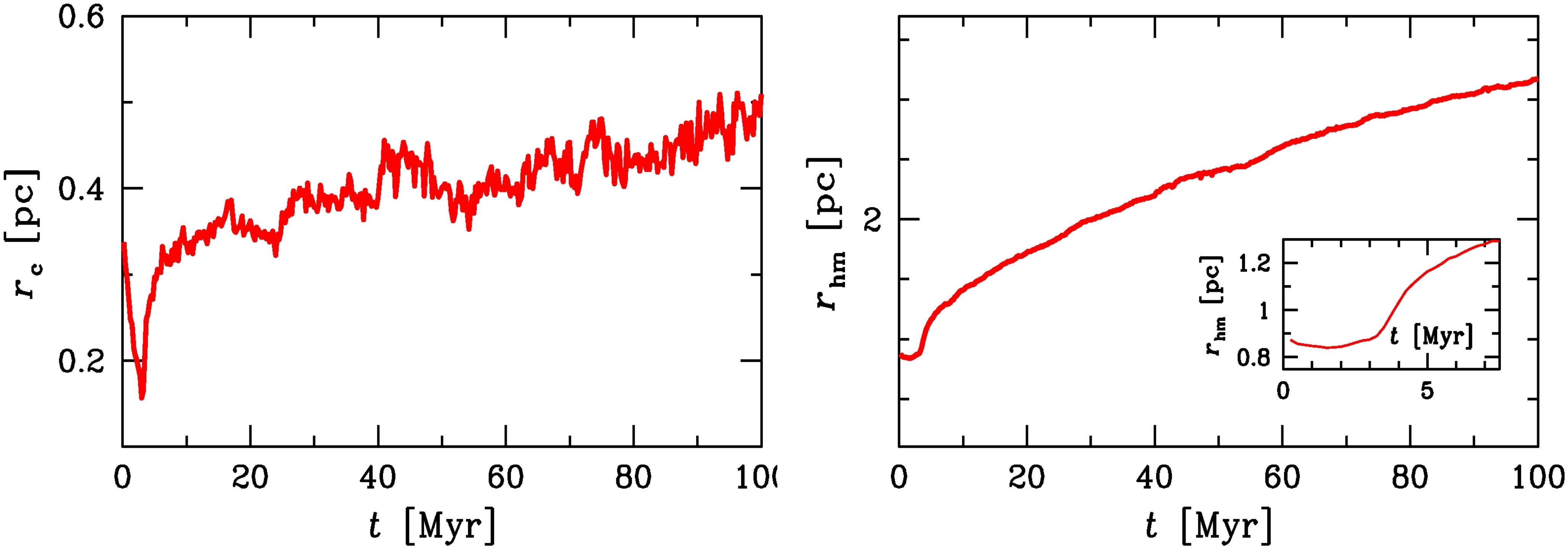
6) Tools for numerical simulations of collisional systems

7) Three-body and planets

8) Nuclear star clusters

4) Effect of metallicity on cluster evolution

Note: ONLY IF TIMESCALE FOR MASSIVE STELLAR EVOLUTION IS SIMILAR TO CORE COLLAPSE – RELAXATION TIME



From N-body simulations with metal-dependent stellar evolution and recipes for stellar winds (MM & Bressan 2013; Trani, MM, Bressan 2014)

N=5500 stars, M=3000 – 4000 $M_\odot$, r_c=0.4 pc, r_h~0.8 pc, Kroupa IMF

→ t_{rlx}~10 Myr, t_{cc}~2 Myr (LIFETIME of >100 $M_\odot$ stars!)

4) Effect of metallicity on cluster evolution

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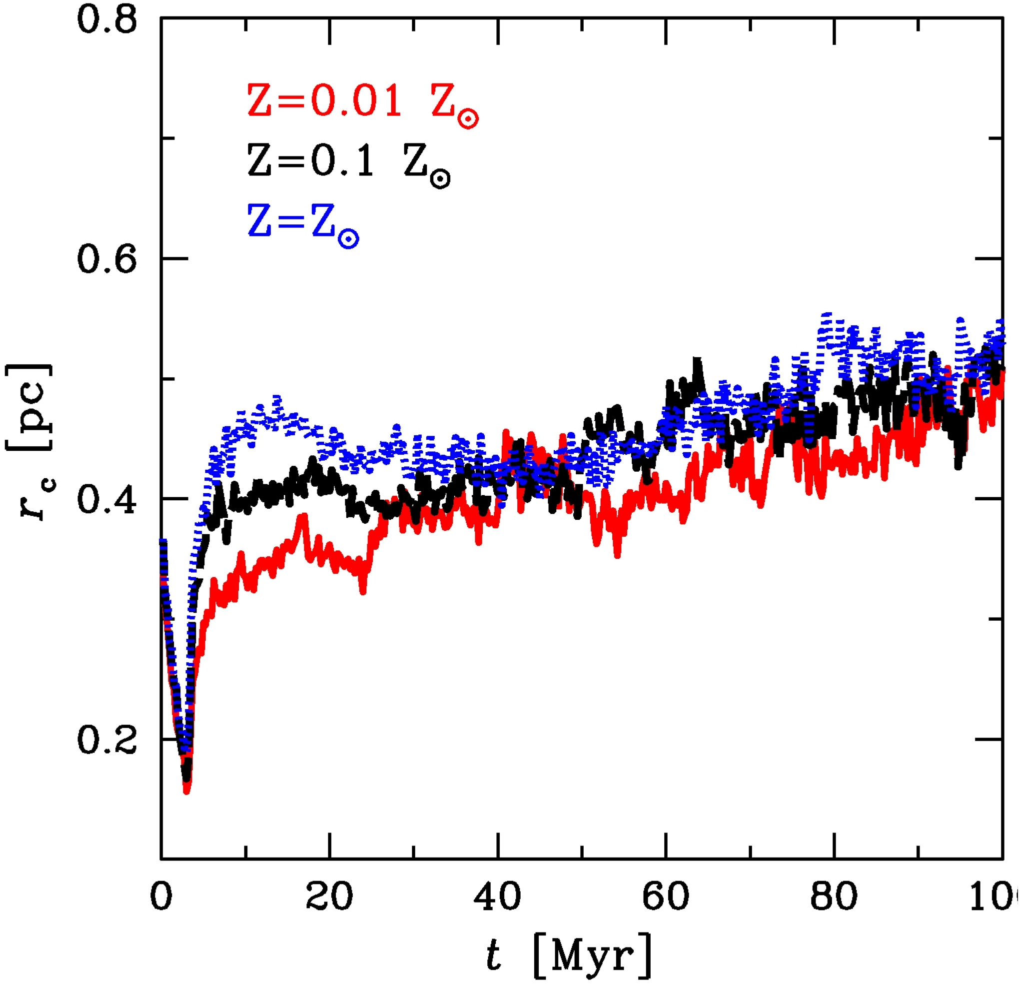

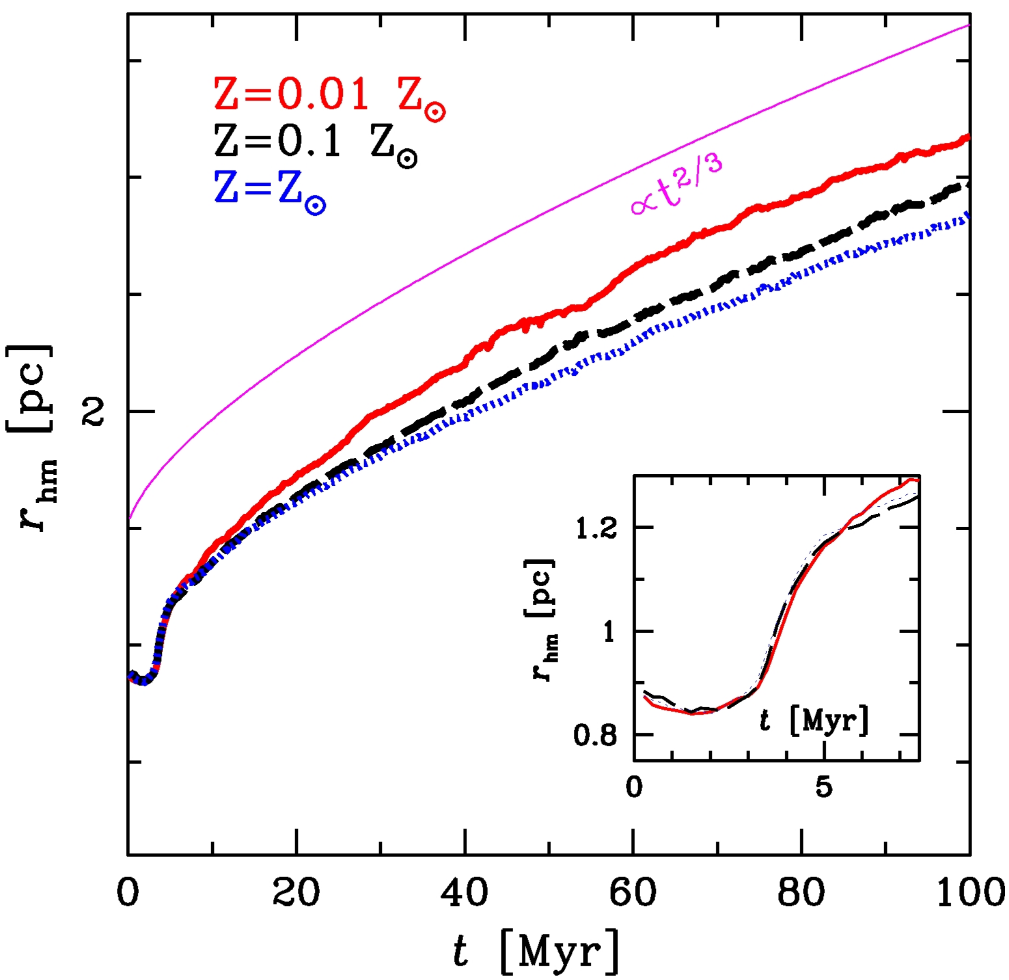

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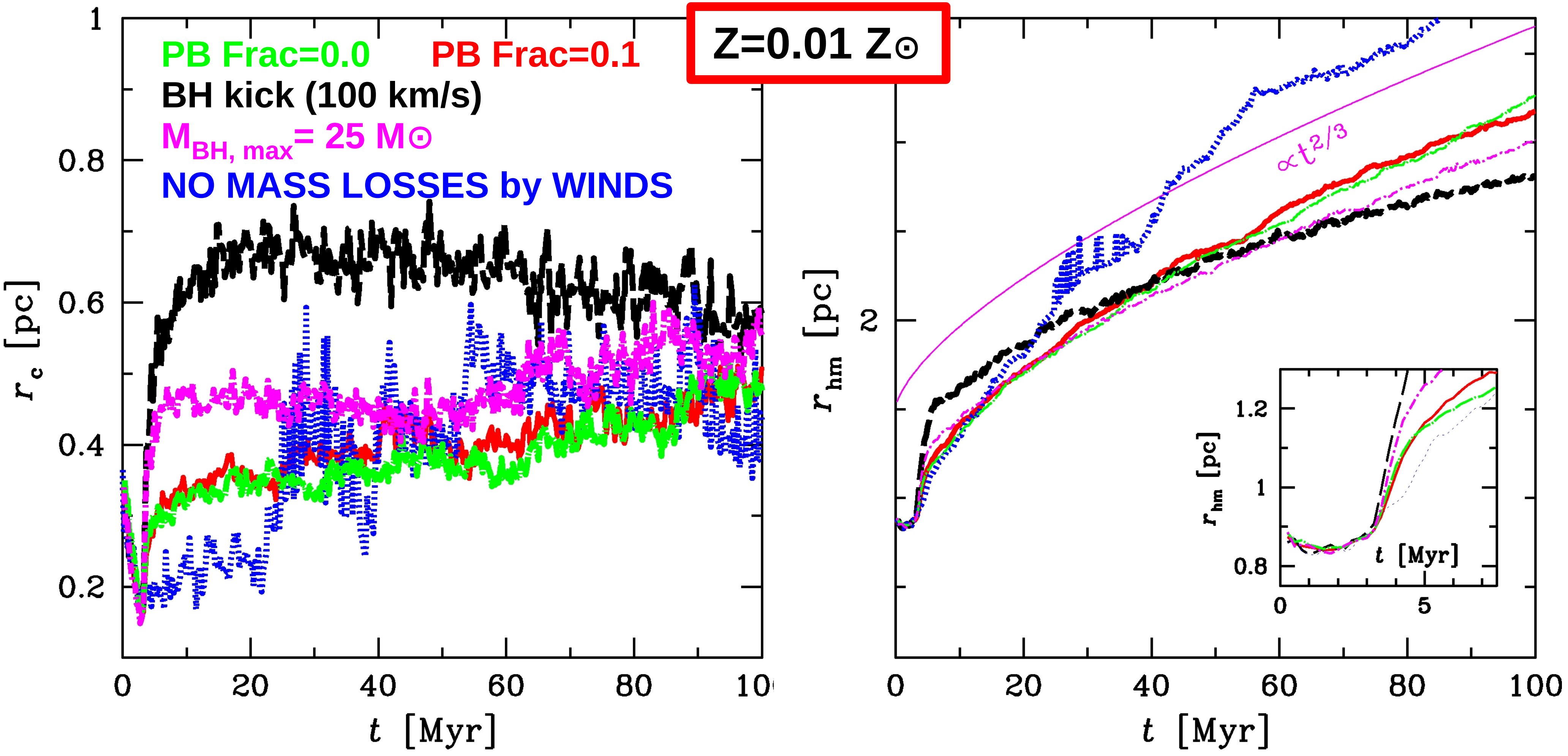


From N-body simulations with metal-dependent stellar evolution and recipes for stellar winds (MM & Bressan 2013; Trani, MM & Bressan 2014)

4) Effect of metallicity on cluster evolution

Observations?

