

LECTURES on COLLISIONAL DYNAMICS:

4. HOT TOPICS on COLLISIONAL DYNAMICS

Part 1



- 1) IMBHs: runaway collapse, repeated mergers, ...**
- 2) BHs eject each other?**
- 3) Effects of 3-body on X-ray binaries (formation and escape)**
- 3b) Gravitational waves**
- 4) Effect of metallicity on cluster evolution**
- 5) Formation of blue straggler stars**
- 6) Tools for numerical simulations of collisional systems**
- 7) Three-body and planets**
- 8) Nuclear star cluster dynamics**

1) Intermediate-mass BHs (IMBHs)

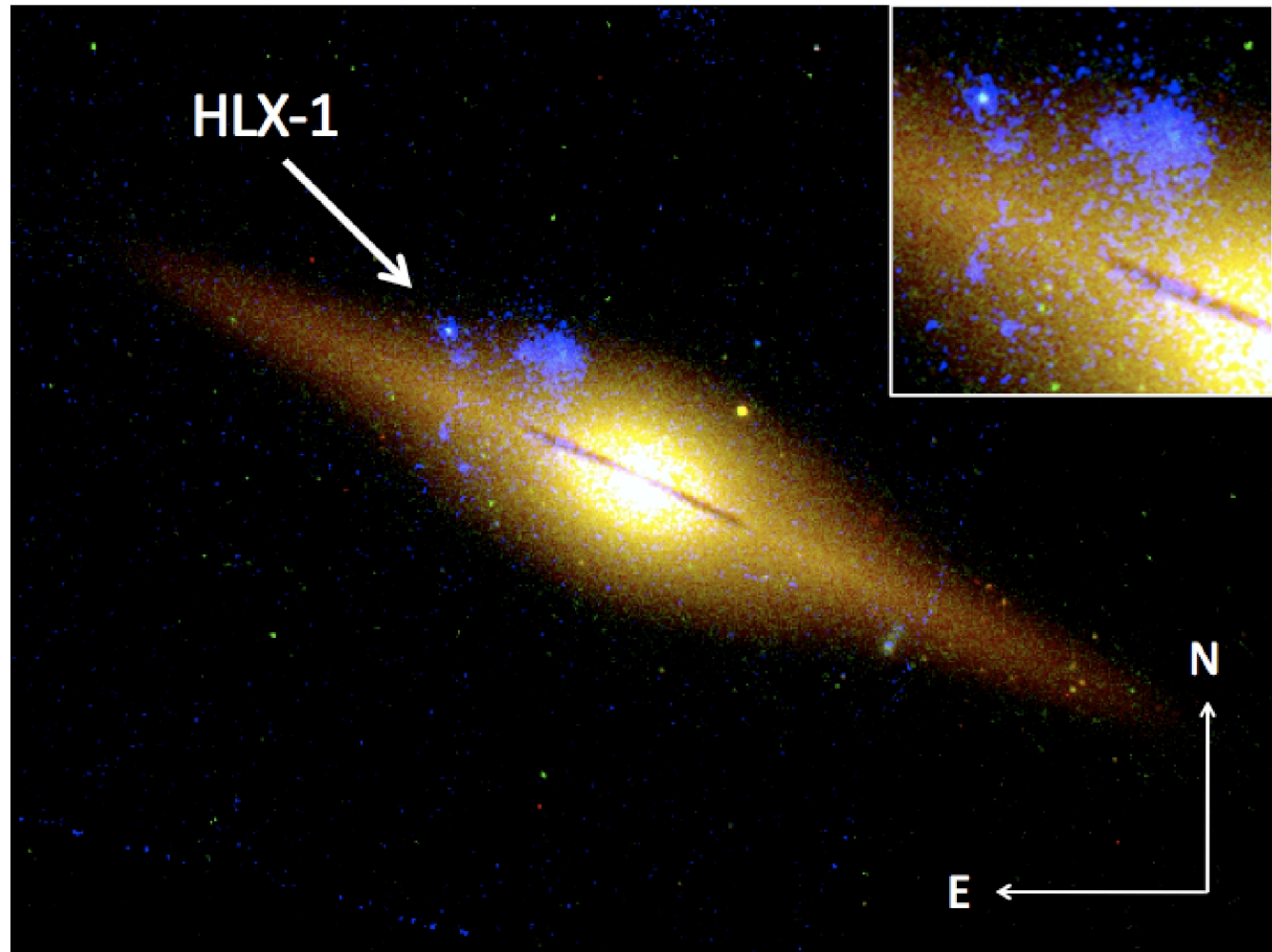
definition: BHs with mass $10^{2-5} M_{\odot}$

OBSERVATIONAL EVIDENCES: none, just hints

1* Hyperluminous X-ray source HLX-1 close to ESO 243-49

peak $L_x \sim 10^{42}$ ergs,
X-ray VARIABILITY,
redshift consistent
with ESO 243-49
(not a background object)
→ BH mass $\sim 10^4 M_{\odot}$

Farrell+ 2009, 2012;
Soria+ 2010, 2012;
MMi+ 2012, 2013

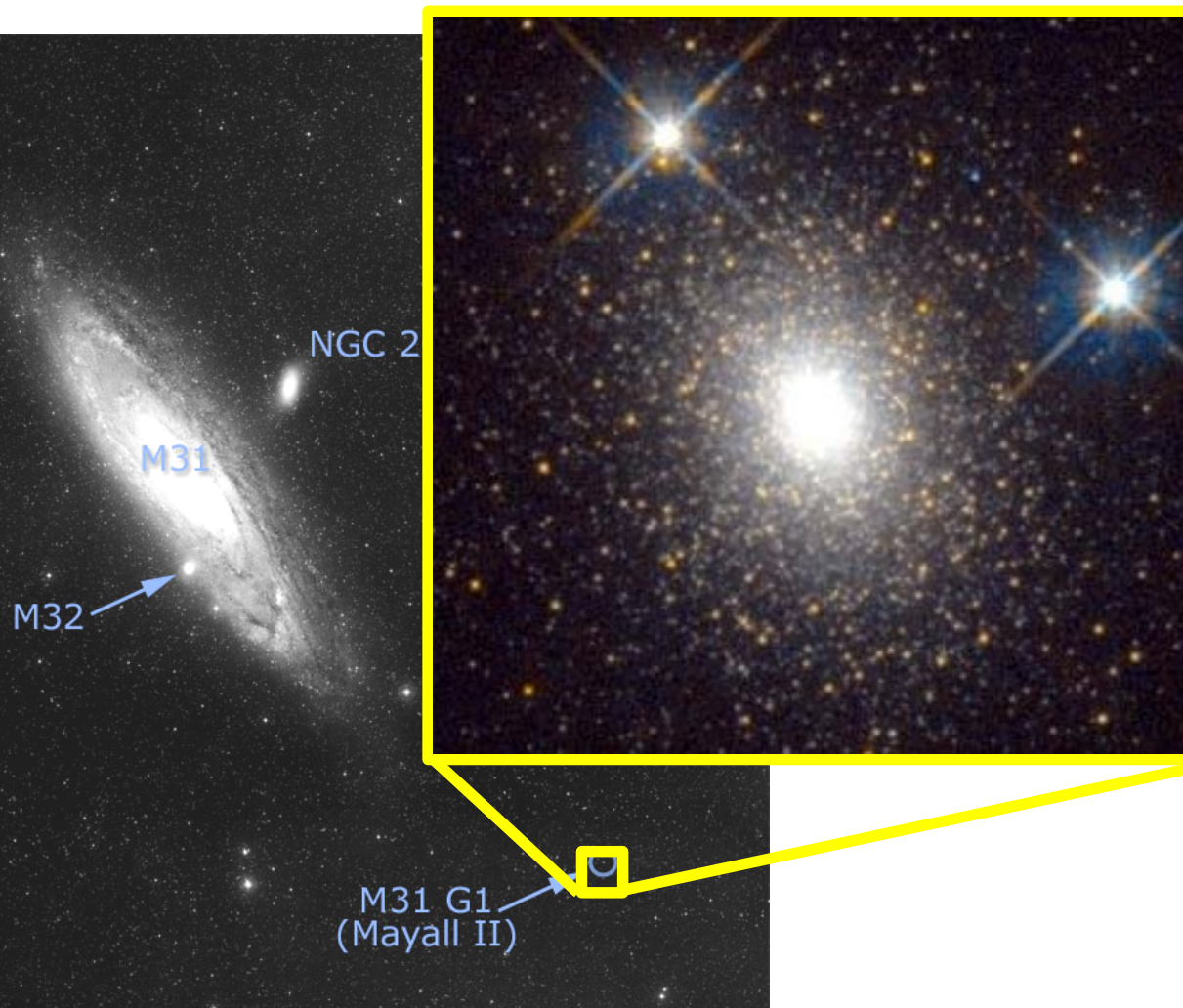
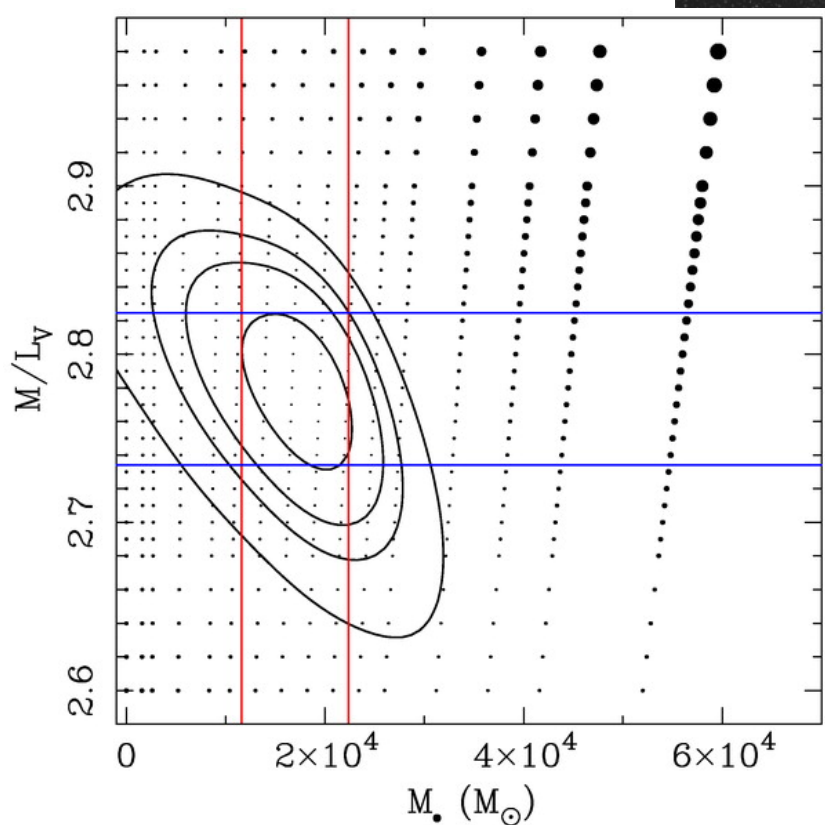


1) Intermediate-mass BHs (IMBHs)

definition: BHs with mass 10^2 - $10^5 M_{\odot}$

2* centre of G1 globular cluster (dwarf nucleus?) in Andromeda

Central velocity distribution
+central M/L ratio
suggest BH mass $\sim 10^4 M_{\odot}$



1) Intermediate-mass BHs (IMBHs)

How do IMBHs form?