Half-light radius of metal-poor GCs ~20% larger than half-light radius of metal-rich GCs (Kundu & Whitmore 1998; Jordàn et al. 2005; Woodley & Gòmez 2010; Strader et al. 2013, arXiv:1210.3621)

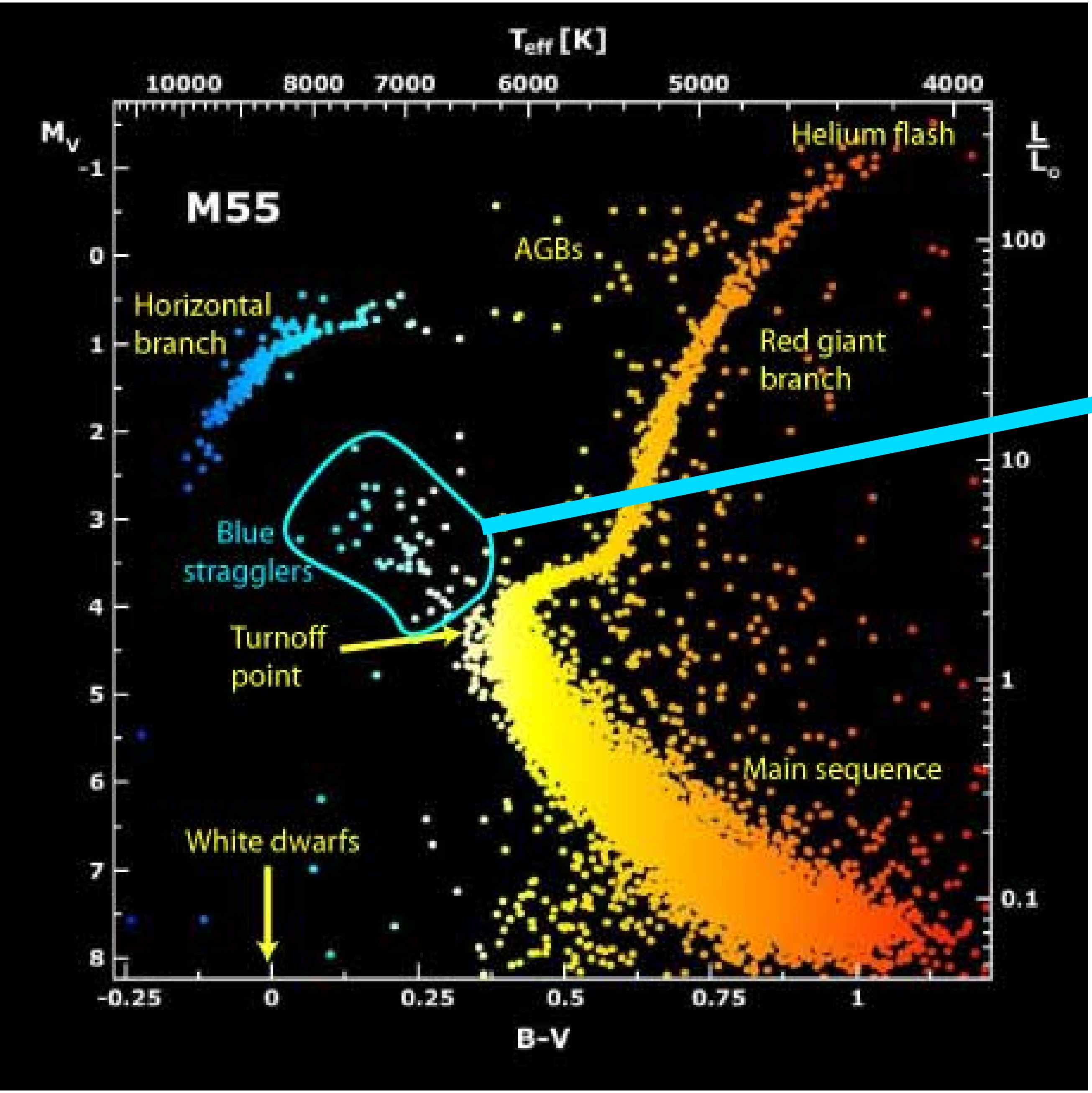
BUT GCs are very different from our simulated clusters!!!

t_{rlx}>>100 Myr → MASS LOSSES by stellar winds occur BEFORE core collapse and do NOT affect significantly cluster evolution

Sippel et al. (2012, arXiv:1208.4851) simulate GCs and find no differences in half-mass radius, but differences in half-LIGHT radius, due to BRIGHTER LOW-MASS METAL-POOR STARS vs metal-rich stars and to REMNANT MASS

DO YOUNG DENSE STAR CLUSTERS SHOW DIFFERENCE IN HALF-MASS RADIUS? STILL TO BE CHECKED!!!
It may be that TIDAL EFFECTS wash everything

5) Formation of blue straggler stars



BSSs: above and blue-ward the MS

If H burning mBSS > mTO

REJUVENATED STARS!!!

5) Formation of blue straggler stars

2 scenarios for rejuvenation:

MASS TRANSFER in BINARIES with efficient mixing

(McCrea 1964)

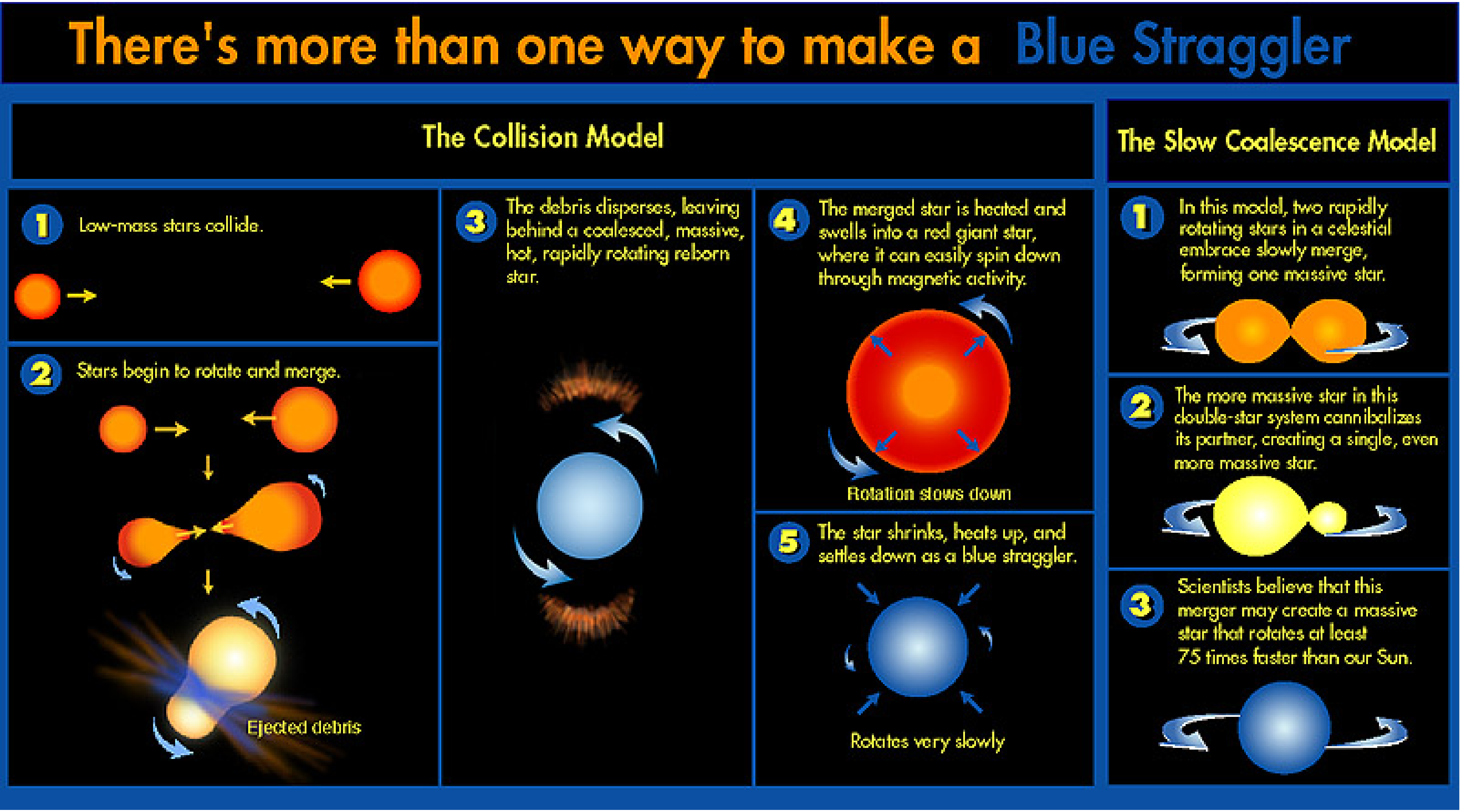
STELLAR COLLISION triggered by 3-body encounters

(e.g., Sigurdsson+1994)

EXCLUSIVE or COEXIST??

IN WHICH ENVIRONMENTS??

5) Formation of blue straggler stars



Credit: R. Saffer (Villanova University), D. Zurek (STScI) and NASA

5) Formation of blue straggler stars

Hybrid MonteCarlo + direct 3 body simulations with BEV (Binary EVolution code, Sigurdsson & Phinney 1995, MM+2004)

Integration of BSS candidates in a multi-mass King SC (potential, distant 2-body encounters, dynamical friction, 3-body encounters)

BSS properties in the code:

for all BSSs lifetime ~ 1-4 Gyr

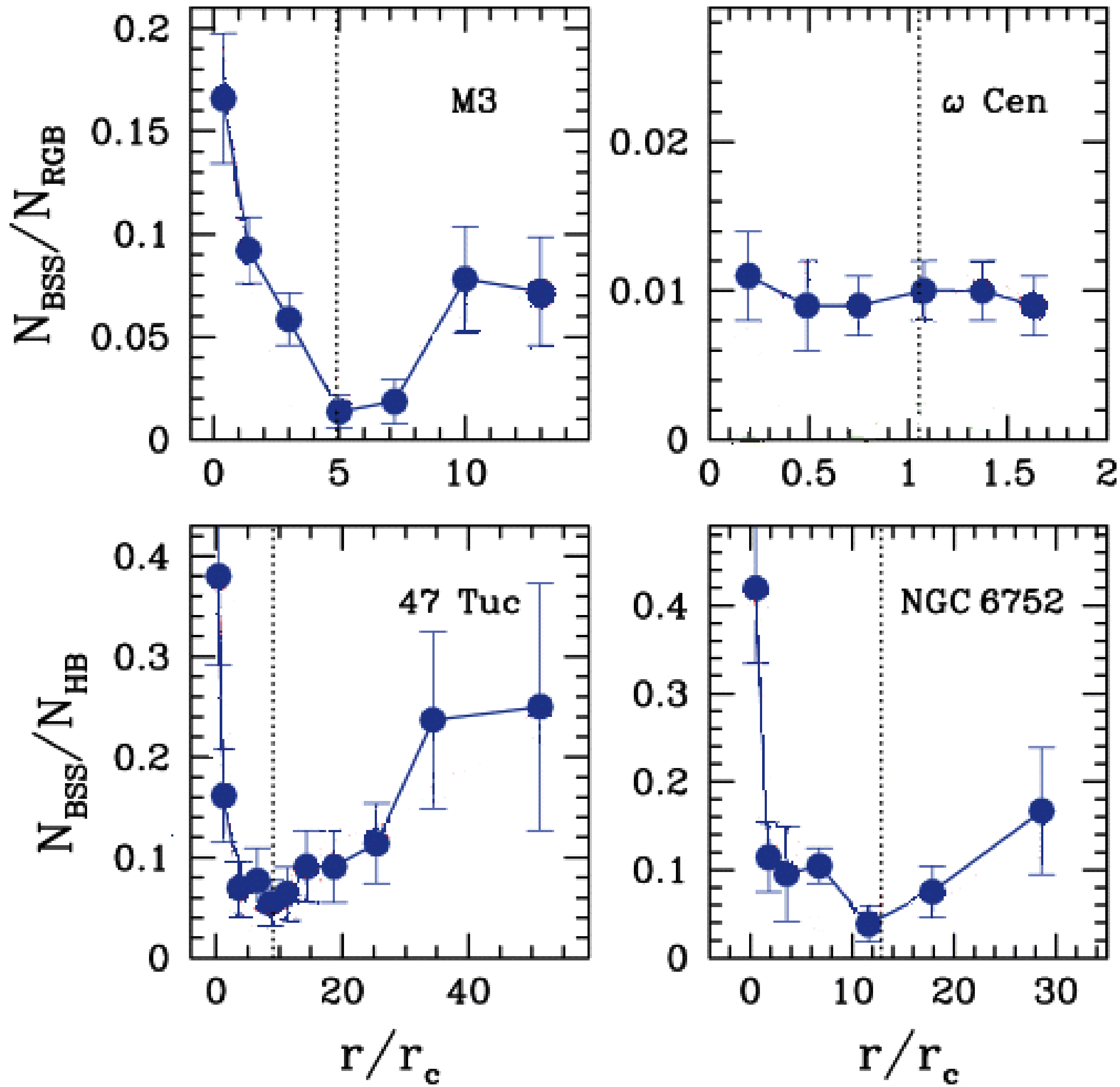
COLLISIONAL BSS (COL-BSS):
- only in CORE
- with initial kick from 3-body encounters

MASS-TRANSFER BSS (MT-BSS):
- follow King model in entire cluster (as primordial binaries)
- with local velocity dispersion

5) Formation of blue straggler stars

Hybrid MonteCarlo + direct 3 body simulations

Possible comparison with data: RADIAL DISTRIBUTION OF BSSs in Scs



DATA (Ferraro+97, 2004,2006, Sabbi+ 2004)

5) Formation of blue straggler stars

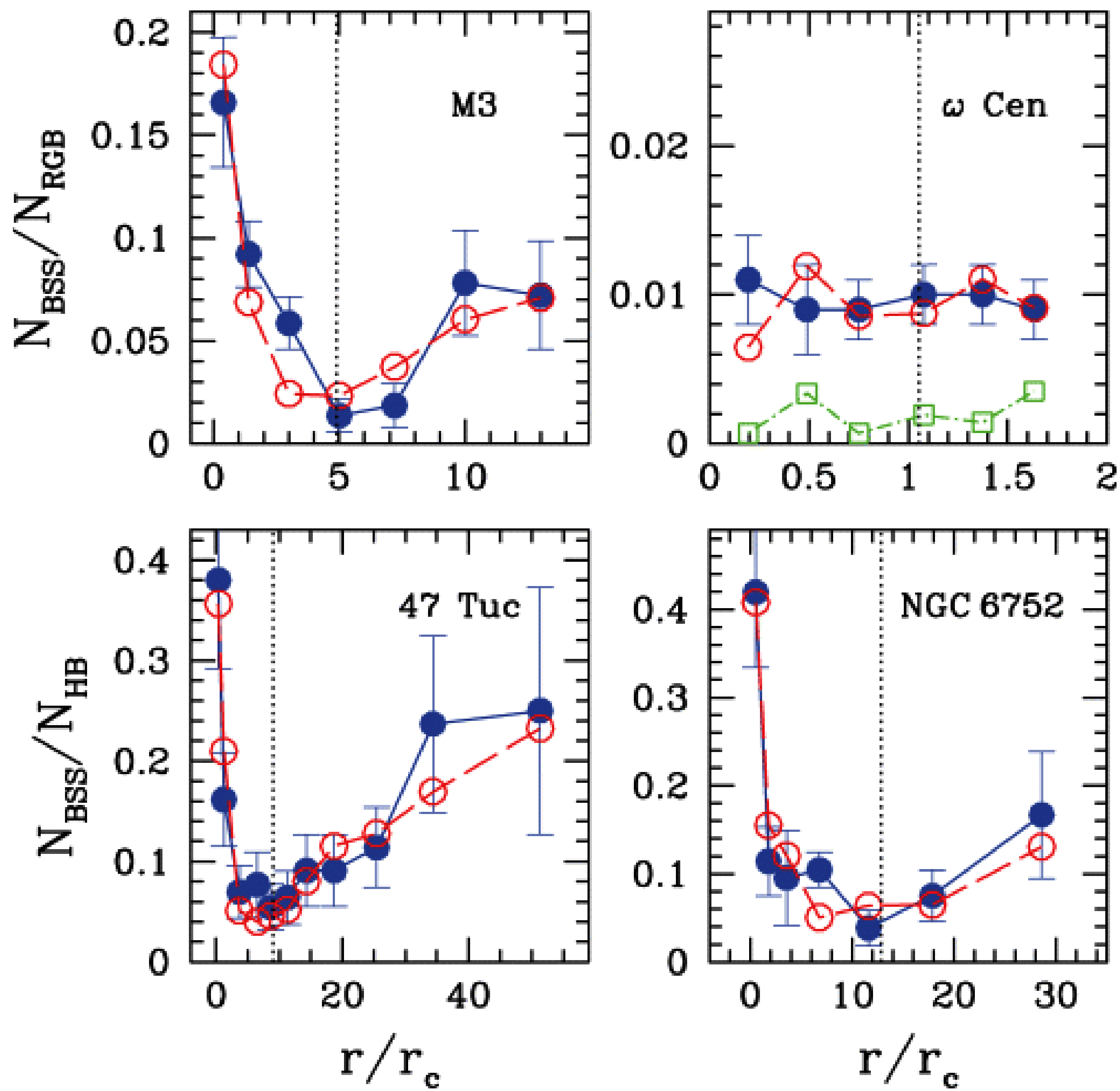
Hybrid MonteCarlo + 3 body simulations

We change η= fraction of MT BSSs over total number BSSs

DATA (Ferraro+97, 2004,2006, Sabbi+ 2004)

SIMULATIONS (MM+2006)

M3	η= 0.4 ±0.2
ω Cen	η= 0.9 ±0.1
47Tuc	η= 0.5 ±0.2
NGC6752	η= 0.4 ±0.1



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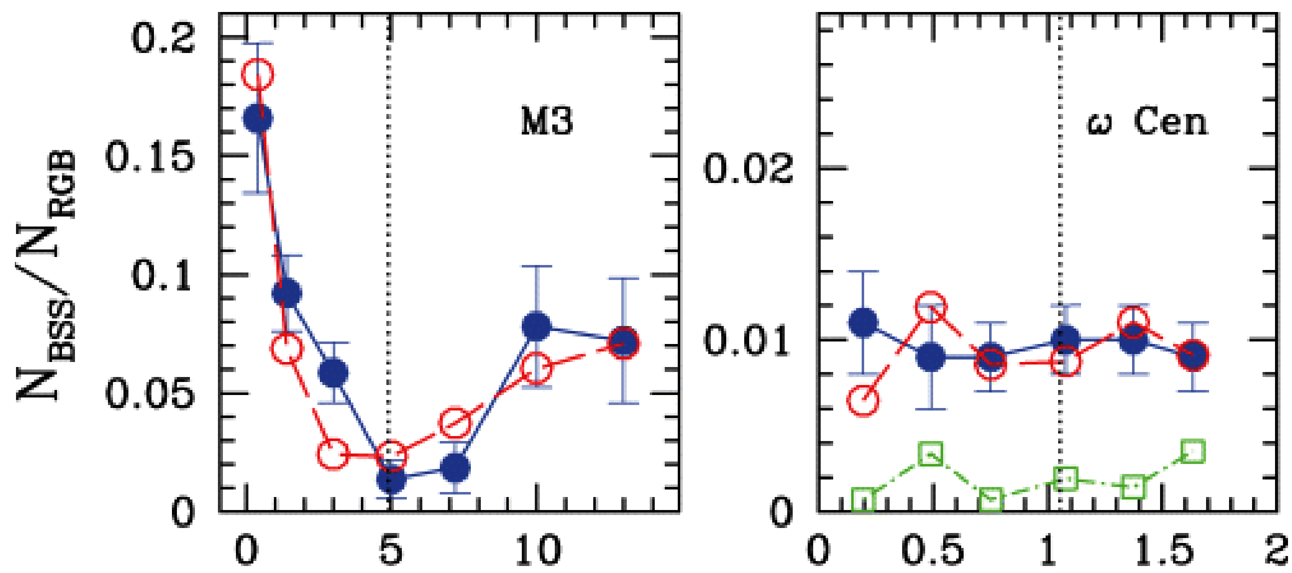


Dotted line: radius where dynamical friction becomes inefficient

(t_{df} longer than cluster age t_{age})

$$t_{df} = \frac{3}{4\,\ln\Lambda\,G^2\,(2\,\pi)^{1/2}}\,\frac{\sigma^3(r)}{m_{BSS}\,\rho(r)} = t_{age}$$

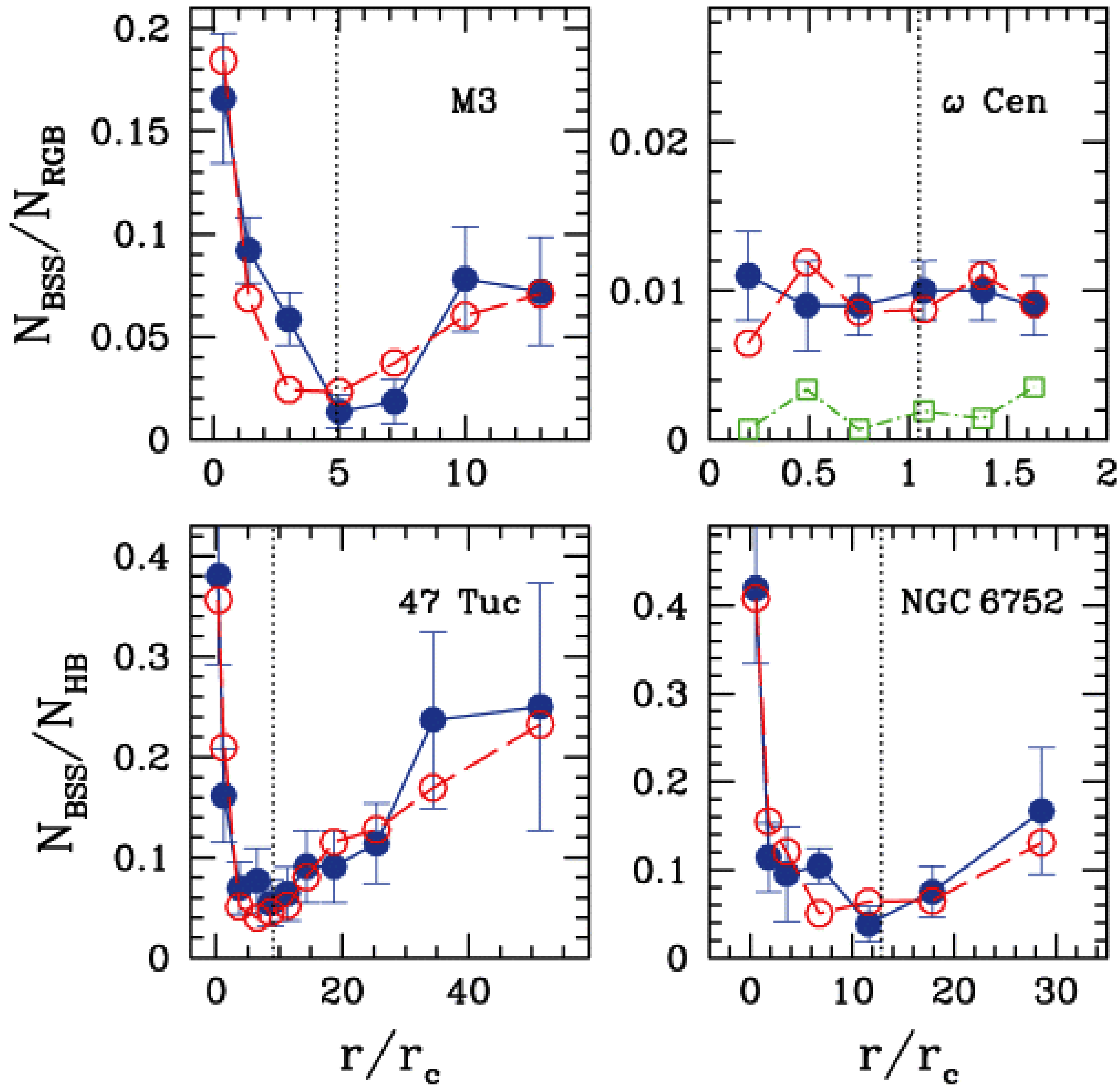
5) Formation of blue straggler stars

Hybrid MonteCarlo + 3 body simulations

COL-BSSs:
central peak
MT-BSS: outer rise

ωCen not relaxed!

SIMULATIONS SHOW THAT COEXISTENCE BETWEEN MT AND COLLISIONS IS NORMAL IN GCs!!



5) Formation of blue straggler stars

Hybrid MonteCarlo + 3 body simulations

COL-BSSs:
central peak
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CENTRAL RELAXATION timescales

$$t_{\rm rlx} = 0.34 \, \frac{\sigma^3}{G^2 \, m \, \rho \ln \Lambda}$$

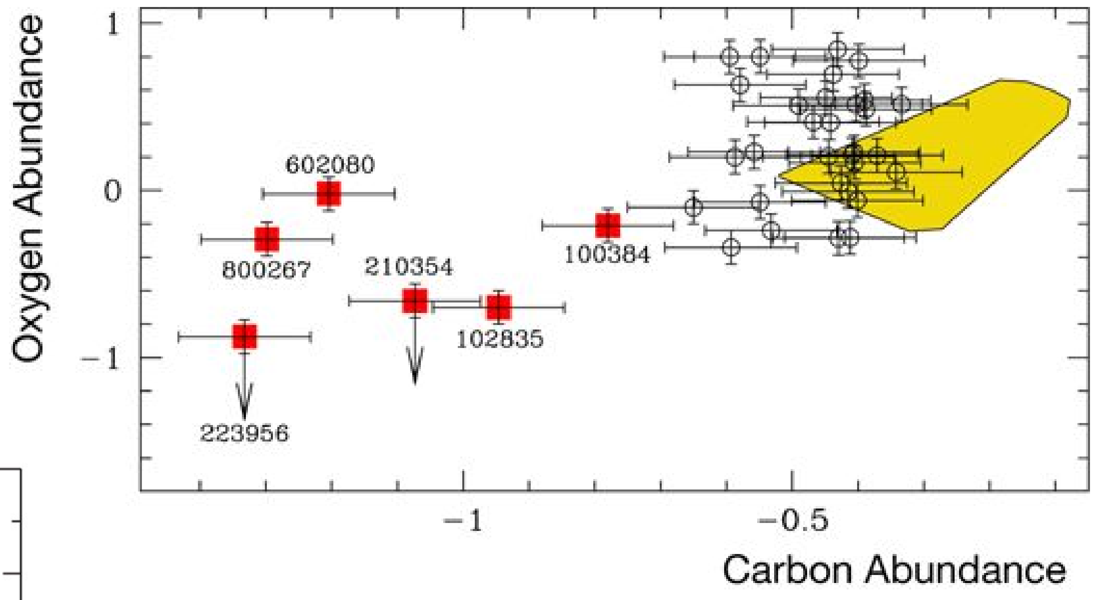
47Tuc:	**4×10^7 yr**
M3:	**2×10^8 yr**
NGC6752:	**5×10^7 – 5×10^8 yr**
Omega Cen:	**8×10^9 yr**

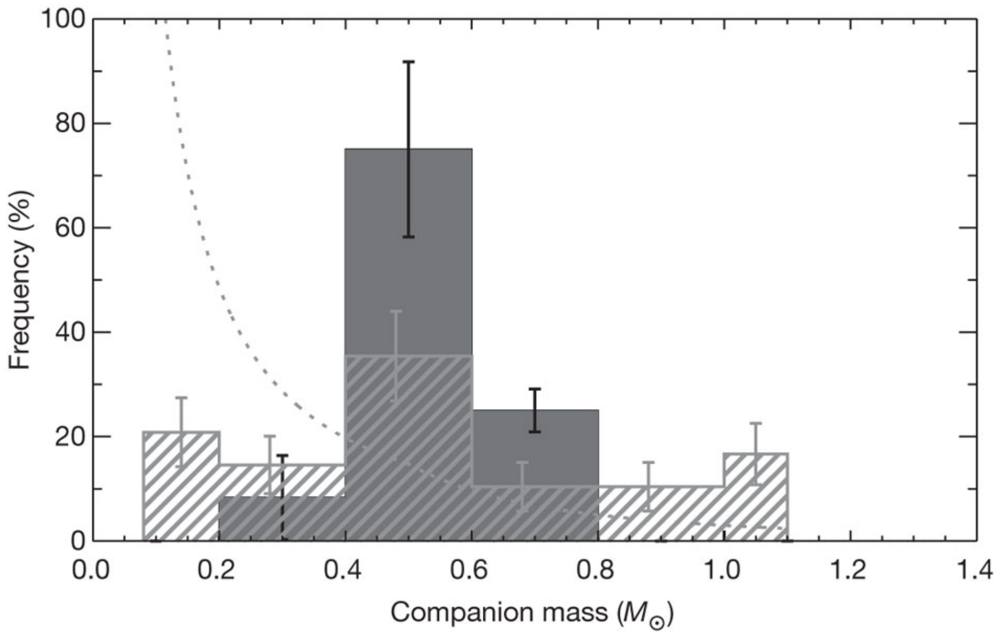
Using Table 1 of MM+2006

5) Formation of blue straggler stars

Observational support for MT BSSs

8 Carbon deficient BSSs
in 47 Tuc :
allowed only for MT in binaries
Ferraro et al. 2006, ApJ 647, L53





Geller A. M., Mathieu R. D., 2011, Nat, 478, 356

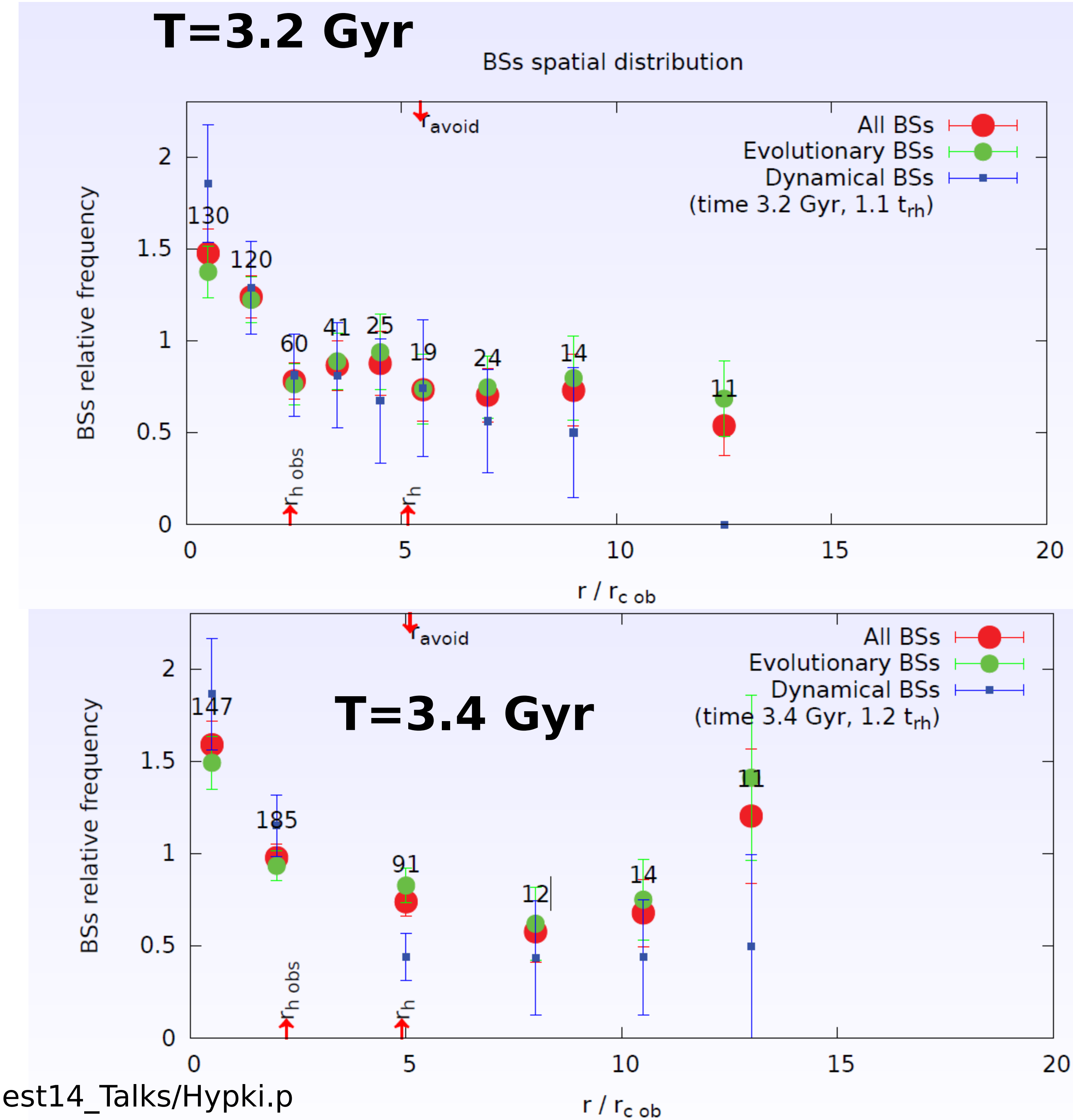
ONLY MT BSS
in the open cluster NGC188:
all observed BSS have a companion (binary) and companion is small

OPEN ISSUE: are we sure we saw COLLISIONAL BSS???