- 1- runaway collapse of stars at centre of star cluster (for systems with core collapse time $<$ evolution of massive stars – e.g. young dense star clusters)
- 2- repeated mergers of BHs at centre of star cluster (for systems with core collapse time \gg evolution of massive stars – e.g. globular clusters)
- 3- remnants of extremely metal poor stars (independent of environment)
- 4- low mass end of super-massive BHs (needs gas physics \rightarrow different PhD course)

1) Intermediate-mass BHs (IMBHs)

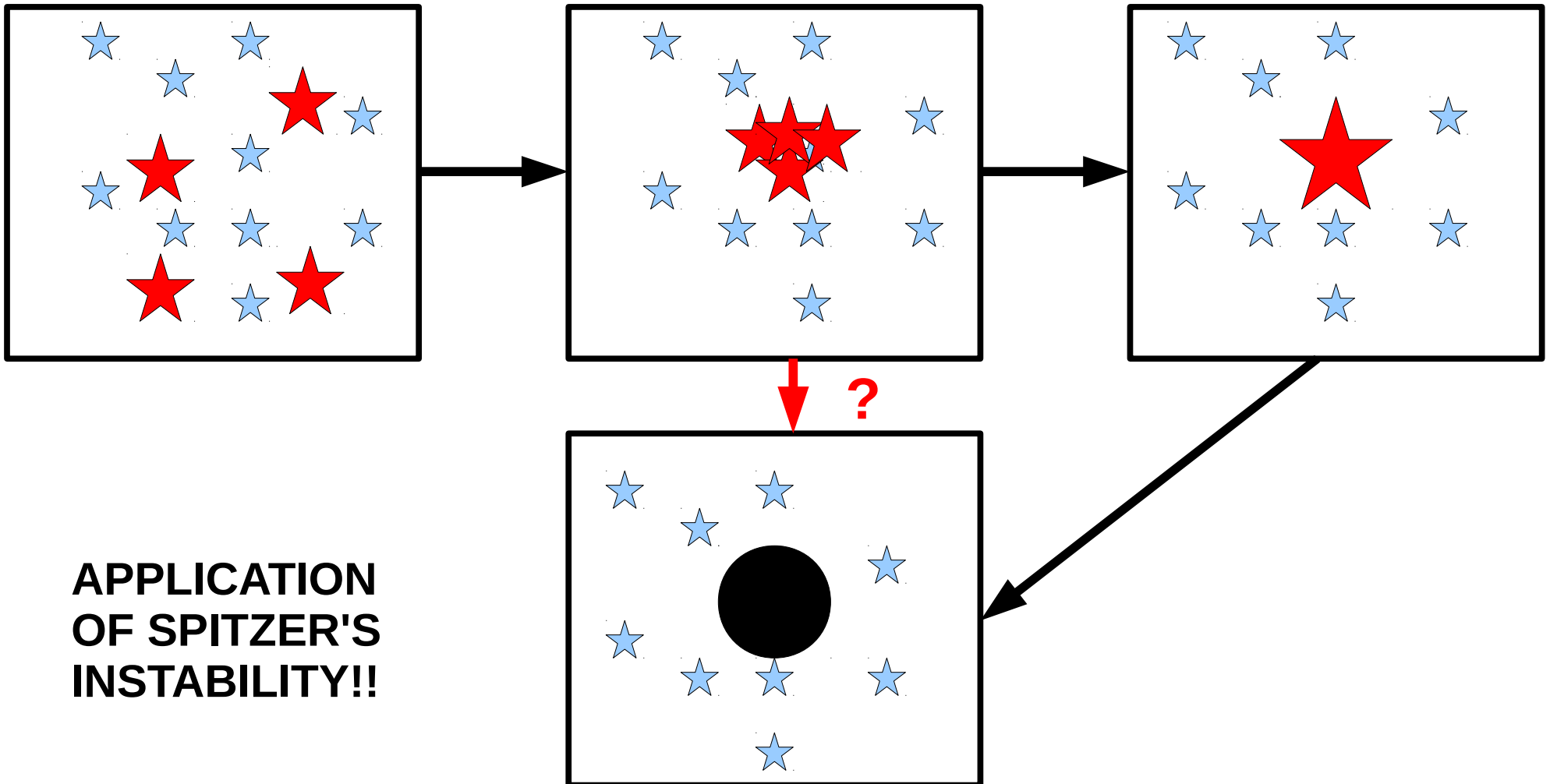
1- runaway collapse of stars at centre of star cluster

IDEA: mass segregation brings very massive stars to the centre

If timescale for mass segregation < timescale for stellar evolution

+ if encounter rate sufficiently high

Massive stars collide, merge and form a super-massive star, which collapses to a BH



1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster
Formalism by Portegies Zwart & McMillan 2002

IDEA: very hard binaries sink to the centre and likely collide with other stars/binaries unless they are ejected.

The product of the first collisions is SO MASSIVE that it triggers other collisions (=is the main collision target)
Starting a RUNAWAY PROCESS

→ Maximum mass that can be grown in a dense star cluster
If all collisions involve the same star

$$\frac{dM_{\text{runaway}}}{dt} = R_{\text{coll}} \delta m_{\text{coll}}$$

Where R_{coll} = collision rate, δm_{coll} = mass transferred per collision on average

1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster
 Formalism by Portegies Zwart & McMillan 2002

IDEA: very hard binaries sink to the centre and likely collide with other stars/binaries unless they are ejected

ESTIMATE of R_{coll}

Maximum recoil velocity for a binary not to be ejected

$$v_{rec} = v_{esc} = 2\sigma$$

Definition of v_{rec}

$$v_{rec} = \frac{m_3}{m_T} v_{fin} = \frac{m_3}{m_T} \sqrt{\frac{m_3 (m_1 + m_2)}{m_e (m_a + m_b)} v_\infty^2 + \frac{2 m_T}{m_e (m_a + m_b)} \Delta E_b}$$

$$\frac{m}{3m} \sqrt{\frac{m(2m)}{m(2m)} \sigma^2 + \frac{2(3m)}{m(2m)} \Delta E_b} = \frac{1}{3} \sqrt{\sigma^2 + \frac{3}{m} \frac{m}{2m} E_b}$$

Nearly equal mass cluster

$$\Delta E_b = \frac{m}{2m} E_b$$


1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster
Formalism by Portegies Zwart & McMillan 2002

$$v_{rec} = v_{esc} = 2 \sigma \quad (1)$$

$$v_{rec} = \frac{1}{3} \sqrt{\sigma^2 + \frac{3}{2m} E_b} \quad (2)$$

Combining (1) and (2) $36 \sigma^2 = \sigma^2 + \frac{3}{2m} E_b$

$$E_b = \frac{70 m}{3} \sigma^2 = 70 k_B T \sim 10^2 k_B T$$
$$\frac{1}{2} m \sigma^2 = \frac{3}{2} k_B T$$


E_b is the binding energy exchanged by a hard binary during its life (i.e. before it is ejected).

1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster
Formalism by Portegies Zwart & McMillan 2002

We calculate now the number of binaries necessary to reverse core collapse (estimated as 10% of the total potential energy of the cluster, Goodman 1987):

$$N_{bin} 10^2 k_B T \sim 0.1 |W| = 0.1 \left(2 \frac{3}{2} N k_B T \right)$$
$$\longrightarrow N_{bin} \sim 10^{-3} N$$

Hard binary formation rate:

$$R_{bin} \sim 10^{-3} \frac{N}{t_{rlx}}$$

Assuming that ~ each hard binary undergoes ≤ 1 collision, we estimate the collision rate

$$R_{coll} \sim 10^{-3} f_{coll} \frac{N}{t_{rlx}}$$

1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster
Formalism by Portegies Zwart & McMillan 2002

IDEA: very hard binaries sink to the centre and likely collide with other stars/binaries unless they are ejected

ESTIMATE of δm_{coll}

From dynamical friction timescale $t_{df} \sim \frac{\langle m \rangle}{m_{df}} t_{rlx}$

where $\langle m \rangle$ = average star mass, M = total cluster mass, N = number of stars

We estimate the minimum mass of star that can sink to the centre in a time t

$$\delta m_{coll} \geq m_{df} \sim \langle m \rangle \frac{t_{rlx}}{t_{df}}$$

the mass that can be acquired after a collision (!!!)