5) Formation of blue straggler stars

New Monte Carlo simulations (live background)

OPEN ISSUE: is the radius of avoidance a transient feature?



Hypki et al., in prep.
http://www.astro.uni-bonn.de/~sambaran/DS2014/Modest14_Talks/Hypki.pdf

6) Simulating collisional systems

You must resolve SINGLE STARS (softening based codes cannot be used)
Solving (i) equations of motion and
possibly ***(ii) stellar and binary evolution***

(i) EQUATIONS of MOTION:

$$\ddot{\vec{r}}_i = -G \sum_{j \neq i} m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

Or

$$\begin{cases} \dot{\vec{r}}_i & = \vec{v}_i \\ \dot{\vec{v}}_i & = -G \sum_{j \neq i} m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3} \end{cases}$$

6) Simulating collisional systems

You must resolve SINGLE STARS (softening based codes cannot be used)
Solving (i) equations of motion

→ **DIRECT N-BODY CODES**

1* Forces on binaries are stronger and change more frequently
→binaries need to be updated more frequently than single stars
→ we need a criterion for different timesteps
Timesteps for BINARIES and THREE-BODY ENCOUNTERS << timesteps for other bodies!

2* Solve Newton's equations for EACH star directly → scale as N^2
+ relaxation time scales as N
→time complexity $t_{CPU} \propto N^3$

(cfr with tree codes and Monte Carlo $\propto N \ln N$)

6) Simulating collisional systems

INTEGRATION SCHEME

If interactions (and especially close interactions) between stars are important
→ integrator must be HIGH ACCURACY even over SHORT TIMES
(integrate perturbations in < 1 orbit)
→ AT LEAST FOURTH-ORDER ACCURACY

4th ORDER PREDICTOR-CORRECTOR HERMITE SCHEME

Based on **JERK** (time derivative of acceleration)

$$\vec{a}_i = G \sum_{j \neq i} \frac{M_j}{r_{ji}^3} \vec{r_{ij}}$$

$$\frac{d\vec{a}_i}{dt} = \vec{j}_i = G \sum_{j \neq i} M_j \left[\frac{\vec{v}_{ji}}{r_{ji}^3} - 3 \frac{(\vec{r}_{ji} \cdot \vec{v}_{ji}) \; \vec{r}_{ji}}{r_{ji}^5} \right]$$

6) Simulating collisional systems

INTEGRATION SCHEME

4th ORDER PREDICTOR-CORRECTOR HERMITE SCHEME

Based on **JERK** (time derivative of acceleration)

BETTER ADD A SOFTENING
(often is the PHYSICAL RADIUS OF STARS)

$$\vec{a_i} = G \sum_{j \neq i} \frac{M_j \, \vec{r_{ij}}}{\left(r_{ji}^2 + \epsilon^2\right)^{3/2}}$$

$$\frac{d\vec{a_i}}{dt} = \dot{\vec{j_i}} = G \sum_{j \neq i} M_j \left[\frac{\vec{v_{ij}}}{\left(r_{ji}^2 + \epsilon^2\right)^{3/2}} + \frac{3(\vec{v_{ij}} \cdot \vec{r_{ij}}) \, \vec{r_{ij}}}{\left(r_{ji}^2 + \epsilon^2\right)^{5/2}} \right]$$

6) Simulating collisional systems

Let us start from 4th order derivative of Taylor expansion:

$$\begin{cases} x_1 = x_0 + v_0\,\Delta t + \frac{1}{2}\,a_0\,\Delta t^2 + \frac{1}{6} j_0\,\Delta t^3 + \frac{1}{24}\dot{j}_0\,\Delta t^4 & (1) \\ v_1 = v_0 + a_0\,\Delta t + \frac{1}{2} j_0\,\Delta t^2 + \frac{1}{6}\dot{j}_0\,\Delta t^3 + \frac{1}{24}\ddot{j}_0\,\Delta t^4 & (2) \\ a_1 = a_0 + j_0\,\Delta t + \frac{1}{2}\dot{j}_0\,\Delta t^2 + \frac{1}{6}\ddot{j}_0\,\Delta t^3 & (3) \\ j_1 = j_0 + \dot{j}_0\,\Delta t + \frac{1}{2}\ddot{j}_0\,\Delta t^2 & (4) \end{cases}$$

We use equations (3) and (4) to eliminate the 1st and 2nd derivative of jerk in equations (1) and (2). We obtain

$$x_1 = x_0 + \frac{1}{2}\,(v_0 + v_1)\,\Delta t + \frac{1}{12}\,(a_0 - a_1)\,\Delta t^2 + O(\Delta t^5) \quad (5)$$

$$v_1 = v_0 + \frac{1}{2}\,(a_0 + a_1)\,\Delta t + \frac{1}{12}\,(j_0 - j_1)\,\Delta t^2 + O(\Delta t^5) \quad (6)$$

WHICH ARE 4th order accuracy:
ALL TERMS in d*j*/dt *(snap)* and d²*j*/dt² *(crackle)* disappear: it is 4th order accuracy with only 2nd order terms!!!

But IMPLICIT for a_1, v_1 and j_1 → we need something to predict them

6) Simulating collisional systems

1) PREDICTION: we use the 3rd order Taylor expansion to PREDICT x_1 and v_1

$$x_{p,1} = x_0 + v_0\,\Delta t + \frac{1}{2}\,a_0\,\Delta t^2 + \frac{1}{6}\,j_0\,\Delta t^3 \qquad v_{p,1} = v_0 + a_0\,\Delta t + \frac{1}{2}\,j_0\,\Delta t^2$$

2) FORCE EVALUATION:

we use these PREDICTIONS to evaluate PREDICTED acceleration and jerk ($a_{p,1}$ and $j_{p,1}$), from Newton's formula.

3) CORRECTION:

we then substitute $a_{p,1}$ and $j_{p,1}$ into equations (5) and (6):

$$x_1 = x_0 + \frac{1}{2}\,(v_0 + v_{p,1})\,\Delta t + \frac{1}{12}\,(a_0 - a_{p,1})\,\Delta t^2$$

$$v_1 = v_0 + \frac{1}{2}\,(a_0 + a_{p,1})\,\Delta t + \frac{1}{12}\,(j_0 - j_{p,1})\,\Delta t^2$$

This result is only 3rd order in positions! But there is a dirty trick to make it 4th order: we calculate v_1 first and then use the result into x_1

$$v_1 = v_0 + \frac{1}{2}\,(a_0 + a_{p,1})\,\Delta t + \frac{1}{12}\,(j_0 - j_{p,1})\,\Delta t^2$$

$$x_1 = x_0 + \frac{1}{2}\,(v_0 + v_1)\,\Delta t + \frac{1}{12}\,(a_0 - a_{p,1})\,\Delta t^2$$

6) Simulating collisional systems

TIME STEP

We can always choose the SAME TIMESTEP for all PARTICLES

BUT: highly expensive because a few particles undergo close encounters → force changes much more rapidly than for other particles

→ we want different timesteps:
longer for 'unperturbed' particles
shorter for particles that undergo close encounter

A frequently used choice:

BLOCK TIME STEPS (Aarseth 1985)

6) Simulating collisional systems

1. Initial time-step calculated as for a particle *i*
   η = 0.01 – 0.02 is good choice

$$\Delta t_i = \eta \frac{a_i}{\dot{j}_i}$$

2. system time is set as $t := t_i + \min(\Delta t_i)$
   All particles with time-step = $\min(\Delta t_i)$ are called ACTIVE PARTICLES
   At time *t* the predictor-corrector is done only for active particles

3. Positions and velocities are PREDICTED for ALL PARTICLES

4. Acceleration and jerk are calculated ONLY for ACTIVE PARTICLES

5. Positions and velocities are CORRECTED ONLY for active particles (for the other particles predicted values are fine)

After force calculation, new timesteps evaluated as 1. and everything is repeated

BUT a different t_i for each particles is VERY EXPENSIVE and system loses coherence

6) Simulating collisional systems

$$\Delta t_i = \eta \frac{a_i}{\dot{j}_i}$$

A different Δt_i for each particles is VERY EXPENSIVE and the system loses coherence

→ BLOCK TIME STEP SCHEME consists in grouping particles by replacing their individual time steps Δt_i with a

BLOCK TIME STEP $\Delta t_{i,b} = (1/2)^n$

where n is chosen according to

$$\left(\frac{1}{2}\right)^n \leq \Delta t_i < \left(\frac{1}{2}\right)^{n-1}$$

This imposes that $t/\Delta t_{i,b}$ be an integer → good for synchronizing the particles at some time

Often it is set a minimum Δt_{min} = $2^{\wedge -23}$

6) Simulating collisional systems

REGULARIZATION

Definition:

mathematical trick to remove the singularity in the Newtonian law of gravitation for two particles which approach each other arbitrarily close.

Is the same as softening????

NO, it is a CHANGE OF VARIABLES,
that removes singularity without affecting the physics

Most used regularizations in direct N-body codes:

-Kustaanheimo-Stiefel (KS) regularisation
a regularization for binaries and 3-body encounters

-Aarseth's CHAIN regularization
a regularization for small N-body problems

6) Simulating collisional systems

You must resolve SINGLE STARS (softening based codes cannot be used)
Solving stellar and binary evolution
NOTE: NO SUB-GRID PHYSICS, BUT RESOLVED PHYSICS

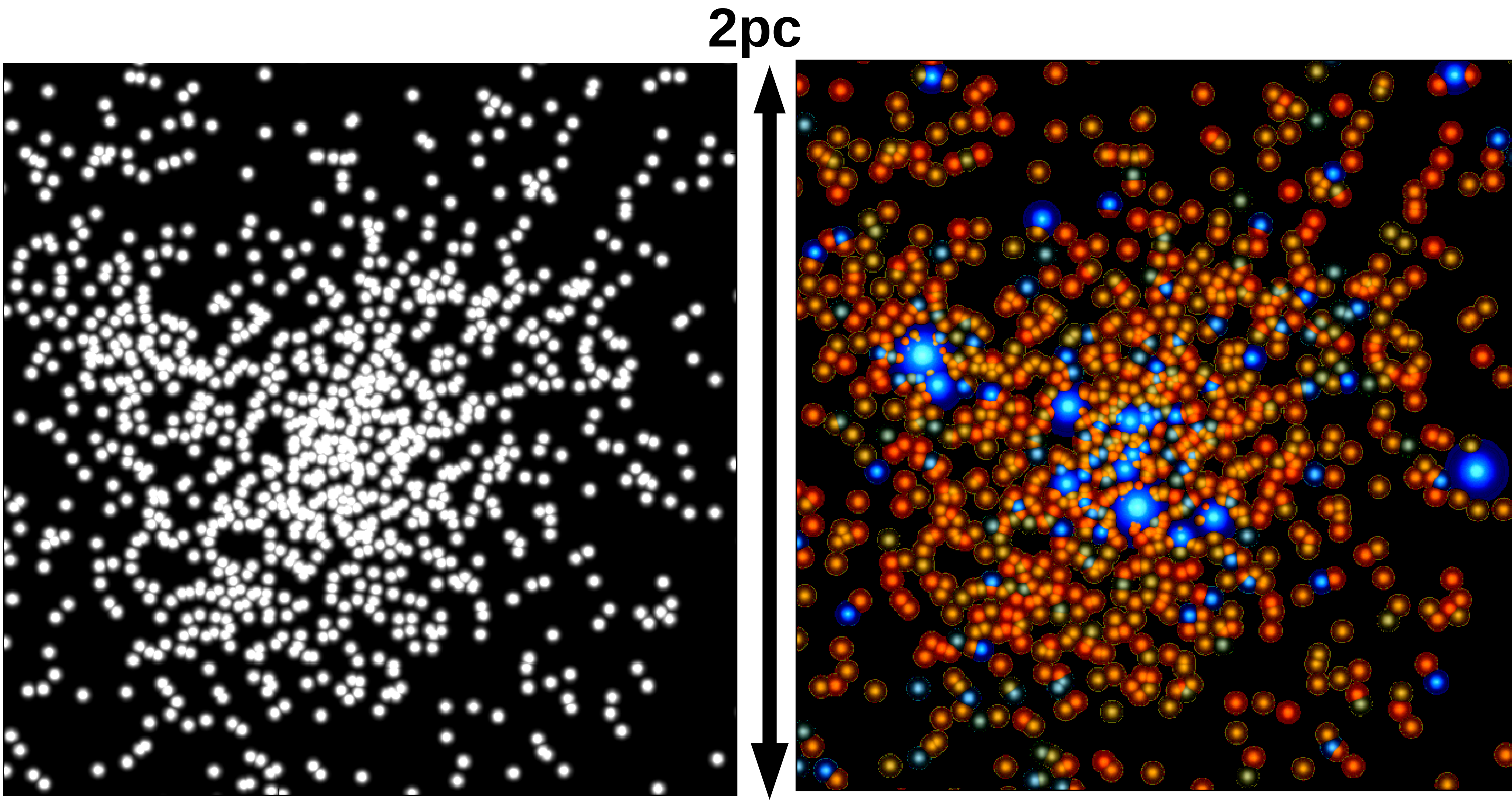
STARS

- Each star has a physical radius, temperature and luminosity (often Hurley, Pols & Tout 2000, MNRAS, 315, 543)
- Can be MS or post-MS (even WR and LBV)
- Mass losses by stellar winds
- Can have METALLICITY
- Can merge with other stars
- Undergoes SN and becomes REMNANT