1) Intermediate-mass BHs (IMBHs)

- 1- runaway collapse of stars at centre of star cluster
Formalism by Portegies Zwart & McMillan 2002

$$\frac{dM_{\text{runaway}}}{dt} \sim 10^{-3} f_{\text{coll}} \frac{N}{t} \langle m \rangle$$

$M_{\text{runaway}} \sim 10^{2-3} M_{\odot}$ for a dense young cluster with $N > 10^5$

1st CONDITIO SINE QUA NON:

core collapse time \ll massive star evolution time

$\rightarrow t_{\text{coll}} < 3-25$ Myr

2nd CONDITIO SINE QUA NON:

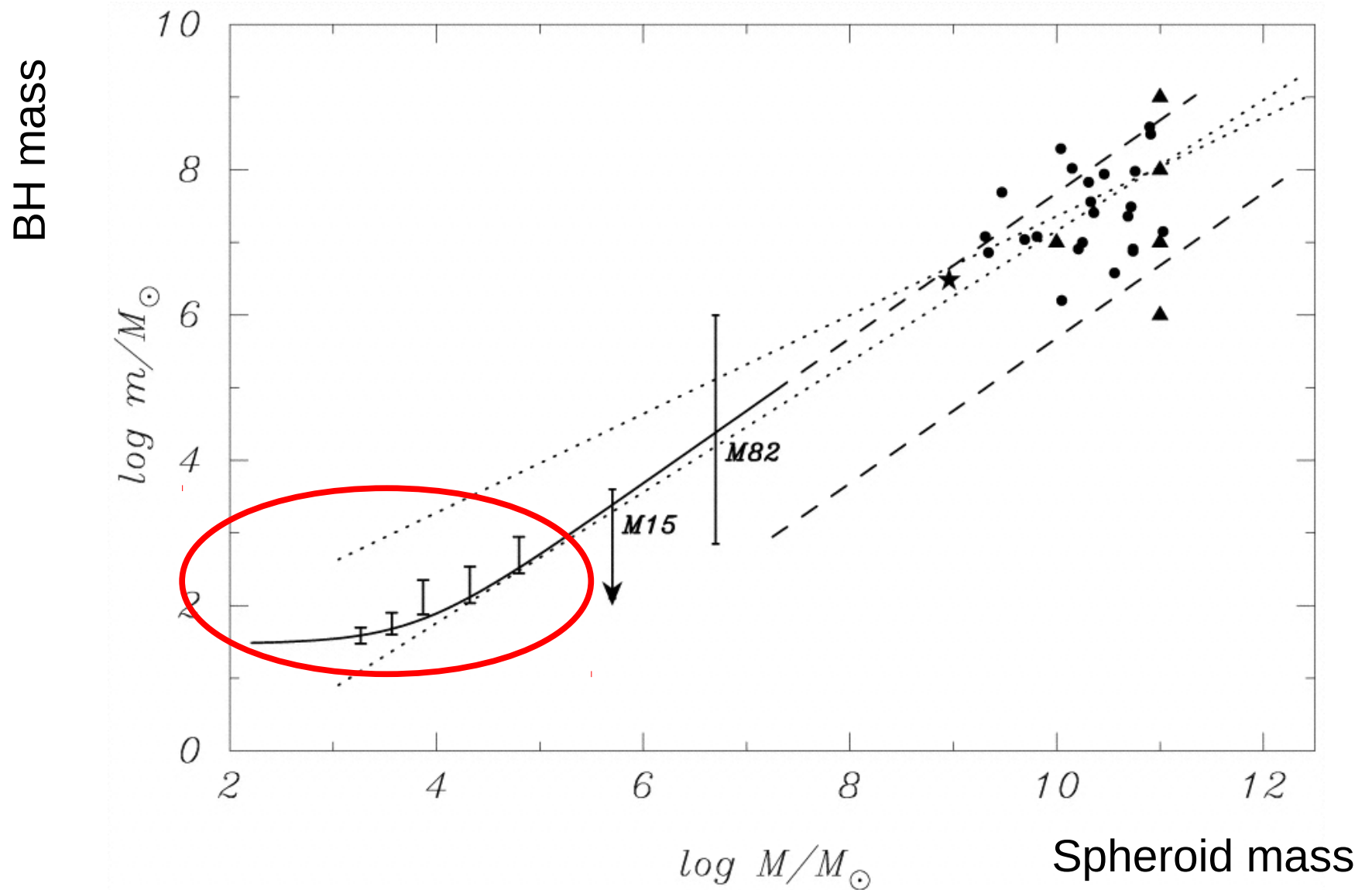
STAR CLUSTER SUFFICIENTLY MASSIVE AND
CONCENTRATED

1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster

Formalism by Portegies Zwart & McMillan 2002

Confirmed by simulations



1) Intermediate-mass BHs (IMBHs)

1- runaway collapse of stars at centre of star cluster

Formalism by Portegies Zwart & McMillan 2002

MAIN ISSUE: MASS LOSSES!!!

(1) during merger

Recent simulations show mass losses

up to 25% of total mass

(Gaburov, Lombardi & Portegies Zwart 2010, MNRAS, 402, 105)

(2) by stellar winds

After merger the super-massive star will

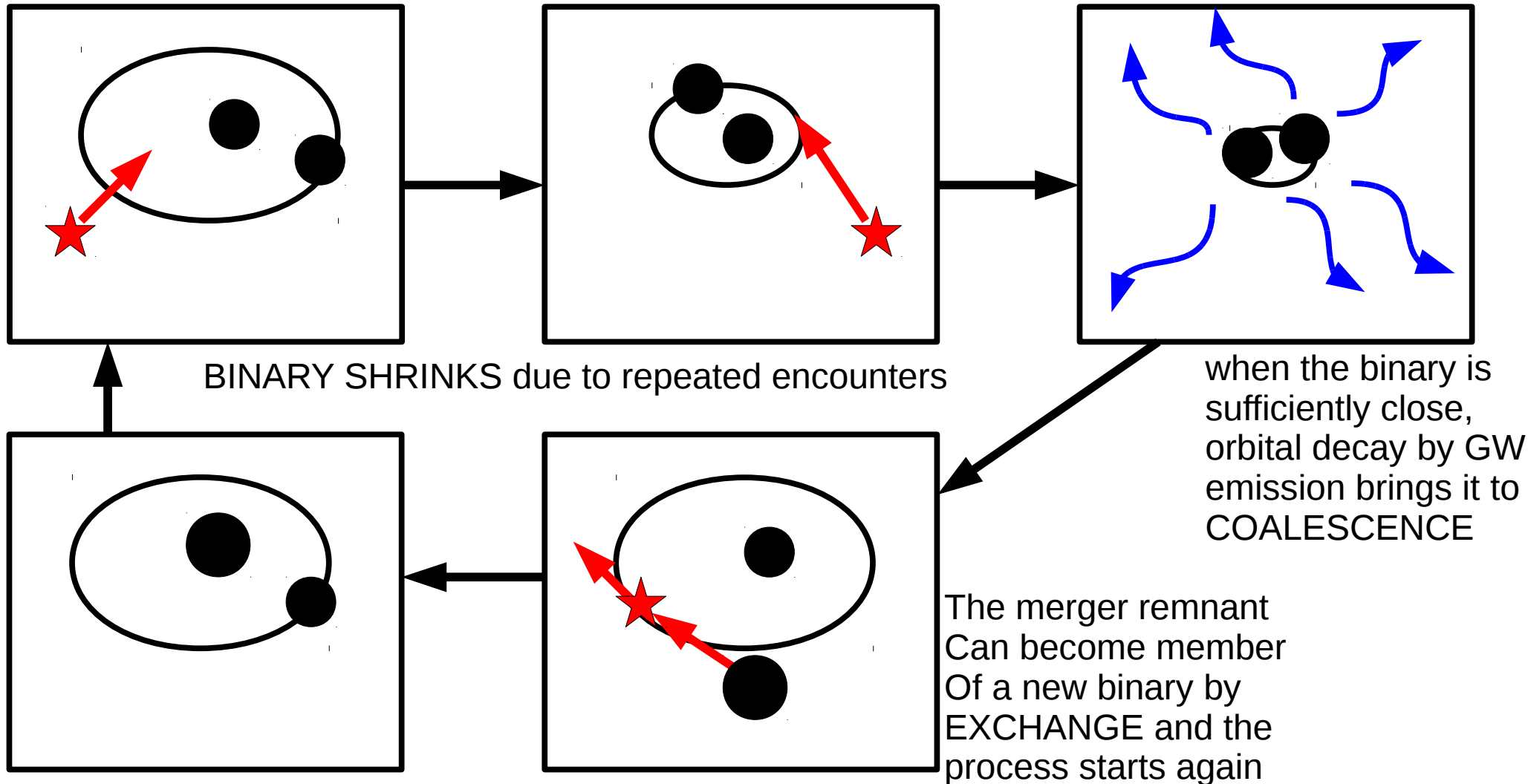
be very unstable (radiation pressure dominated)

1) Intermediate-mass BHs (IMBHs)

2- repeated mergers

Formalism by Miller & Hamilton (2002)

In a old cluster stellar BHs can grow in mass because of repeated mergers with the companion triggered by 3-body encounters



1) Intermediate-mass BHs (IMBHs)

2- repeated mergers

Formalism by Miller & Hamilton (2002)

MAIN PROBLEM: seed BH must avoid ejection before merger

$$v_{rec} = \frac{m_3}{m_T} \sqrt{\frac{m_3 (m_1 + m_2)}{m_e (m_a + m_b)} v_\infty^2 + \frac{2 m_T}{m_e (m_a + m_b)} \Delta E_b} \sim \frac{m_3}{m_T} \sqrt{\frac{2 m_T}{m_3 (m_1 + m_2)} \Delta E_b}$$

\uparrow
 $m_1 + m_2 \gg m_3$

$$\sim \frac{m_3}{m_T} \sqrt{\frac{2 m_T}{m_3 (m_1 + m_2)} \frac{\xi m_3}{(m_1 + m_2)} E_b} \sim \frac{m_3}{m_1 + m_2} \sqrt{\frac{2 \xi}{m_T} E_b}$$

We find the minimum binding energy for EJECTION ($E_{b,min}$) by imposing

$$v_{rec} = v_{esc} \Rightarrow E_{b,min} \sim \frac{(m_1 + m_2)^3}{2 \xi m_3^2} v_{esc}^2$$

where we assumed $m_1 + m_2 \sim m_T$

$$E_{b,min} \sim 2 \times 10^{50} \text{ erg} \left(\frac{m_1}{50 M_\odot} \right)^3 \left(\frac{m_3}{10 M_\odot} \right)^{-2} \left(\frac{\xi}{0.2} \right)^{-1} \left(\frac{v_{esc}}{50 \text{ km s}^{-1}} \right)^2$$

1) Intermediate-mass BHs (IMBHs)

2- repeated mergers

Formalism by Miller & Hamilton (2002)

MAIN PROBLEM: seed BH must avoid ejection before merger

Orbital separation in merger regime (see lecture 3):

$$a_{GW} \sim 3 \times 10^{11} \text{ cm} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{1/4} \left(\frac{m_1}{50 M_\odot} \right)^{1/2} \left(\frac{m_2}{10 M_\odot} \right)^{1/4}$$

Binding energy in merger regime:

$$E_{b, \text{merg}} = \frac{G m_1 m_2}{2 a_{GW}} \sim 2 \times 10^{50} \text{ erg} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{-1/4} \left(\frac{m_1}{50 M_\odot} \right)^{1/2} \left(\frac{m_2}{10 M_\odot} \right)^{3/4}$$

COMPARING $E_{b, \text{min}}$ with $E_{b, \text{merg}}$:

$$x = \frac{E_{b, \text{min}}}{E_{b, \text{merg}}} \sim \left(\frac{m_1}{50 M_\odot} \right)^{5/2} \left(\frac{m_2}{10 M_\odot} \right)^{-11/4} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{1/4}$$

If $x > 1$ BINARY MERGES BEFORE EJECTION

If $x < 1$ BINARY IS EJECTED BEFORE MERGER

1) Intermediate-mass BHs (IMBHs)

2- repeated mergers

Formalism by Miller & Hamilton (2002)

In a old cluster stellar BHs can grow in mass because of repeated mergers with the companion triggered by 3-body encounters

Number of 3-body encounters for a BH to merge with its companion
(from lecture 3):

$$N_{merg} = \frac{1}{\xi} \frac{m_T}{\langle m \rangle} \ln \left(\frac{a_0}{a_{GW}} \right)$$

Time required for 1 merger:

$$dt = -\frac{\sigma}{2\pi G \xi \rho} \frac{da}{a^2} \quad \longrightarrow \quad \int_0^{t_{GW}} dt = -\frac{\sigma}{2\pi G \xi \rho} \int_{a_0}^{a_{GW}} \frac{da}{a^2}$$

$$t_{GW} = \frac{\sigma}{2\pi G \xi \rho} \left(\frac{1}{a_{GW}} - \frac{1}{a_0} \right)$$

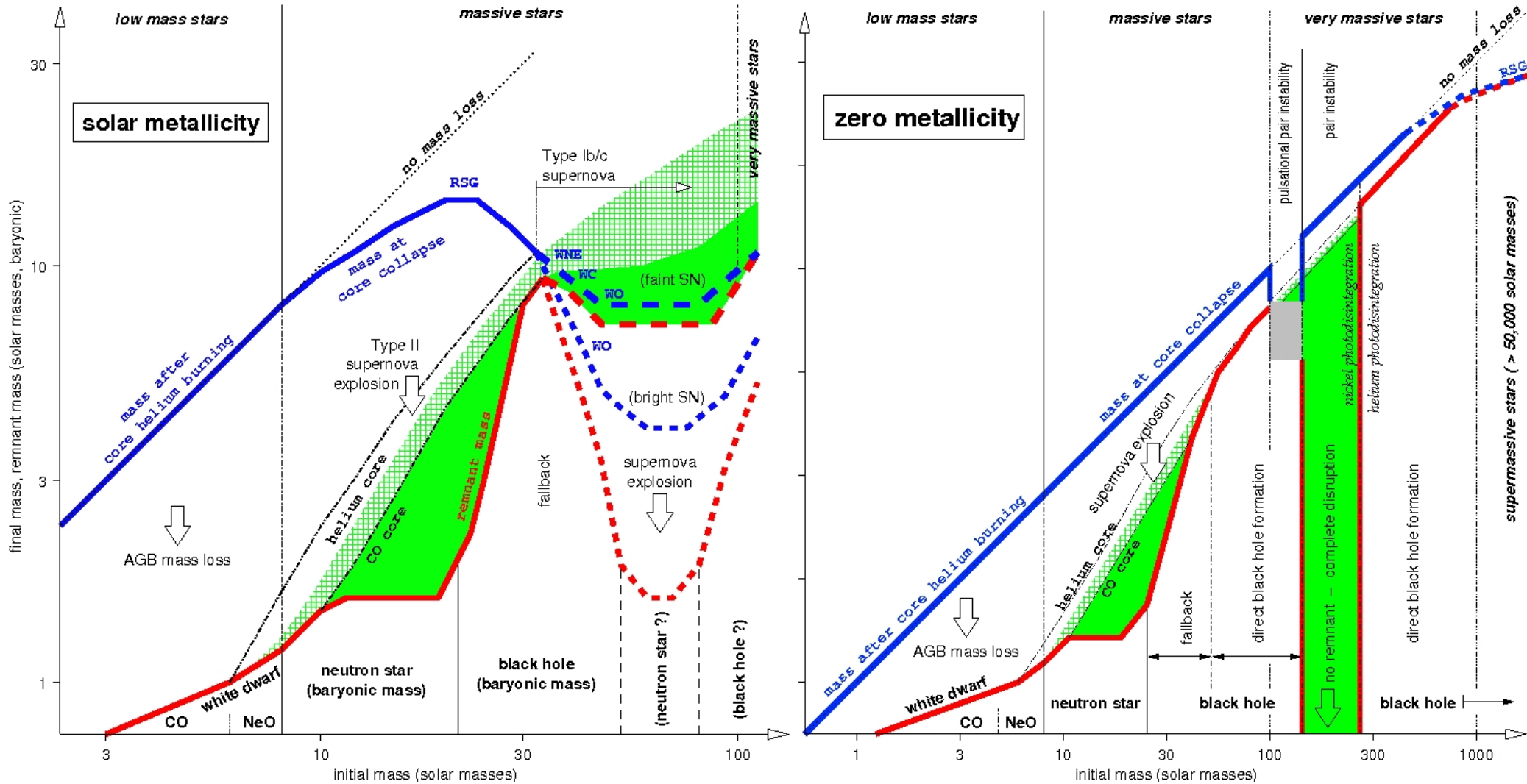
$$t_{GW} \sim 3 \times 10^8 \text{ yr} \left(\frac{\sigma}{10 \text{ km s}^{-1}} \right) \left(\frac{\xi}{0.2} \right)^{-1} \left(\frac{\rho}{10^6 M_{\odot} \text{ pc}^{-3}} \right)^{-1} \left(\frac{a_{GW}}{1 \text{ AU}} \right)^{-1}$$

INEFFICIENT!!!!

1) Intermediate-mass BHs (IMBHs)

3- remnants of extremely metal-poor stars

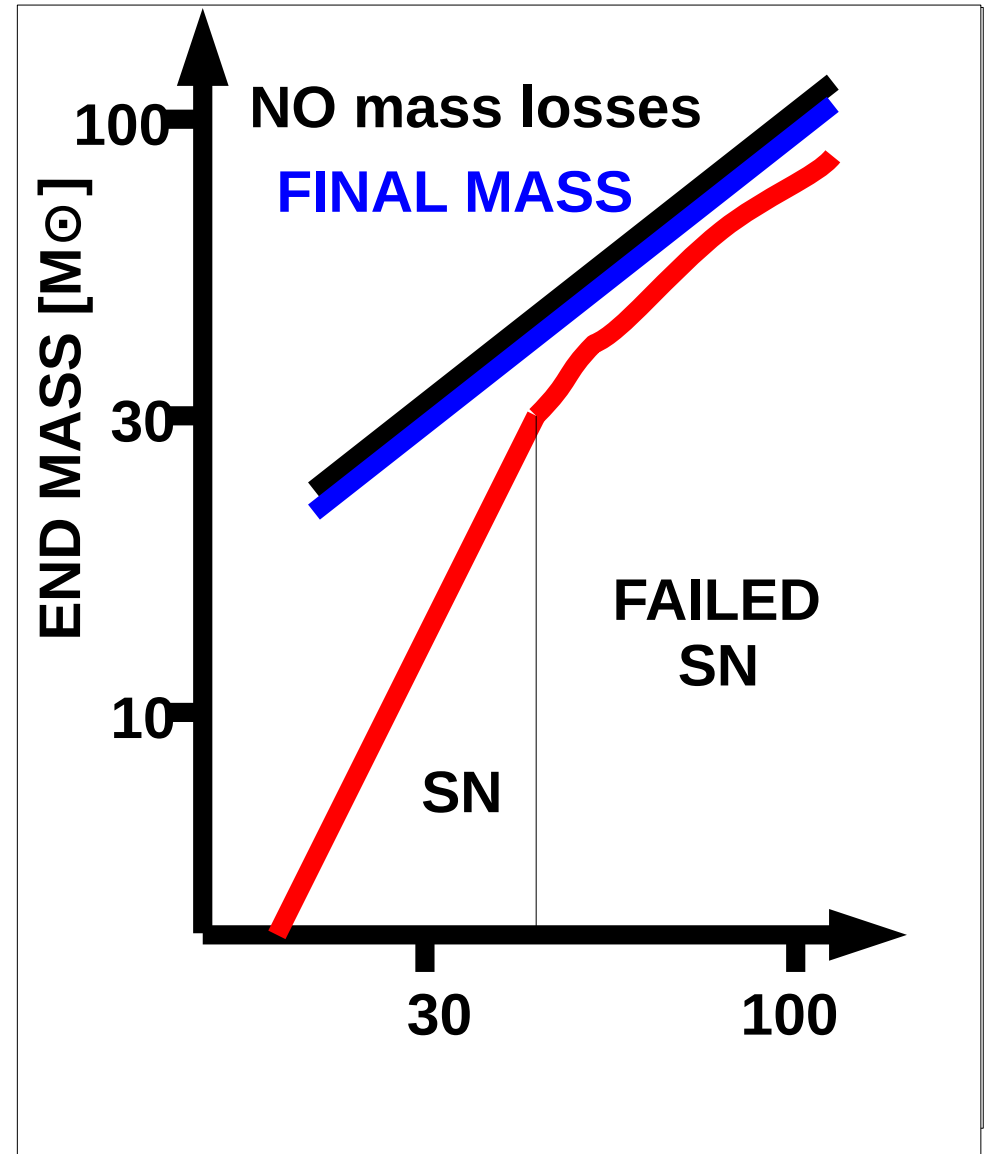
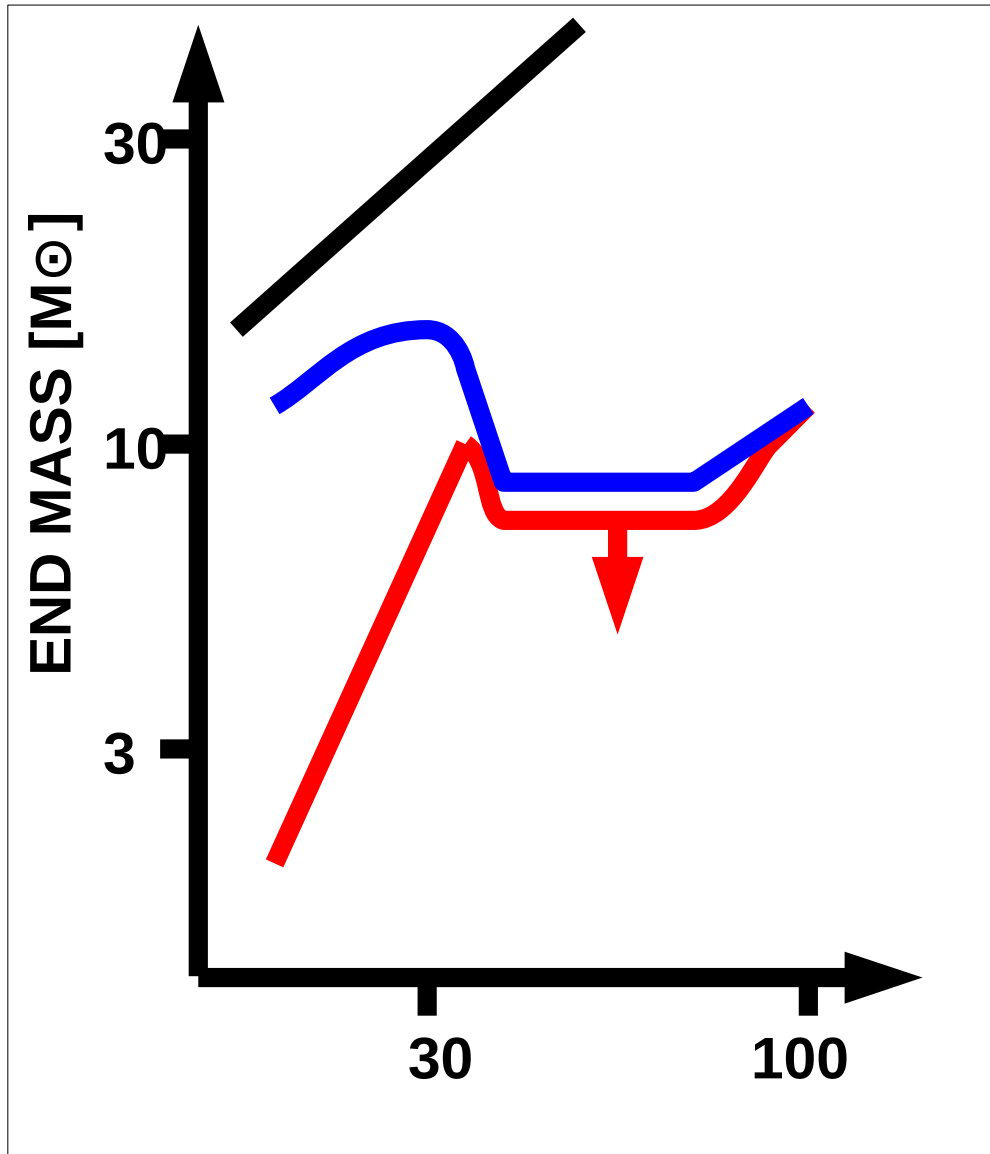
Formalism by Heger et al. (2003)