BINARIES

- Each binary can undergo mass transfer
- rules for circularization
- rules for merger

6) Simulating collisional systems

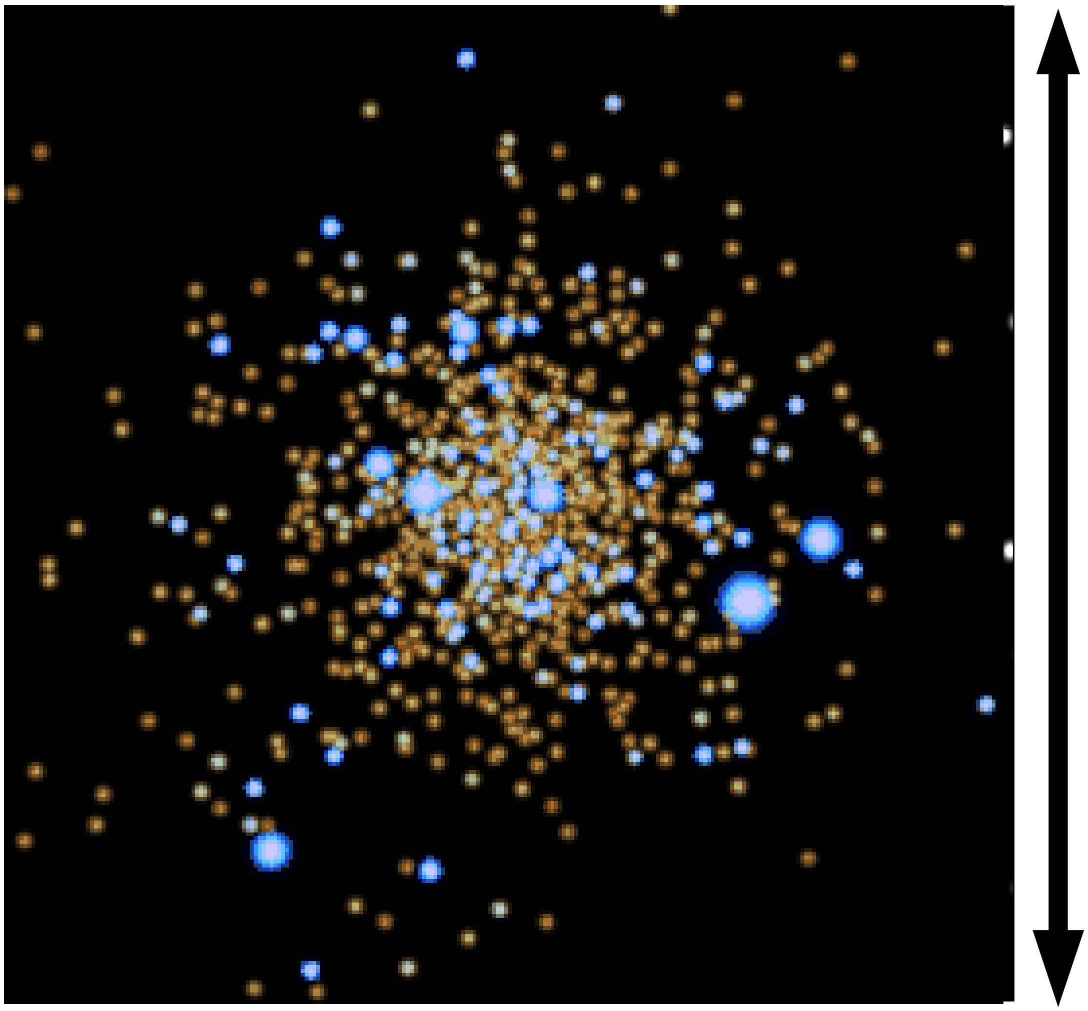


GRAVITY ONLY

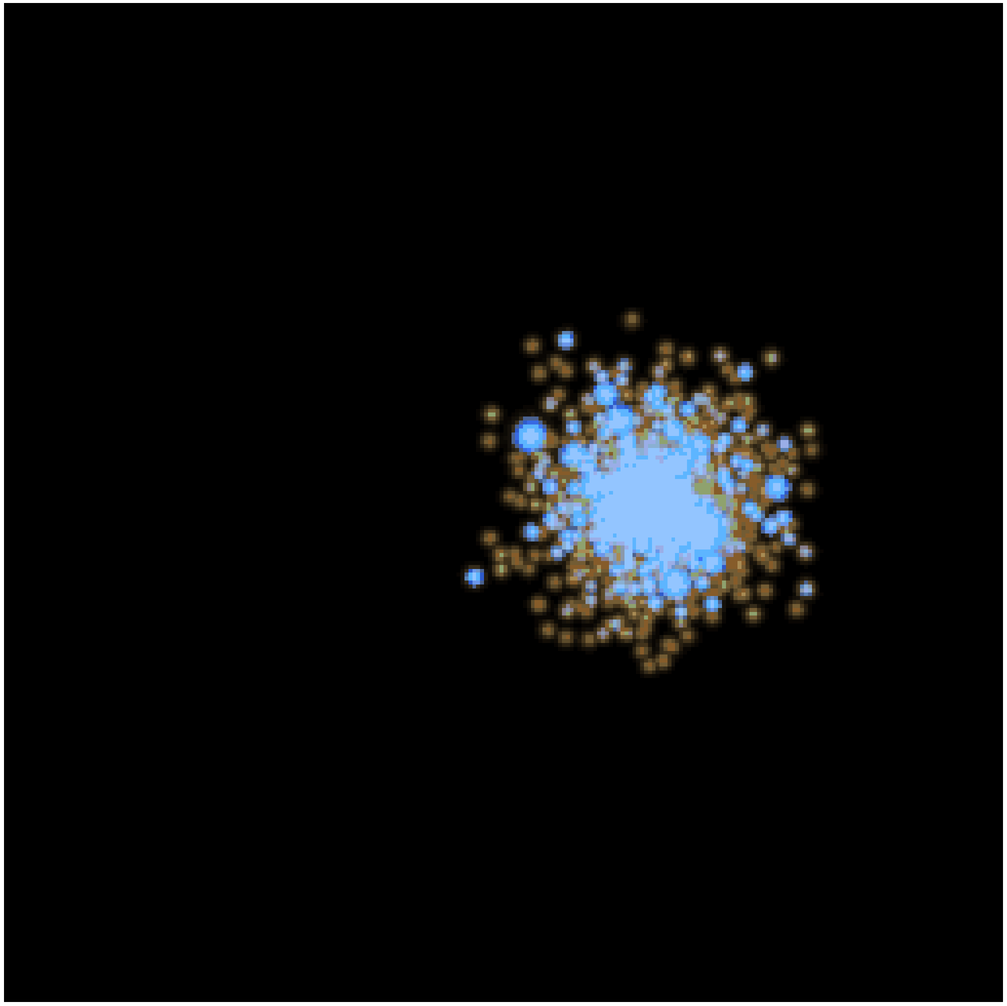
Each star has a physical radius, temperature and luminosity

6) Simulating collisional systems

2pc



ISOLATED

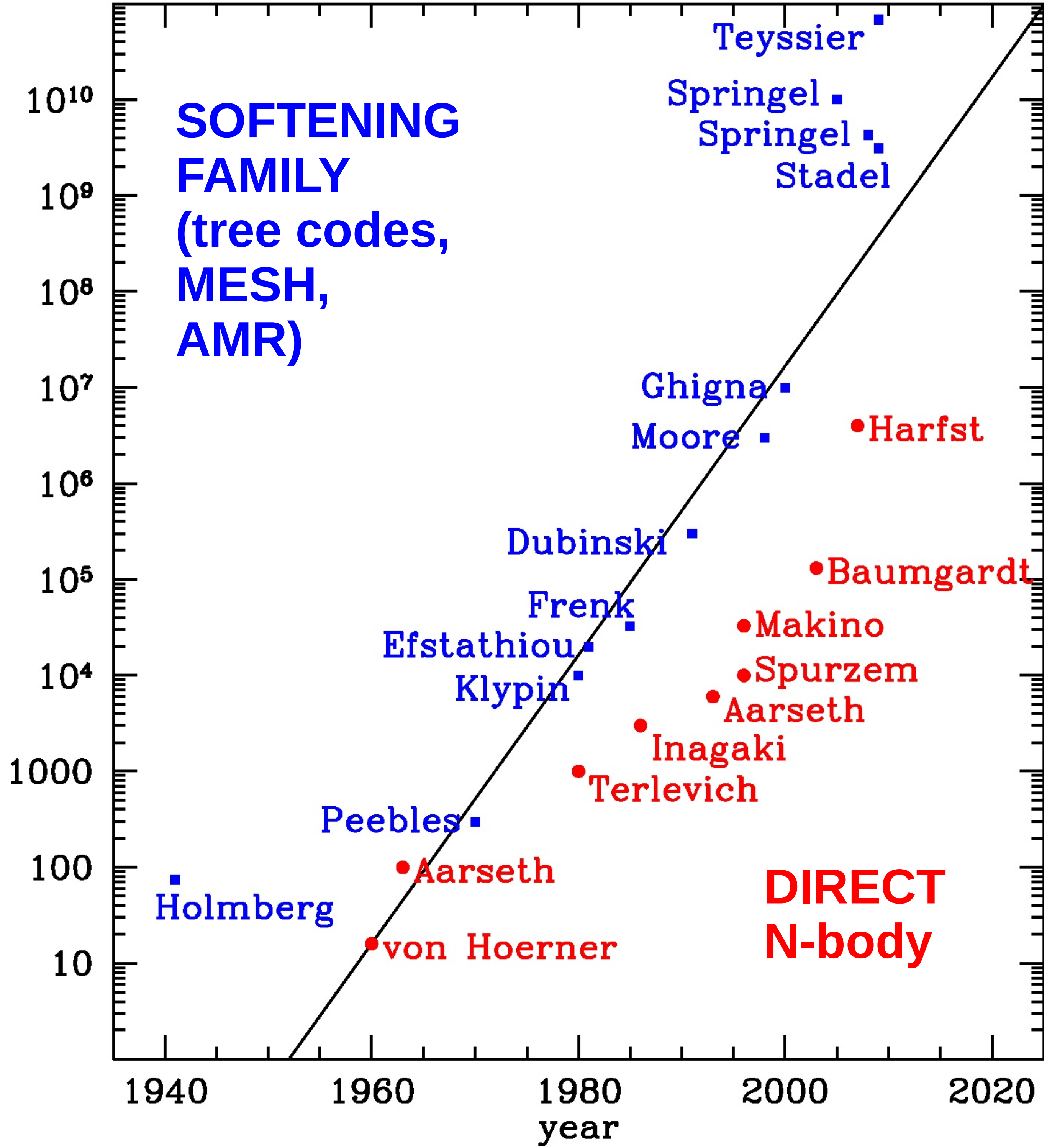


In tidal field

6) Simulating collisional systems

Moore's Law for advances in computational Astrophysics

(from Dehnen & Read 2011, arXiv:1105.1082)

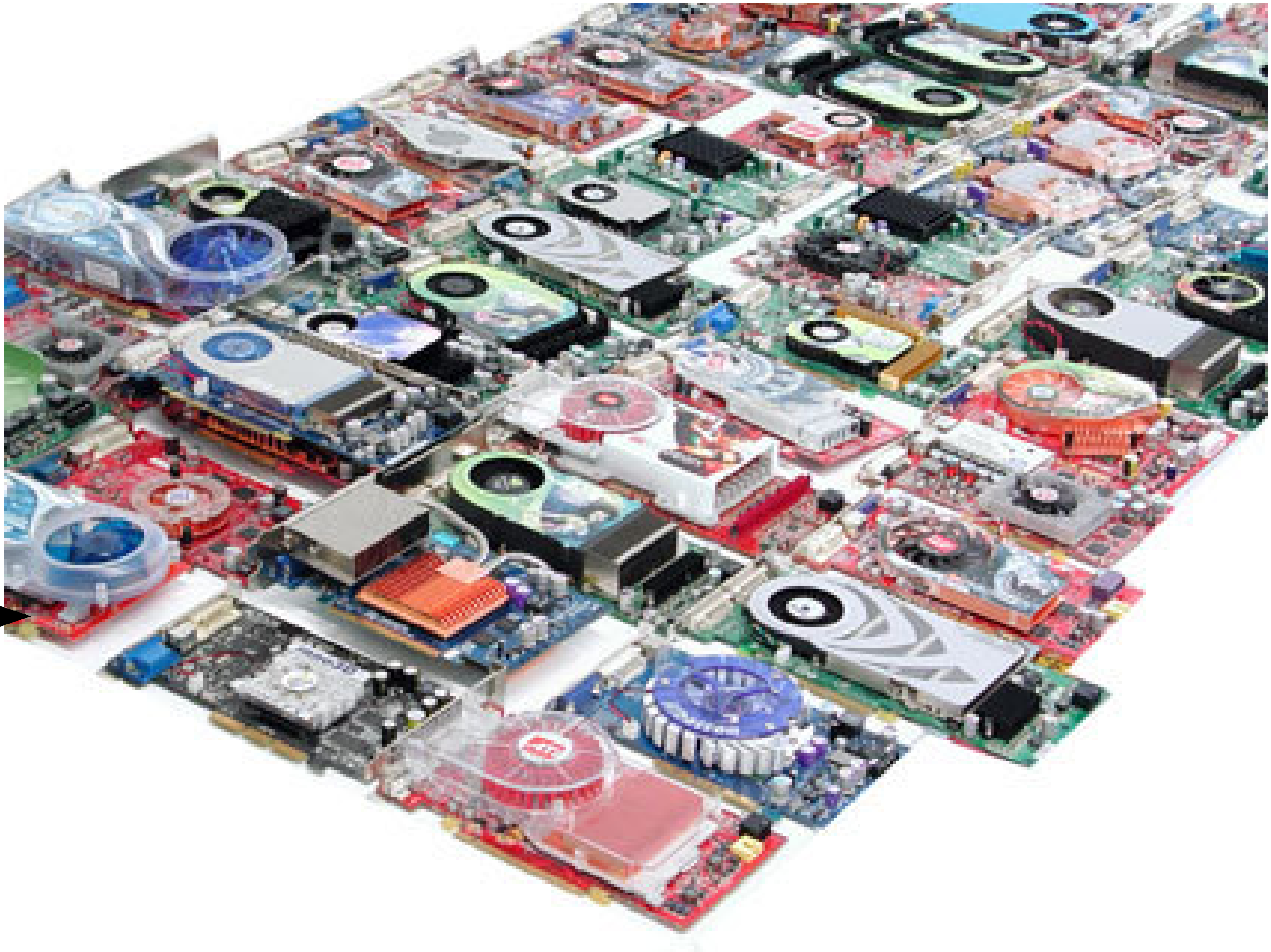


6) Simulating collisional systems: the hardware

GRAPHICS PROCESSING UNITS (GPUs)

Wikipedia's definition: specialized electronic circuit designed to rapidly manipulate and alter memory to accelerate the creation of images in a frame buffer intended for output to a display

Mostly graphics accelerator of the VIDEO CARD, but in some PC are in the MOTHERBOARD



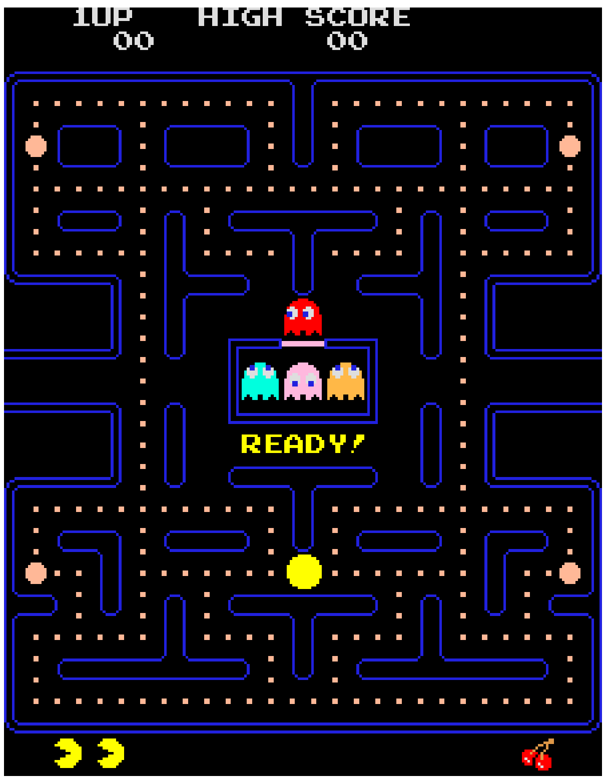
VIDEO CARDS WITH GPUS

6) Simulating collisional systems: the hardware

GRAPHICS PROCESSING UNITS (GPUs)

Born for applications that need FAST and HEAVY GRAPHICS: VIDEO GAMES

BEFORE GPU



AFTER GPU

In ~2004 GPUS WERE FOUND TO BE USEFUL FOR CALCULATIONS: WHY??

6) Simulating collisional systems: the hardware

GRAPHICS PROCESSING UNITS (GPUs)

SIMPLE IDEA:

coloured pixel represented by 4 numbers (R, G, B and transparency)

each pixel does not need information about other pixels (near or far)

→ **when an image must be changed each single pixel can be updated INDEPENDENTLY of the others and SIMULTANEOUSLY to the others**

→ **GPUs are optimized to perform MANY SMALL OPERATIONS (change a single pixel) SIMULTANEOUSLY i.e. MASSIVELY PARALLEL**

THIS IS THE CONCEPT OF **SIMD** TECHNIQUE:

SINGLE INSTRUCTION MULTIPLE DATA

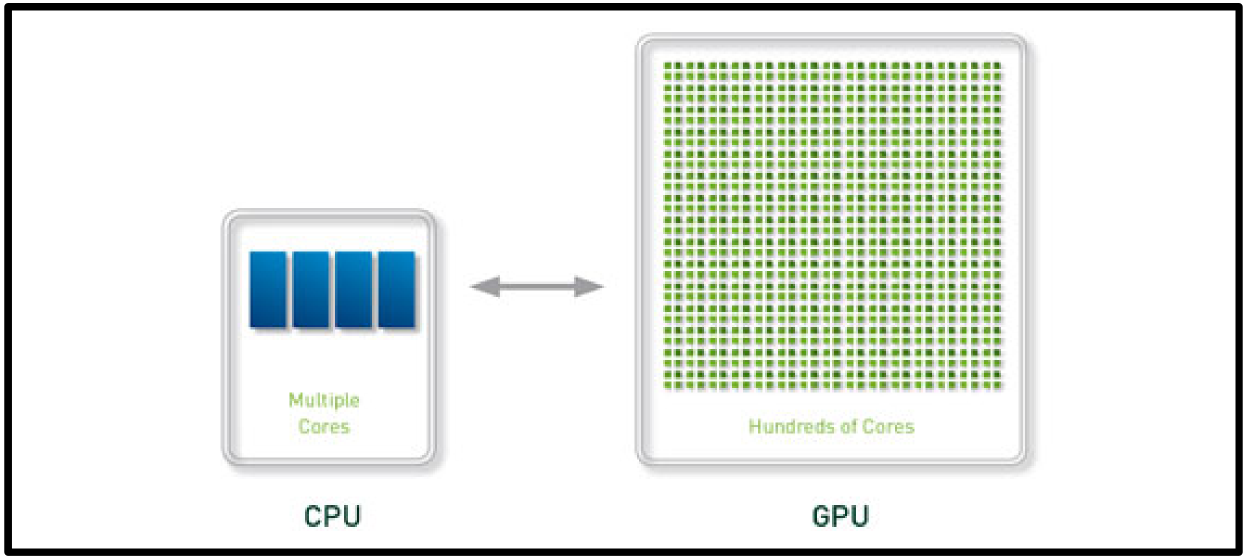
GPUS are composed of many small threads, each able to perform a small instruction (**kerne**l), which is the same for all threads but applied on different data

→ NVIDIA calls it **SIMT**= single instruction multiple **THREAD**

6) Simulating collisional systems: the hardware

SIMD/SIMT TECHNIQUE: SINGLE INSTRUCTION MULTIPLE DATA/THREADS

MANY PROCESSING UNITS PERFORM THE SAME SERIES OF OPERATIONS ON DIFFERENT SUB-SAMPLES OF DATA



Even current CPUs are multiple CORES (i.e. can be multi-threading) but the number of independent cores in GPUs is ~100 times larger!

1M $ QUESTION: WHY IS THIS PARTICULARLY GOOD FOR DIRECT N-BODY CODES?

6) Simulating collisional systems: the hardware

SIMD TECHNIQUE: SINGLE INSTRUCTION MULTIPLE DATA

WHY IS THIS PARTICULARLY GOOD FOR DIRECT N-BODY CODES?

BECAUSE THEY DO A SINGLE OPERATION

(acceleration and jerk calculation)

on MANY PAIRS of PARTICLES

$$\vec{a}_i = G \sum_{j \neq i} \frac{M_j}{r_{ji}^3} \vec{r_{ij}}$$

EACH INTERPARTICLE FORCE BETWEEN A PAIR IS INDEPENDENT OF THE OTHER PAIRS!!

SINGLE INSTRUCTION: ACCELERATION CALCULATION

MULTIPLE DATA: N (N-1)/2 ~ N^2 FORCES

6) Simulating collisional systems: the hardware

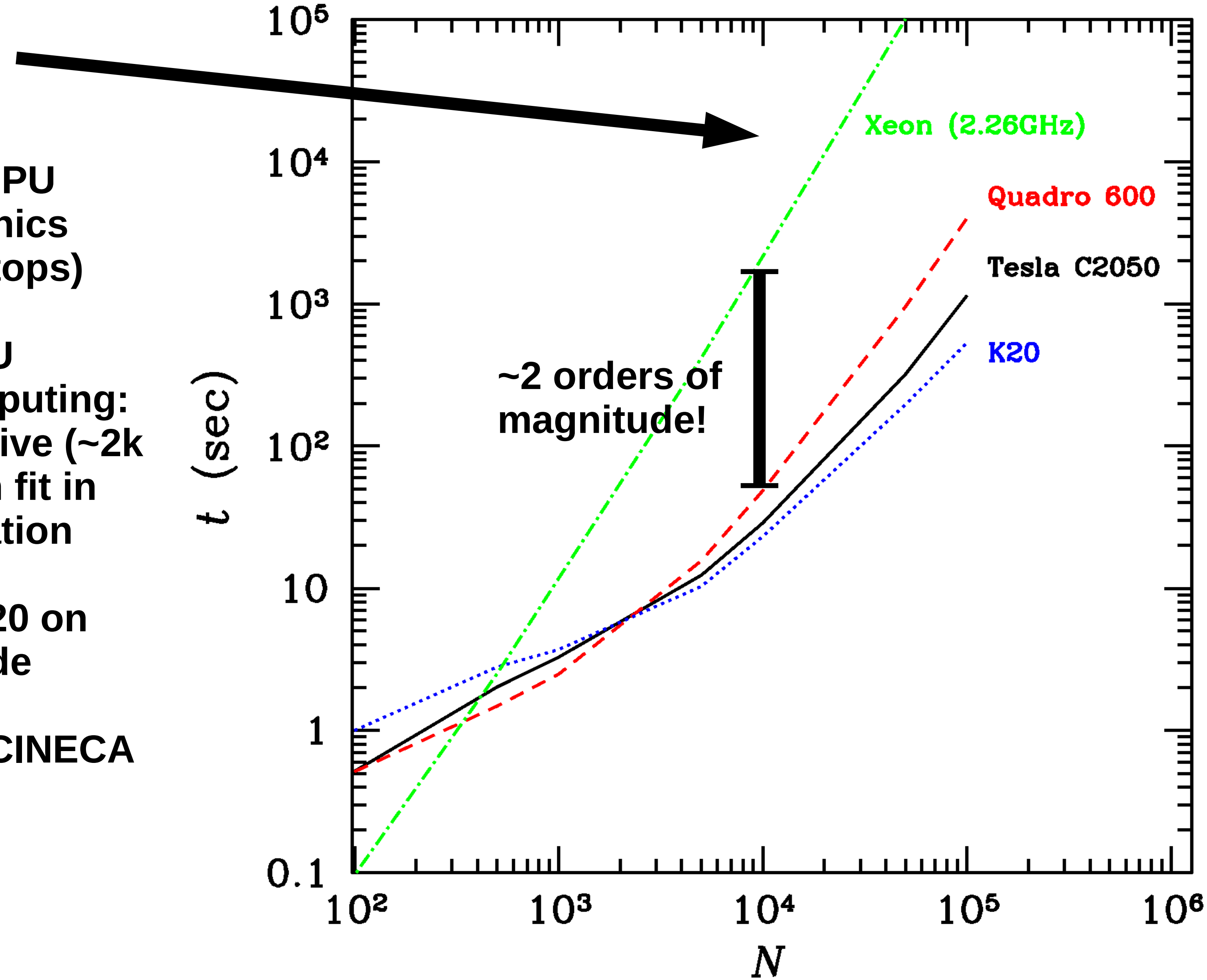
PERFORMANCE TEST:

Single Xeon processor

VS Quadro GPU (typical graphics card of desktops)

VS Tesla GPU
GPU for computing: more expensive (~2k EUR) but can fit in your workstation

VS 2 Kepler 20 on the same node mounted on EURORA @ CINECA



YOU CAN RUN YOUR OWN TESTS @ HOME!

6) Simulating collisional systems

DIFFERENT APPROACH: **Monte Carlo codes**

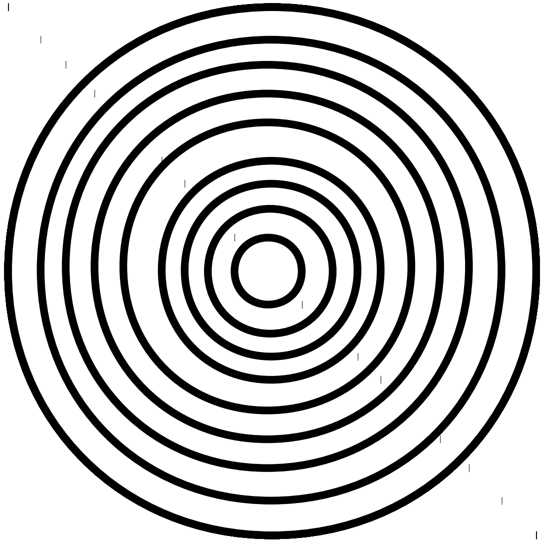
Generate random quantities starting from probability distribution

* Assume dynamical equilibrium (valid for < t_{rlx})

* Assume **spherical symmetry** (each particle is a shell of mass *m*)

Steps in Monte Carlo calculations:

1. Initialize positions and velocities, compute *E*, *L*
2. Order the particles by radius, and compute gravitational potential (in spherical symmetry)
3. Compute effects of two-body encounters
4. Calculate new *E, L* for all particles
5. Reassign radii of all particles
6. Repeat from 2



HYBRID CODES (cfr. Blue straggler stars)

- add N-body code for 3-body encounters
- add stellar and binary evolution

TIME COMPLEXITY as *N* ln *N* !!!! Good **up to 10^7 stars**

7) Three-body and planets

EXCHANGES are important in PLANETARY SYSTEMS, even in the SOLAR SYSTEM (the one we observe better)

Neptune and Triton is one of the most fascinating binaries of our system:

Triton mass ~ 0.004 Earth mass

Neptune mass ~17 Earth mass (~5000 Triton's)

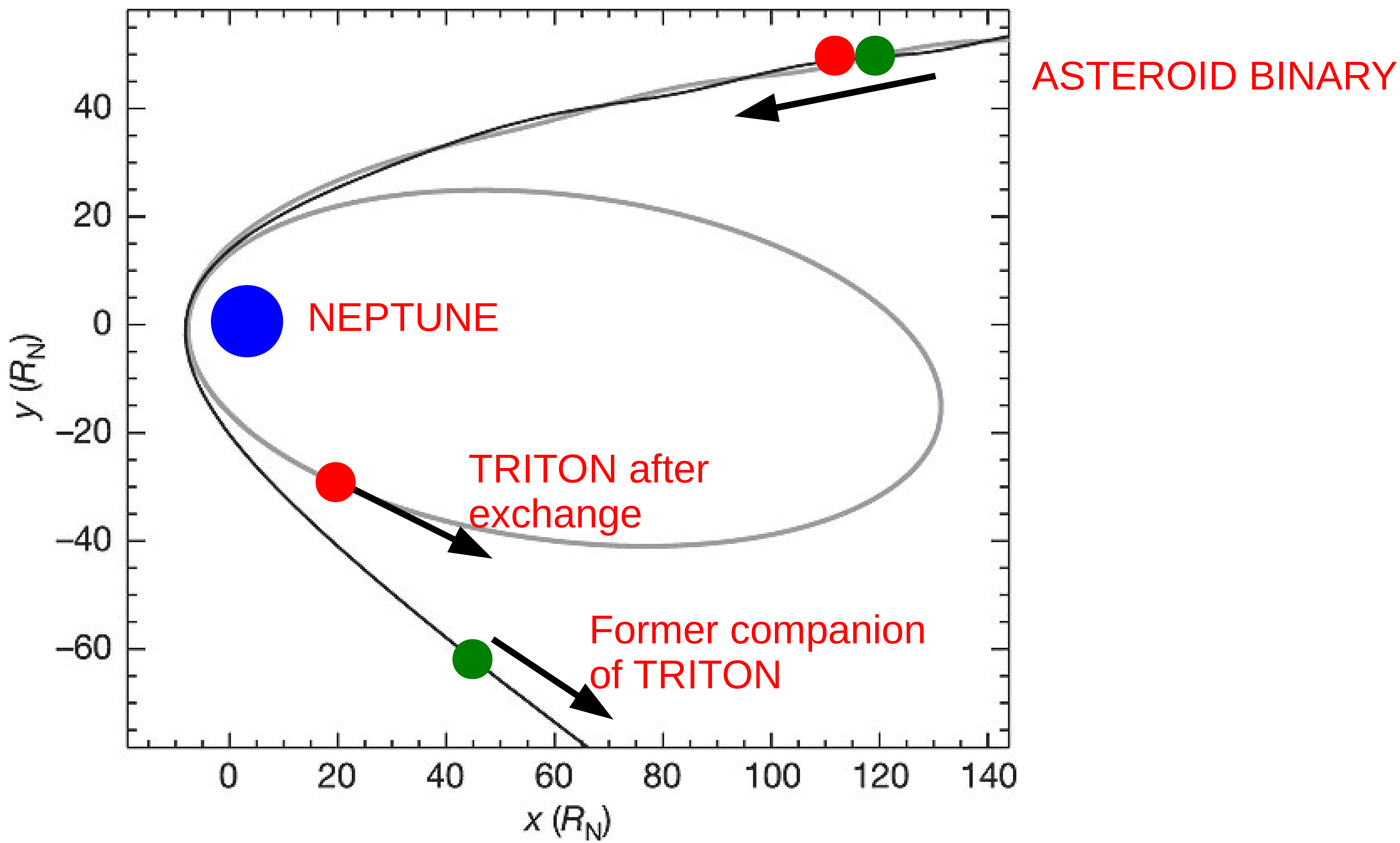
Triton orbit is **retrograde** (versus Neptune rotation)