1) Intermediate-mass BHs (IMBHs)

3- remnants of extremely metal-poor stars

Formalism by Heger et al. (2003)



1) Intermediate-mass BHs (IMBHs)

3- remnants of extremely metal-poor stars
Formalism by Heger et al. (2002)

TWO INGREDIENTS:

1) STELLAR WINDS depend on METALLICITY

$$\dot{M}(Z) \propto \left(\frac{Z}{Z_{\odot}} \right)^{\alpha} \quad \alpha = 0.5 - 0.9$$

at low Z, stars lose less mass by stellar winds!

Vink+ (2001)

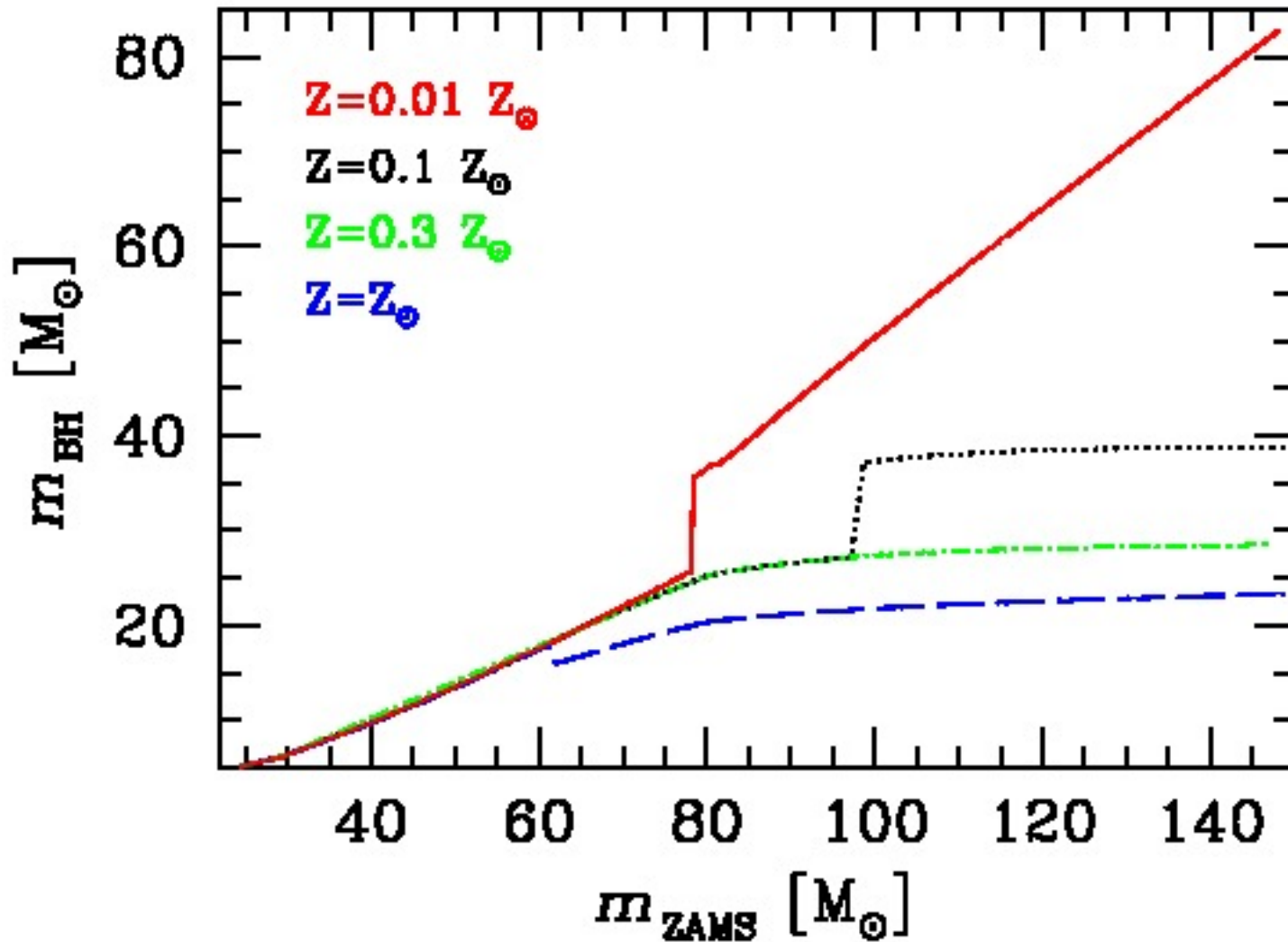
**2) IF FINAL MASS SUFFICIENTLY HIGH (> 40 Msun),
SN EXPLOSION CANNOT SUCCEED:
almost NO EJECTA and direct collapse to BHs
(FAILED SUPERNOVAE, Fryer 1999)**

1) Intermediate-mass BHs (IMBHs)

3- remnants of extremely metal-poor stars

Formalism by Heger et al. (2002)

NOT ONLY AT ZERO METALLICITY



MM+09; Zampieri & Roberts 2009; Belczynski+2010;
Fryer+2012; MM+2013

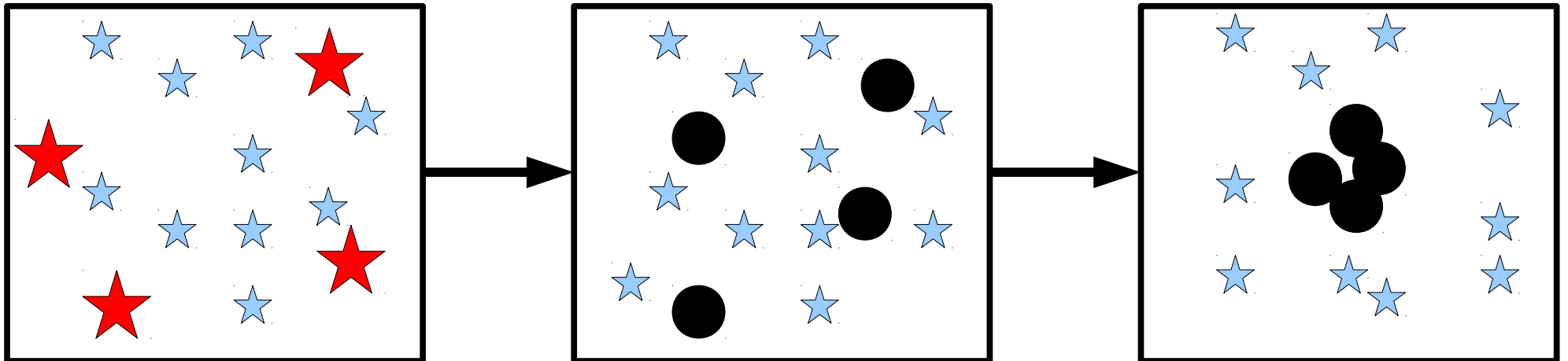
2) BHs eject each other?

Note: valid for globular clusters! Why? BHs form before progenitors segregate to the centre (no runaway collapse)

$N_{bh} \sim 10^2$ expected to form in GCs

Segregate to the centre in

$$t_{df} \sim 3 \times 10^7 \text{ yr} \left(\frac{\sigma}{10 \text{ km s}^{-1}} \right) \left(\frac{r_c}{1 \text{ pc}} \right)^2 \left(\frac{m_{BH}}{10 M_{\odot}} \right)^{-1}$$



2) BHs eject each other?

What happens when BHs are in the core?

1) Total mass of BHs sufficiently large to have SPITZER'S INSTABILITY →

- * formation of a **dynamically decoupled core of BHs!!!**
- * **core collapse** for BHs on faster timescale than expected for stars
- * **efficient formation of BH-BH** binaries
- * **fast ejection of all lighter stars** from the BH dominated core
- * ejections of (nearly) all single BHs in halo or out of cluster
- * **ejections of binary BHs** with $x > 1$

$$x = \frac{E_{b, min}}{E_{b, merg}} \sim \left(\frac{m_1}{50 M_{\odot}} \right)^{5/2} \left(\frac{m_2}{10 M_{\odot}} \right)^{-11/4} \left(\frac{t_{GW}}{10^6 \text{ Myr}} \right)^{1/4}$$

How many BHs left in the cluster?? 0, 1, 2, boh...