Tidally captured by Neptune? Low probability Because Triton can hardly survive a tidal interaction

Asteroid BINARY FRACTION is quite HIGH (~10%) → **Agnor & Hamilton (2006, Nature, 441, 192)** propose EXCHANGE between a binary of small bodies (including Triton) and Neptune

7) Three-body and planets

Agnor & Hamilton (2006, Nature, 441, 192) propose EXCHANGE between a binary of small bodies (including Triton) and Neptune



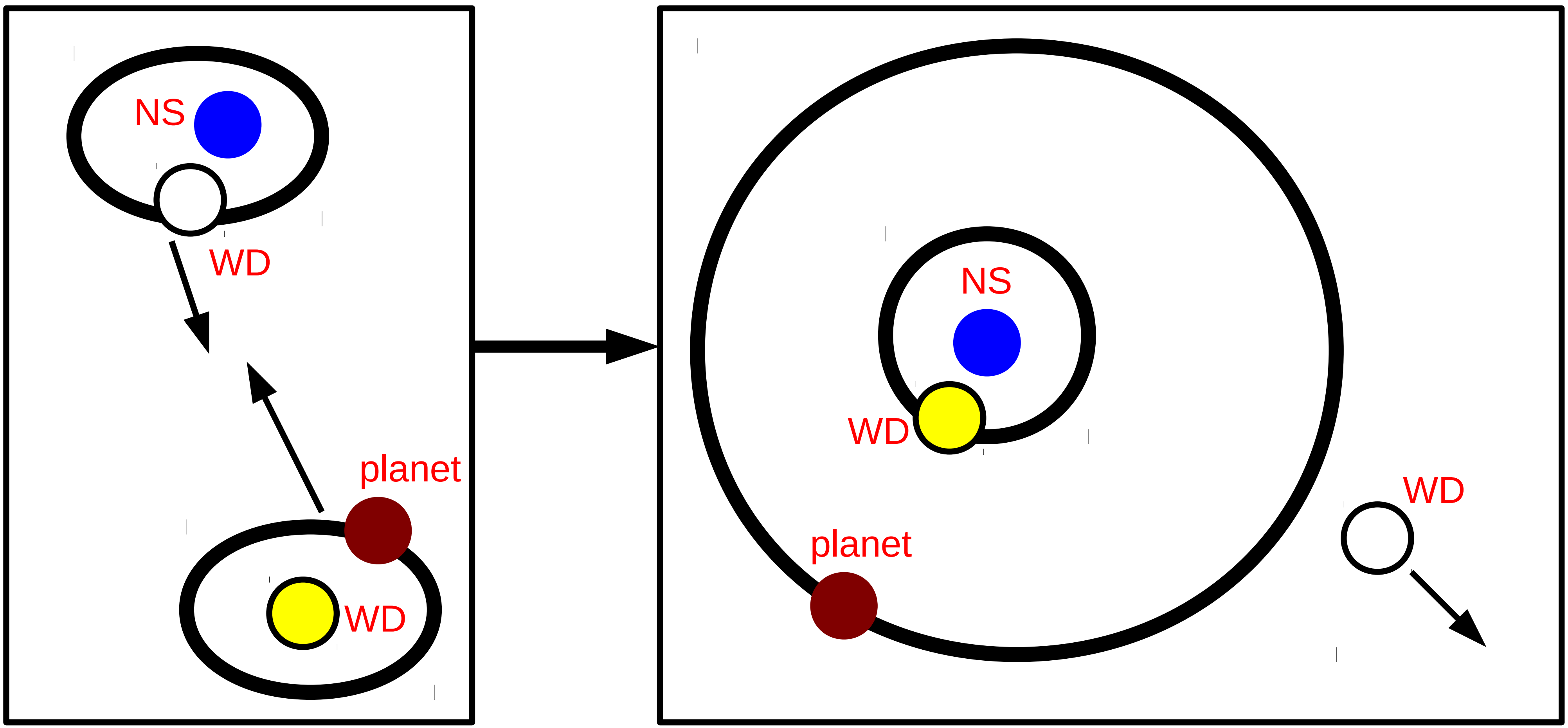
More likely than tidal capture of single body
if Triton-like binary fraction > 3×10^{-4}

7) Three-body and planets

Other important case: the PLANET in the TRIPLE SYSTEM in M4
MILLISECOND PULSAR B1620-26 [very good clock for planets!]
+WHITE DWARF (0.3-0.5 Msun, Period 191 days)
+ circumbinary planet

(e.g. Sigurdsson et al. 2003, Science, 301, 193)

system shows anomalies (e.g. pulsar period high-order derivatives) that cannot be explained if system was primordial → Needs 4-body encounter



8) Nuclear star clusters

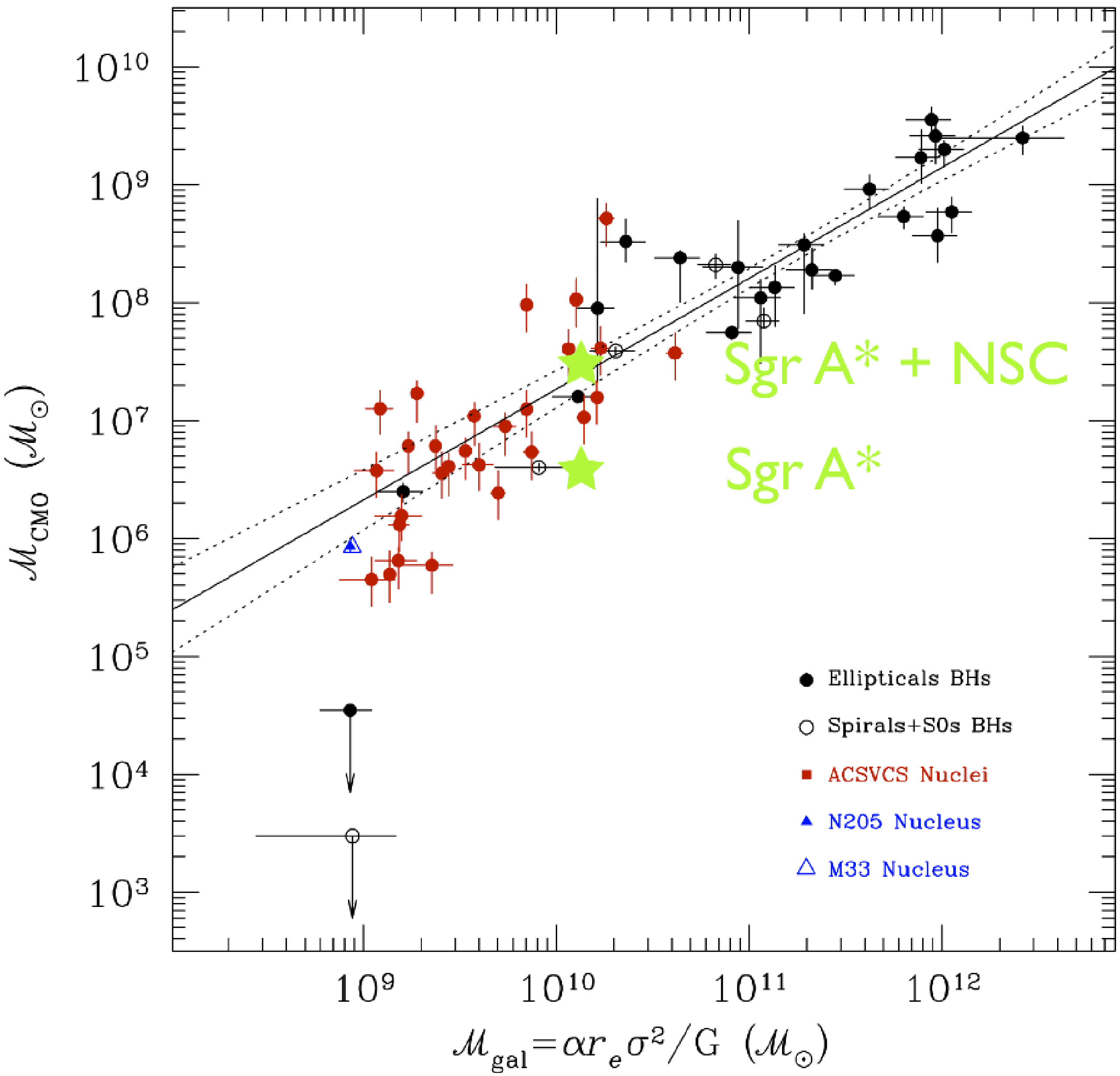
Very open topic!

* more massive than globular clusters (M>10^6 Msun)

* MULTIPLE POPULATION (<1 Gyr up to 13 Gyr)

* in lower-mass spheroids than SMBHs, but sometimes COEXISTENT with the SMBH

* obey SCALING RELATIONs as SMBHs



Schoedel 2010, arXiv:1001.4238

8) Nuclear star clusters

* Their formation is a mistery:
  Collision of star clusters sunk to the centre by DYNAMICAL FRICTION?

  In situ formation by gas clouds in different accretions?

* They are COLLISIONAL SYSTEMS!
  As far as SMBHs are not included (increase local velocity field)

* If there are SMBHs, nuclear star clusters are still COLLISIONAL OUT OF SMBH INFLUENCE RADIUS (inside SMBH dominates gravity)

$$r_{BH} = \frac{G\, m_{BH}}{\sigma^2} = 1.7\,\mathrm{pc}\,\left(\frac{m_{BH}}{10^6\,M_\odot}\right)\,\left(\frac{50\,\mathrm{km\ s}^{-1}}{\sigma}\right)^2$$

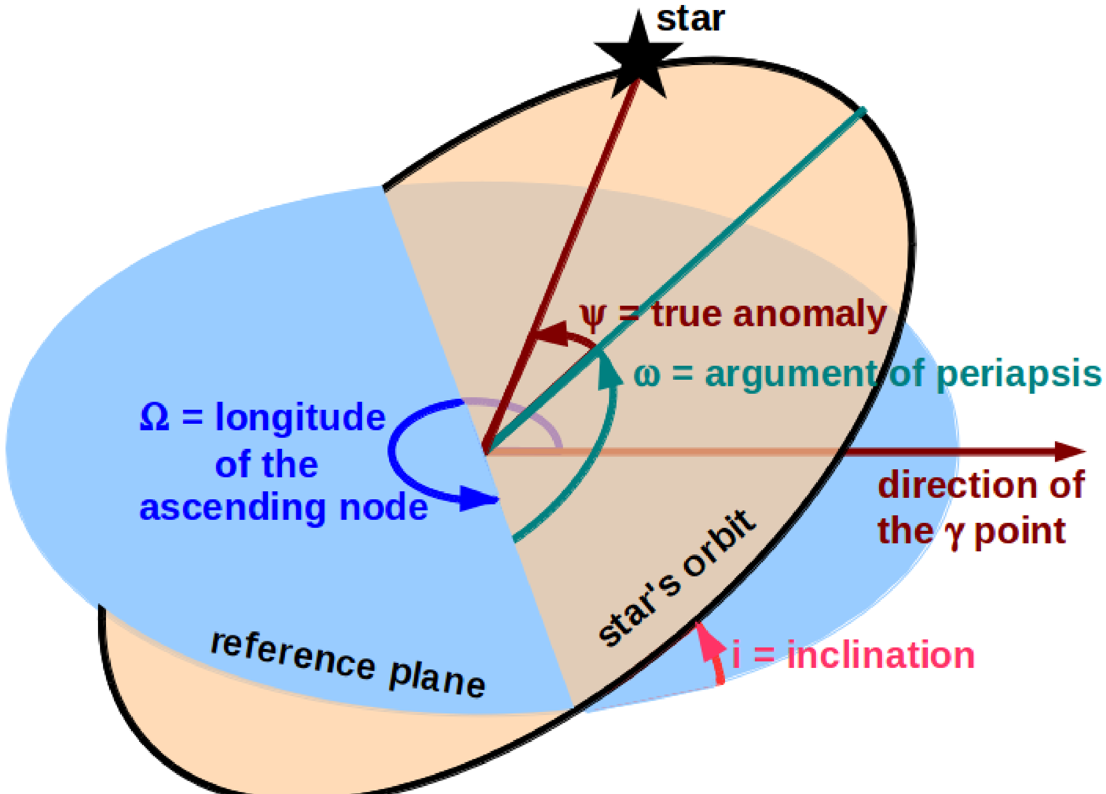
* They can enhance GW events, X-ray sources, **ejection of hypervelocity stars**, ….