How massive? It depends whether mechanism by Miller & Hamilton (2002) is efficient or not – i.e. it depends on the mass of available seeds

From Kulkarni, Hut & McMillan 1993, Nature 364, 421
Sigurdsson & Hernquist 1993, Nature 364, 423

2) BHs eject each other?

What happens when BHs are in the core?

2) Total mass of BHs relatively small with respect to stars →

- * efficient formation of BH-BH binaries on standard core-collapse time
- * ejection of lighter stars from the core (BH-BH binaries harden)
- * ejections of (nearly) all single BHs in halo or out of cluster
- * ejections of binary BHs with $x > 1$

$$x = \frac{E_{b, \min}}{E_{b \text{ merg}}} \sim \left(\frac{m_1}{50 M_\odot} \right)^{5/2} \left(\frac{m_2}{10 M_\odot} \right)^{-11/4} \left(\frac{t_{\text{GW}}}{10^6 \text{ Myr}} \right)^{1/4}$$

How many BHs left in the cluster?? 0, 1, 2, boh...

How massive? It depends whether mechanism by Miller & Hamilton (2002) is efficient or not – i.e. it depends on the mass of available seeds

2) BHs eject each other?

What do data tell us?

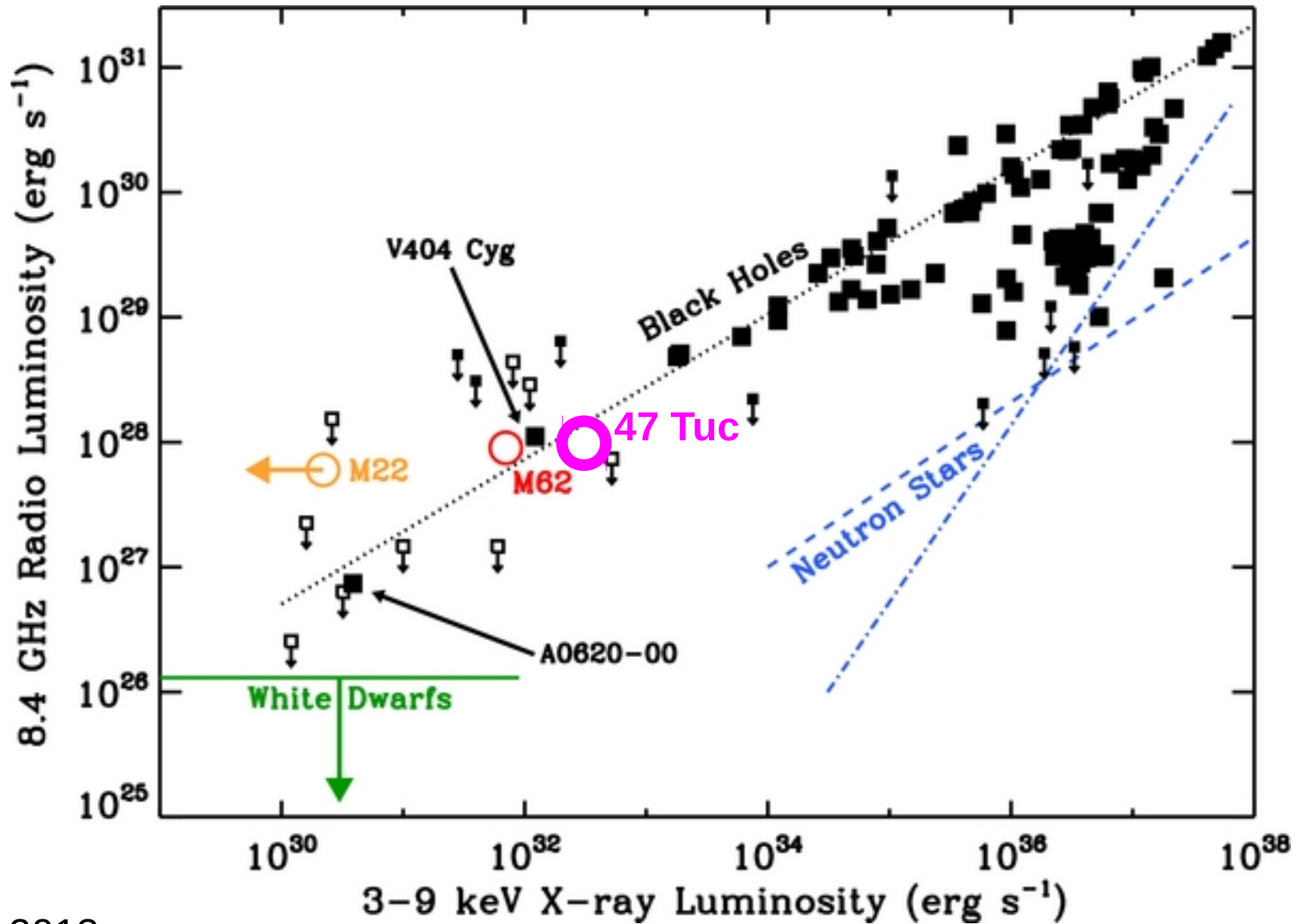
* The only 4 strong **BH candidates in MW GCs** are:

- **2 RADIO SOURCES** in M22 (Strader et al. 2012, Nature, 490, 71).
No X-ray detection ($<10^{30}$ erg/s) $\rightarrow \log L_R/L_X > -2.6$ (too high for NSs)
- 1 radio+X source in M62, Chomiuk et al. 2013
- 1 radio+X source in 47 Tuc, Strader et al., in prep.?, see
http://www.astro.uni-bonn.de/~sambaran/DS2014/Modest14_Talks/Strader.pdf

* 5 sources in **GCs of elliptical galaxies** are strong BH candidates for X-ray variability:

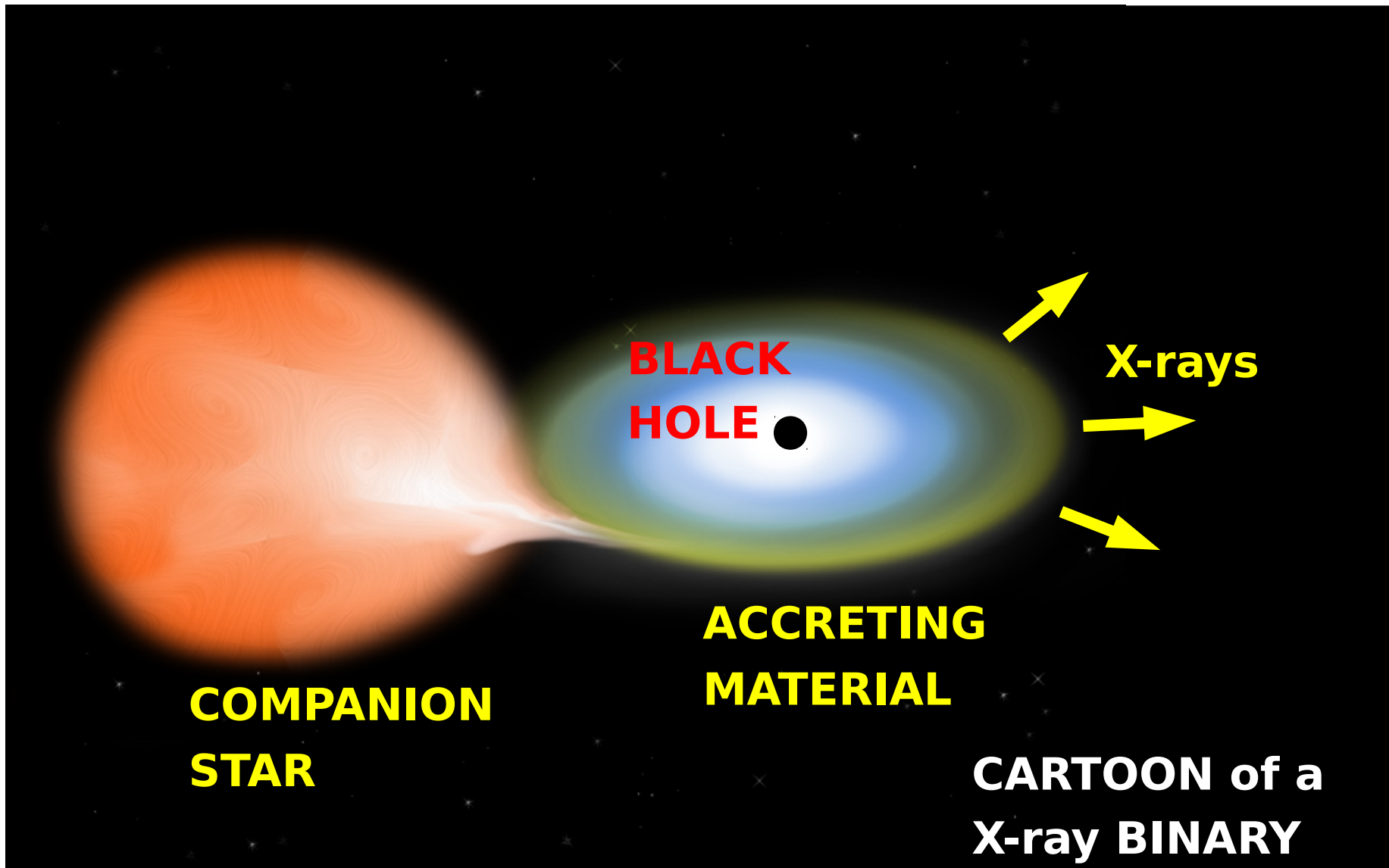
- NGC 4472 (MacCarone et al. 2007, Nature, 445, 183)
- NGC 4472 (2nd source, MacCarone et al. 2011, MNRAS, 410, 1655)
- NGC 3379 (Brassington et al. 2010, ApJ, 725, 1805)
- NGC 1399 (Irwin et al. 2010, ApJ, 712, L1)
- NGC 1399 (2nd source, Shih et al. 2010, ApJ, 721, 323)

2) BHs eject each other?



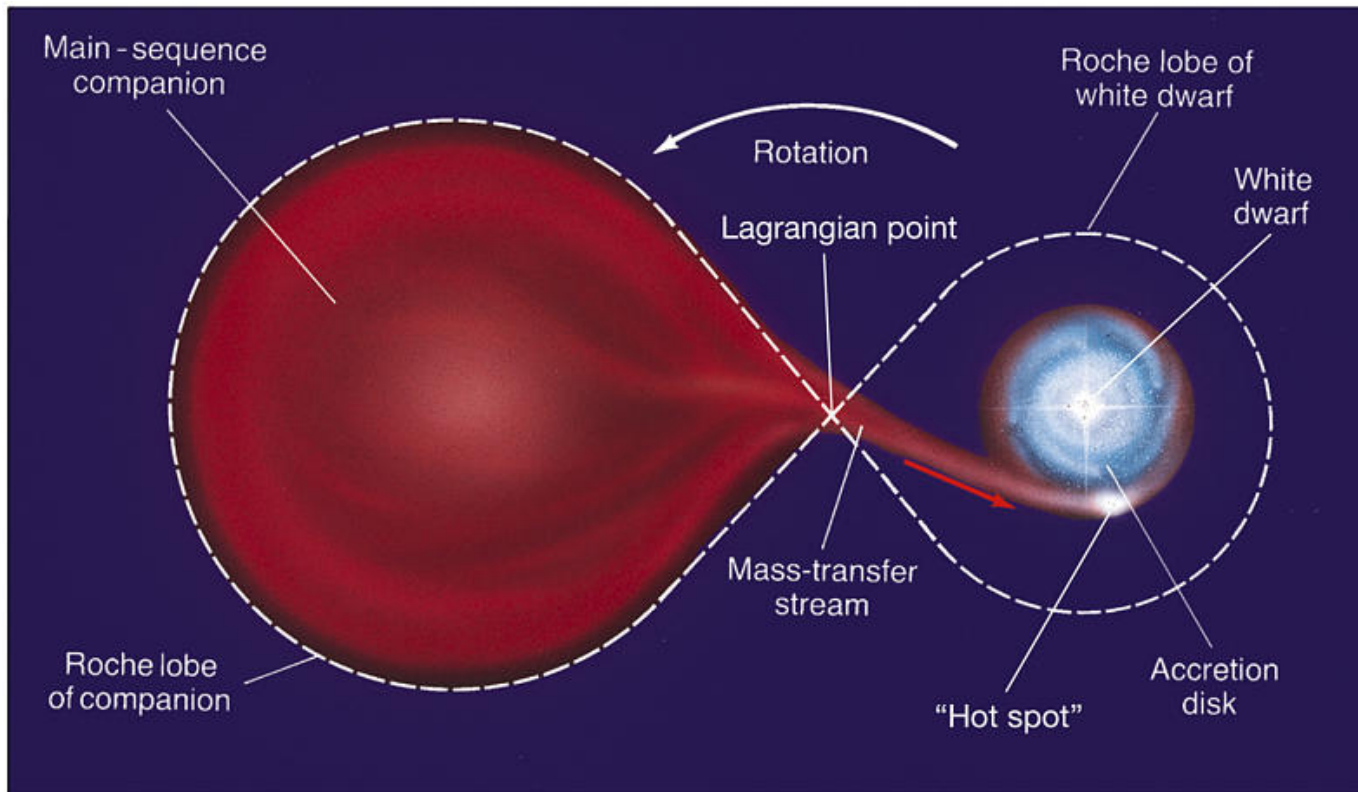
3) Effects of 3-body on X-ray binaries (formation and escape)

Compact object accreting matter from companion star via Roche lobe overflow or stellar winds



3) Effects of 3-body on X-ray binaries

Compact object accreting matter from companion star via Roche lobe overflow or stellar winds



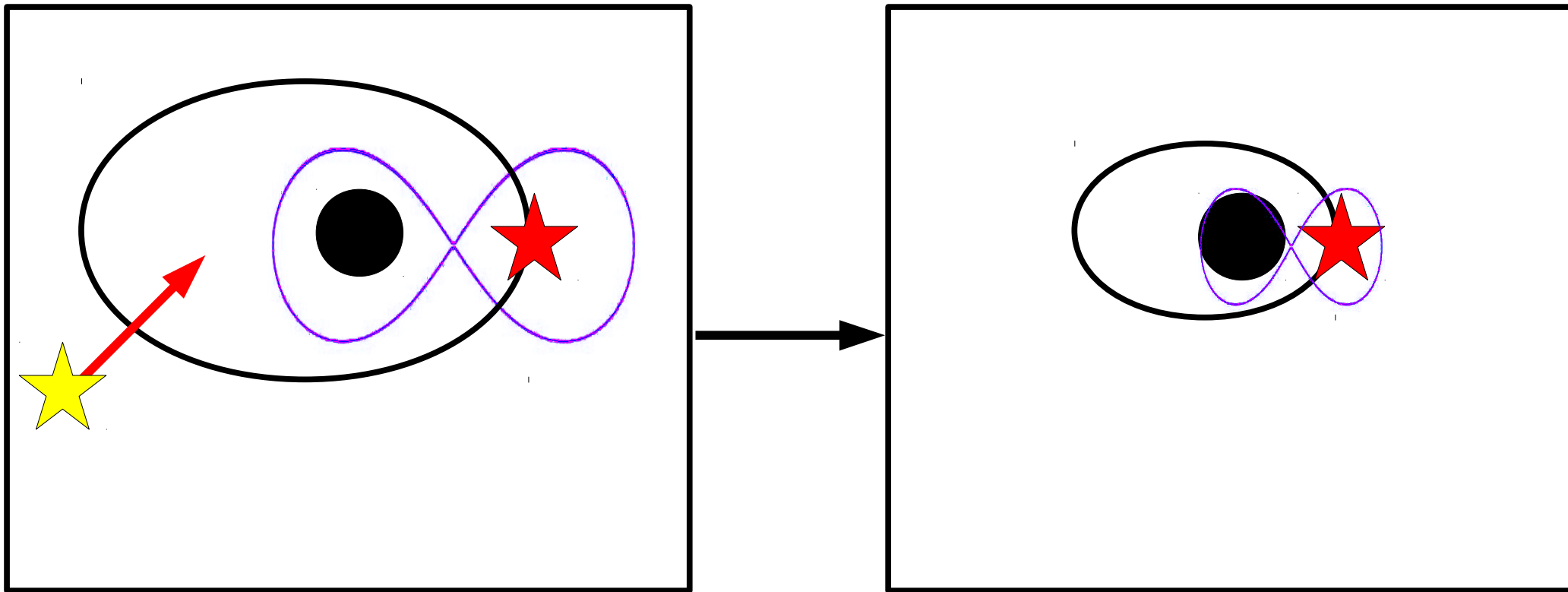
**Luca
Zampieri's
course**

Copyright © 2005 Pearson Prentice Hall, Inc.

$$r_{RL} = \frac{0.49 a (M_1/M_2)^{2/3}}{0.6 (M_1/M_2)^{2/3} + \ln [1 + (M_1/M_2)^{1/3}]}$$

3) Effects of 3-body on X-ray binaries

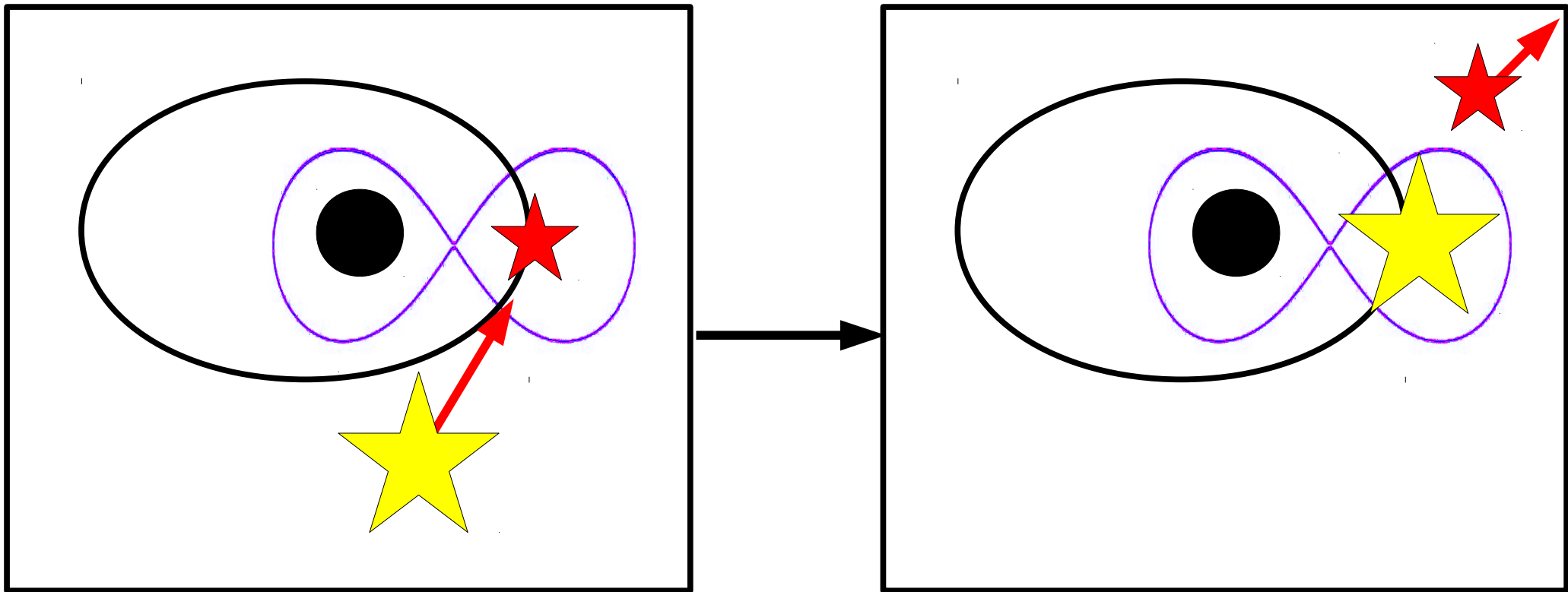
Which is the effect of 3-body encounters on X-ray binaries?