8) Nuclear star clusters: precession

They are COLLISIONAL SYSTEMS!
BUT WITH DIFFERENT PROCESSES INVOLVED with respect to other star clusters:

NEWTONIAN PRECESSION(s)

A star orbiting the SMBH can be described as in Keplerian motion around the SMBH plus an EXTERNAL POTENTIAL (= the old stellar cusp, the other young stars, the CNR)
The external induces PRECESSION



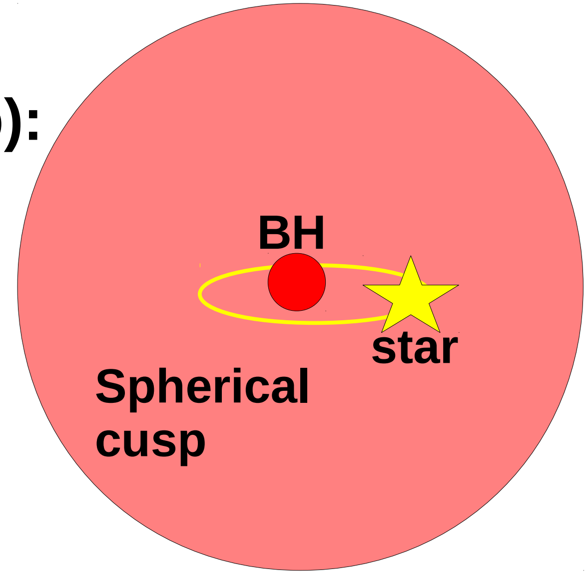
Precession can affect:

- argument of periapsis
- longitude of asc. node
- inclination
- eccentricity

Depending on the structure of the external potential

8) Nuclear star clusters: precession

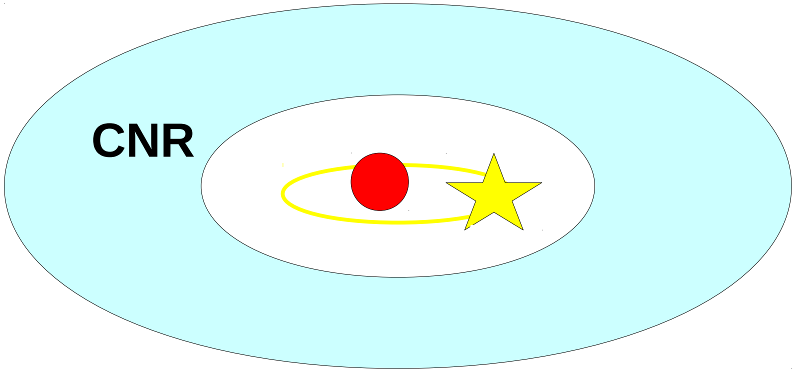
- **SPHERICAL POTENTIAL (e.g. spherical stellar cusp):**



Timescale

$$T_{cusp} = \frac{M_{BH}}{M_{cusp}(a)} P_{orb}\, f(e)$$

Only argument of pericentre



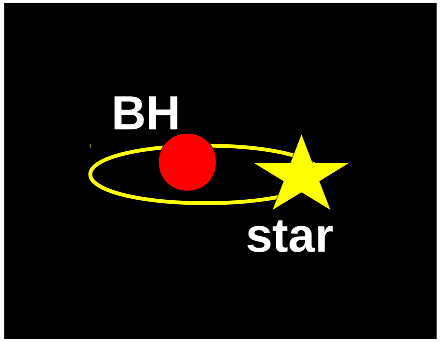
- **AXISYMMETRIC POTENTIAL (e.g. stellar or gas ring)**

Timescale

$$T_K = \frac{M_{BH}}{M_{DISC}} \frac{R^3_{DISC}}{a^{3/2}\sqrt{G\, M_{BH}}}$$

- **if i~0 only longitude of ascending node**
- **if i>>0 also inclination and eccentricity are affected**

8) Nuclear star clusters: precession



RELATIVISTIC PRECESSION:
precession of orbits in general relativity

Caused by the SMBH mass, even if there are no external potentials
Three types (Schwarzschild prec. + 2 precession effects that depend on spin)

Schwarzschild precession (lowest order correction to Newton):

$$T_{\rm RP} = 1.3 \times 10^3 \ {\rm yr} \ \left(1 - {\rm ecc}^2\right) \ \left(\frac{r}{0.001 \ {\rm pc}}\right)^{5/2} \ \left(\frac{4 \times 10^6 \ {\rm M}_\odot}{M_{\rm BH}}\right)^{3/2}$$

- **affects only argument of pericentre**
- **efficient for very small semi-major axis**
- **more efficient for high eccentricity**
- **more efficient for large BH mass**

8) Nuclear star clusters: precession

- relativistic precession important only if a<<0.1 pc

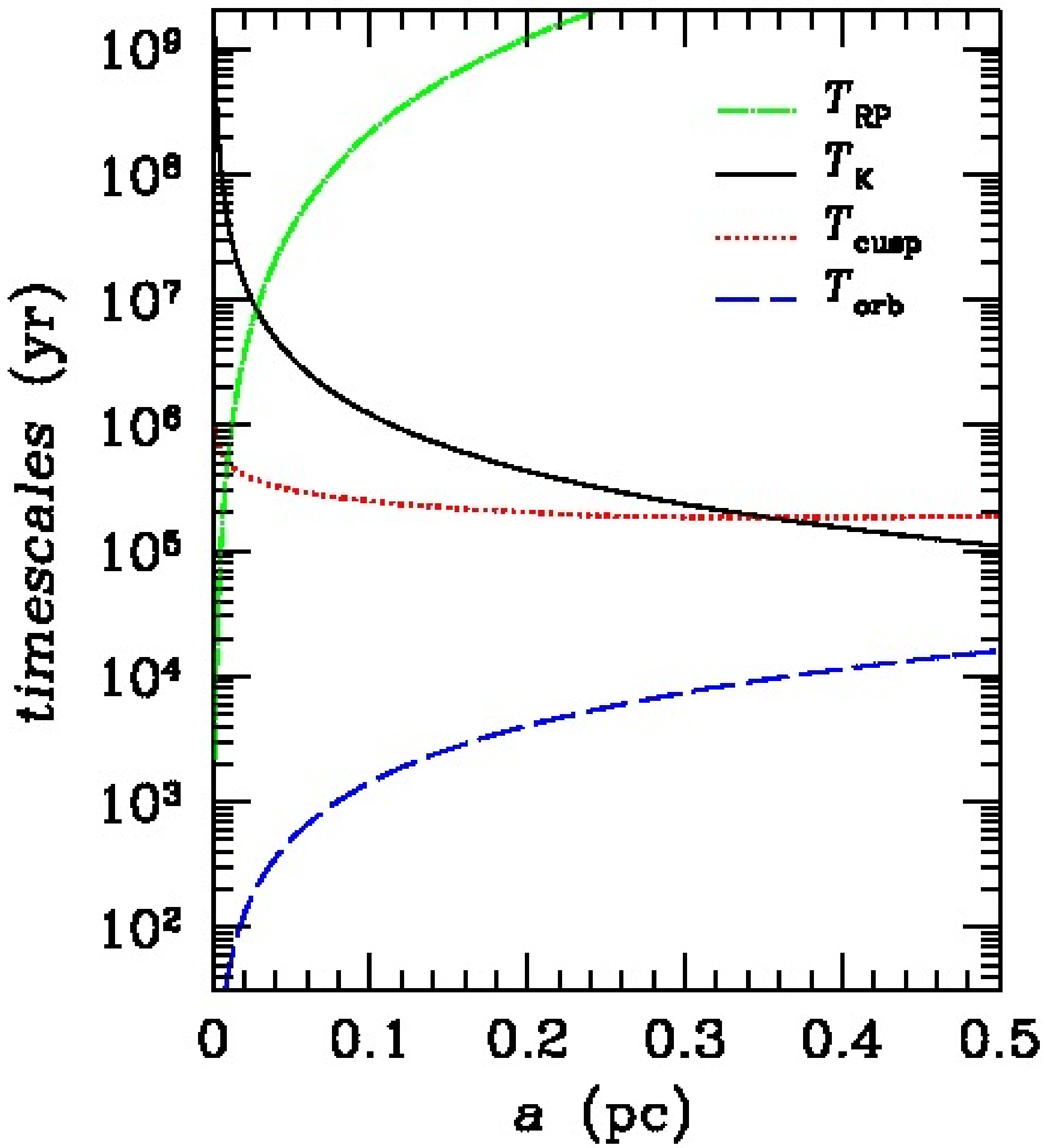
- spherical cusp important at <0.3 pc

- disc important at >0.3

↓

IF SPHERICAL POTENTIAL DOMINATES over AXISYMMETRIC ($T_{cusp} << T_K$),

then only precession of argument of pericentre and of longitude of asc. node are not damped



MM, Gualandris & Hayfield 2013

8) Nuclear star clusters: relaxation

TWO-BODY RELAXATION: changes ENERGY

$$T_{\rm rlx} = 0.34 \times \frac{\sigma^3}{G^2\, m_*\, \rho_*\, \ln\Lambda},$$

RESONANT RELAXATION: changes ECCENTRICITY
NO ENERGY

$$T_{\rm RR} = 10^4\ {\rm yr}\ \left(\frac{r}{0.001\ {\rm pc}}\right)^{3/2} \sqrt{\frac{M_{\rm BH}}{3\times10^6\ {\rm M}_\odot}}\ \left(\frac{10\ {\rm M}_\odot}{m_*}\right)\ \sqrt{\frac{10^3}{N_*}}$$

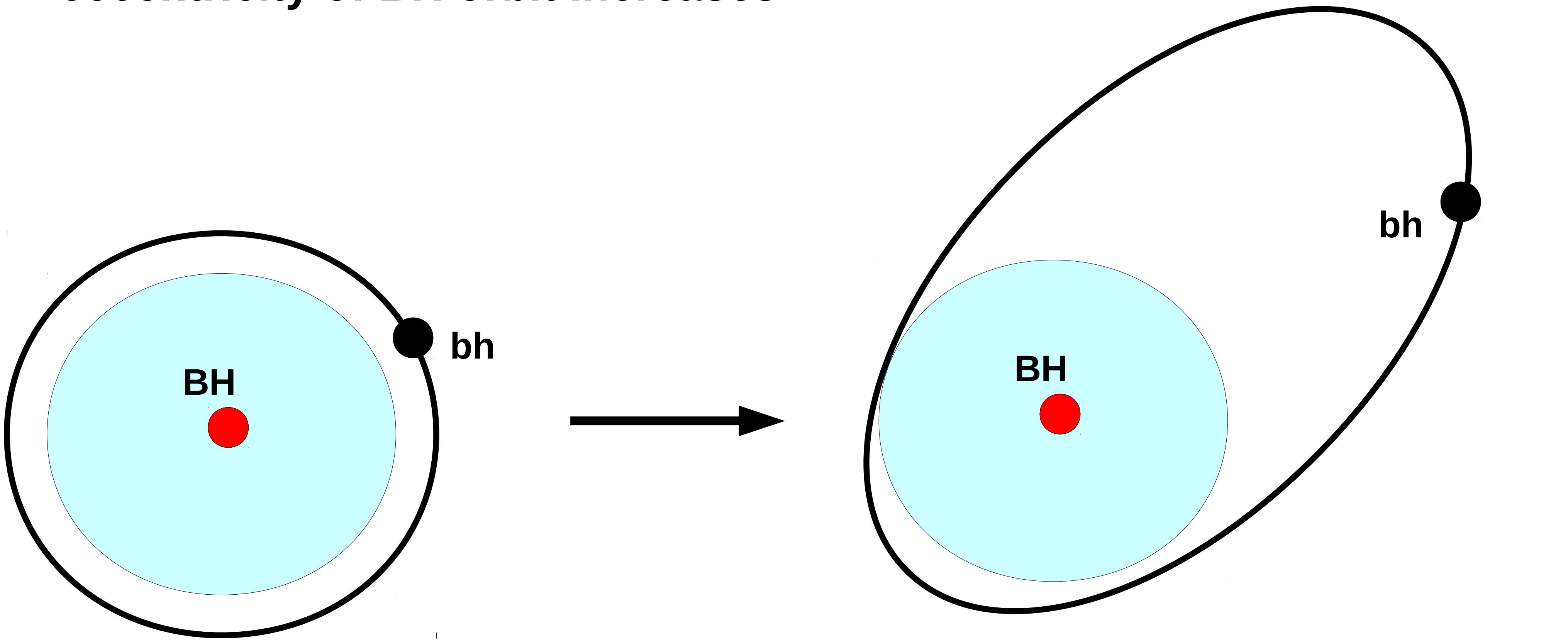
8) Nuclear star clusters: relaxation

RESONANT RELAXATION

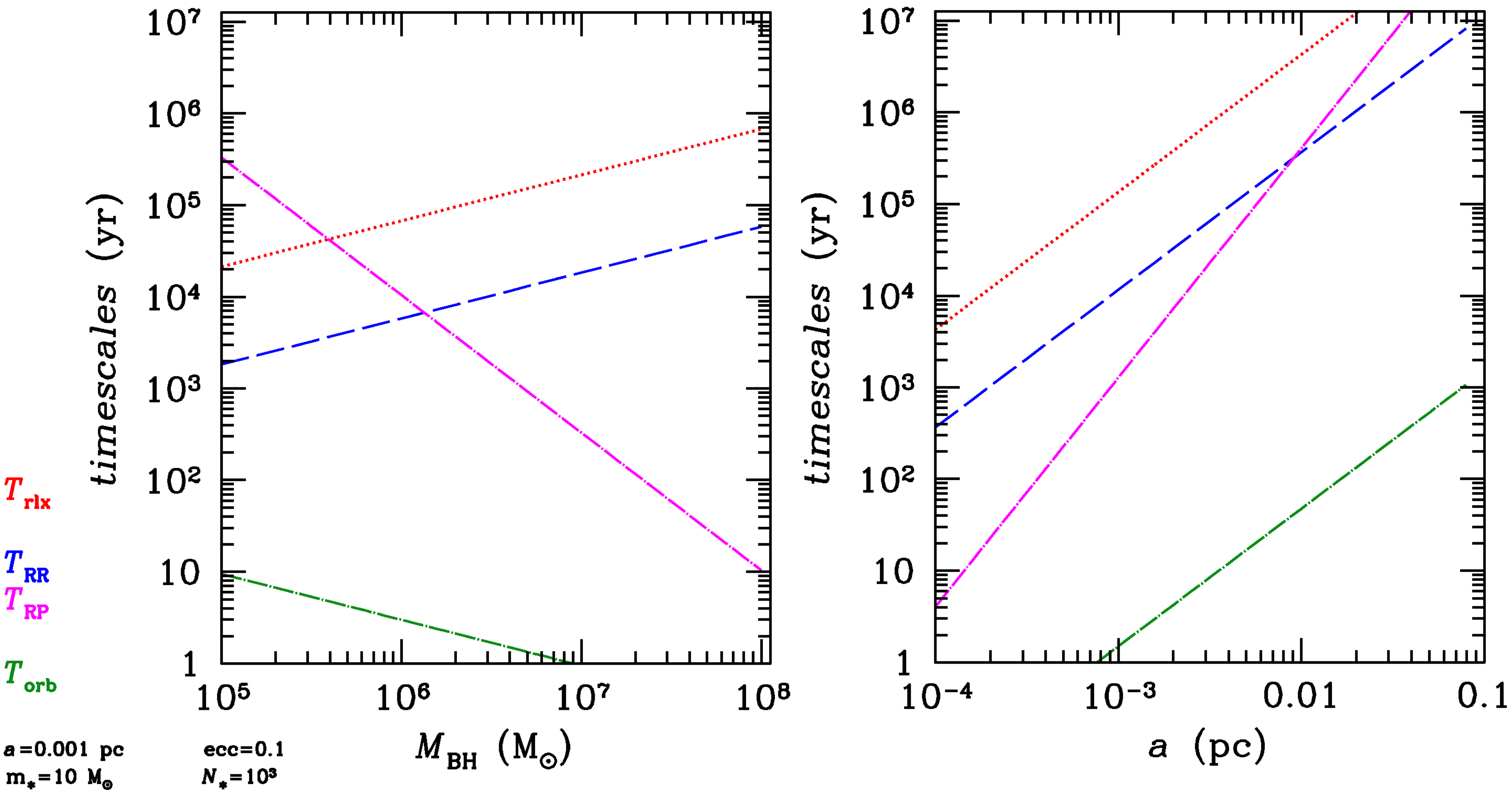
Stars orbiting between the SMBH and the stellar BH exert TORQUES

such torques REDUCE ANGULAR MOMENTUM, not energy

→ eccentricity of BH orbit increases



8) Nuclear star clusters: relaxation + precession



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* Hurley, Pols & Tout 2000, MNRAS, 315, 543

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* Agnor & Hamilton 2006, Nature, 441, 192