After 3-body encounters, the semi-major axis shrinks and the radius of the companion equals the Roche lobe

3) Effects of 3-body on X-ray binaries

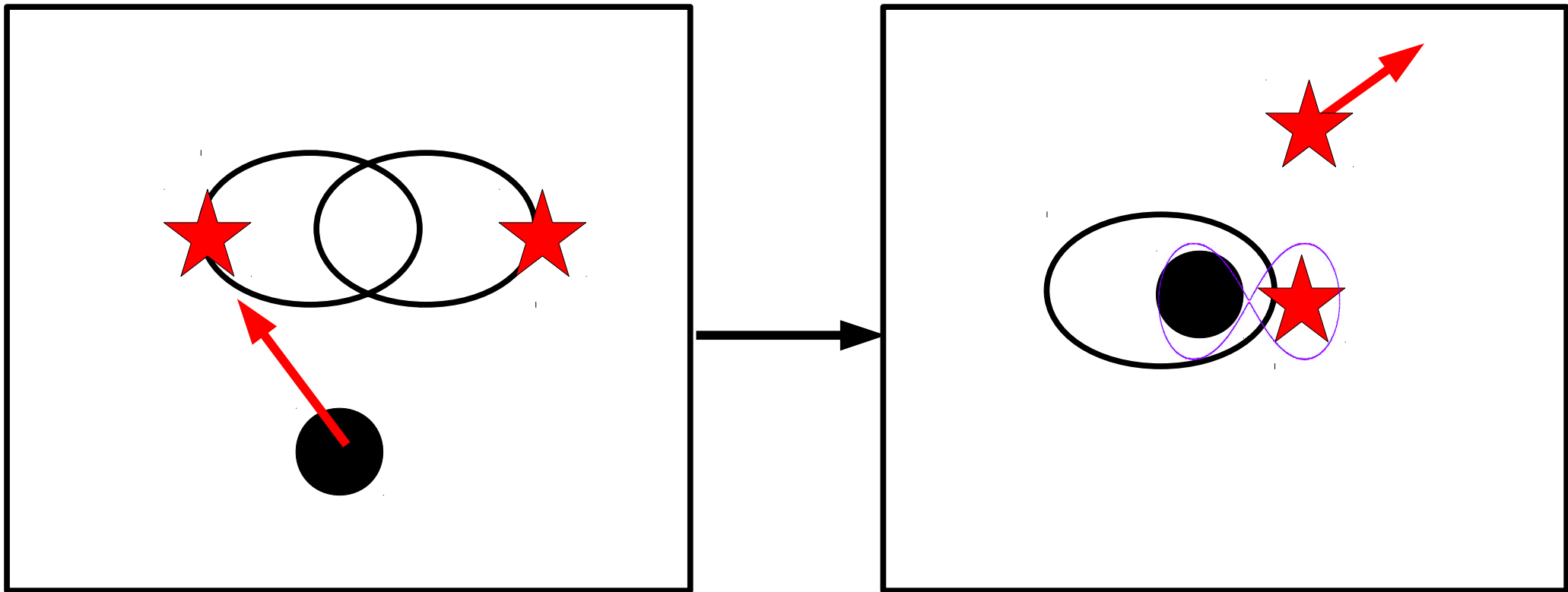
Which is the effect of 3-body encounters on X-ray binaries?



Exchanges are very important: (1) bring stars with larger radius in the binary

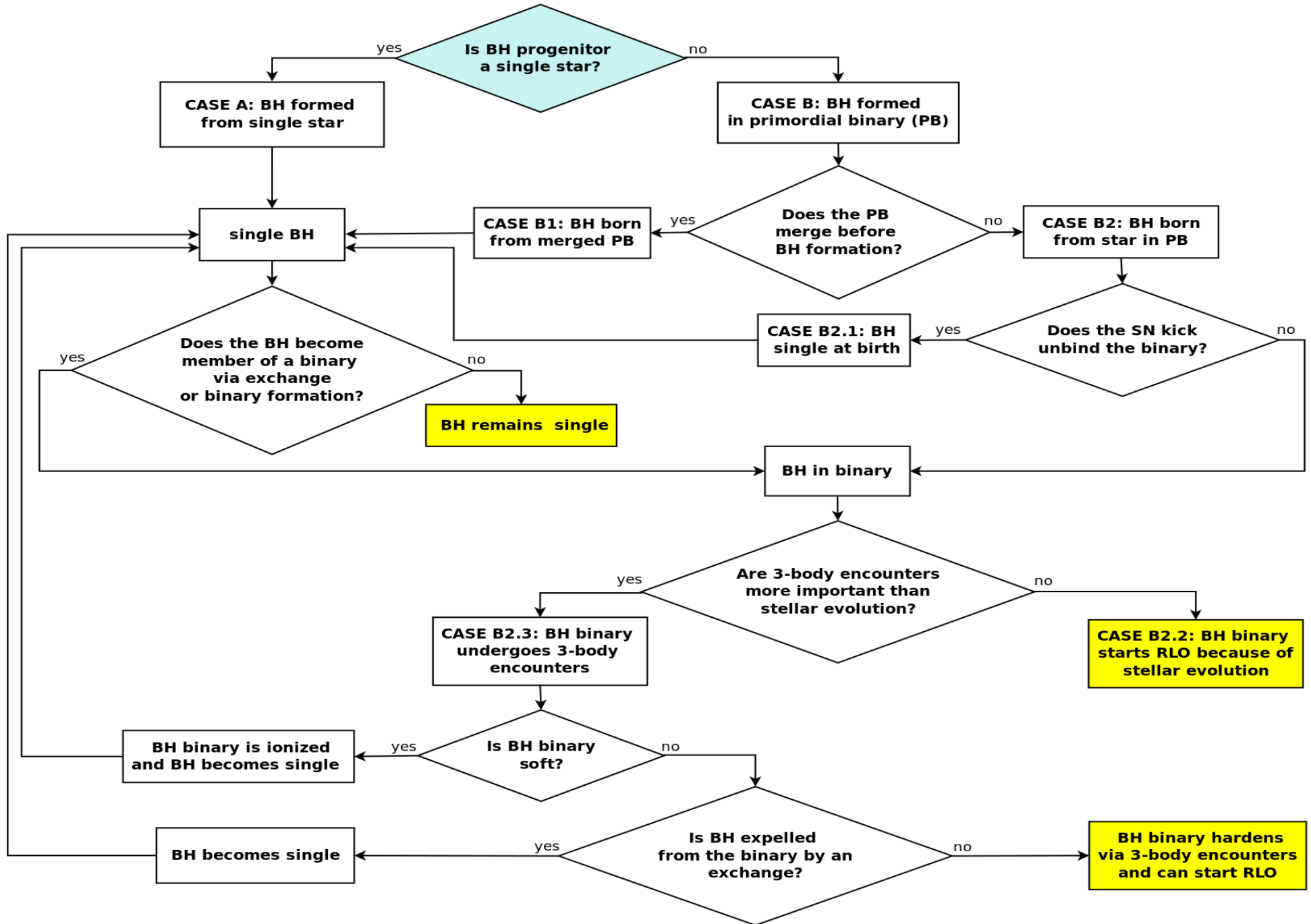
3) Effects of 3-body on X-ray binaries

Which is the effect of 3-body encounters on X-ray binaries?



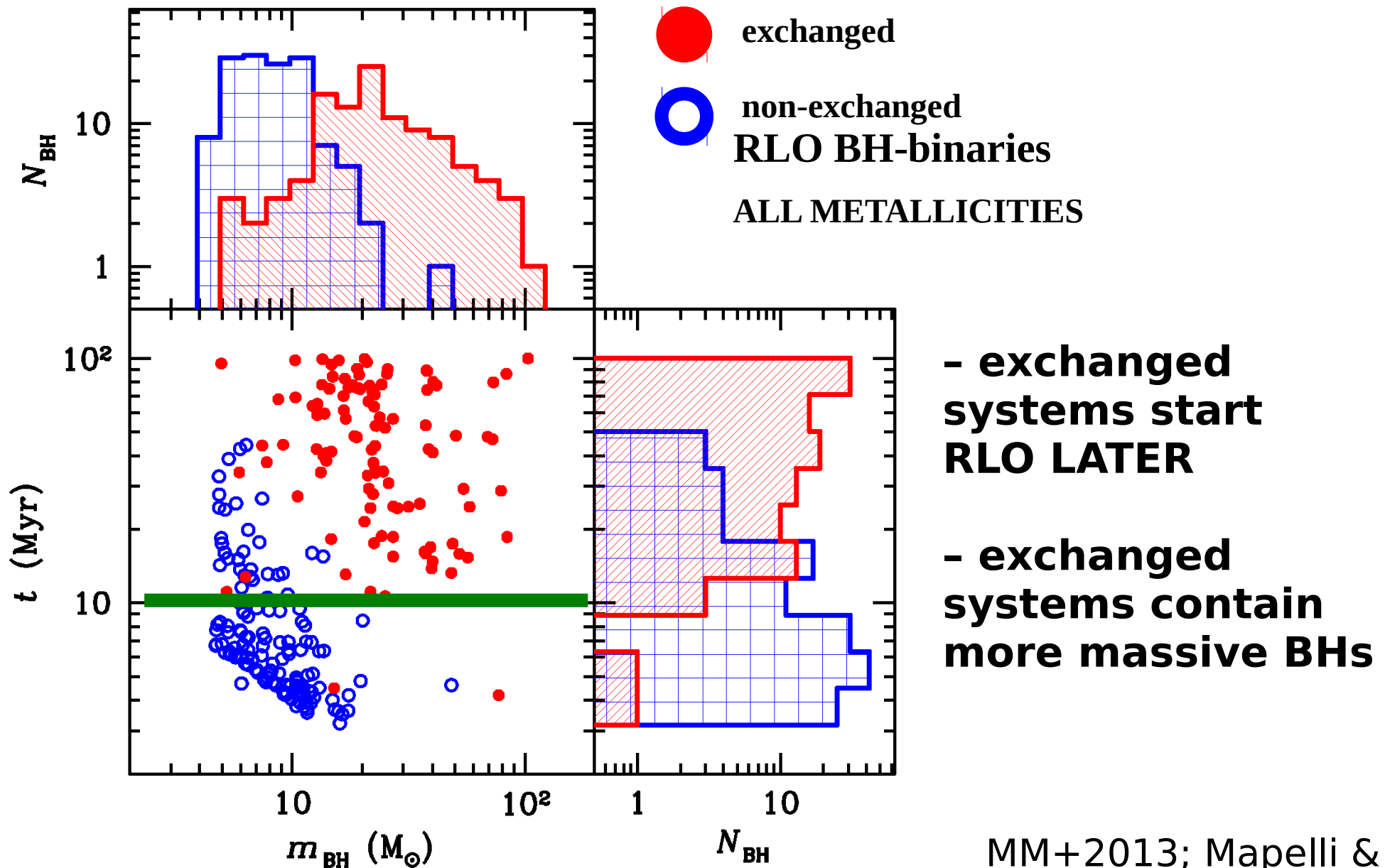
Exchanges are very important: (2) bring single BHs in binaries

3) Effects of 3-body on X-ray binaries



3) Effects of 3-body on X-ray binaries

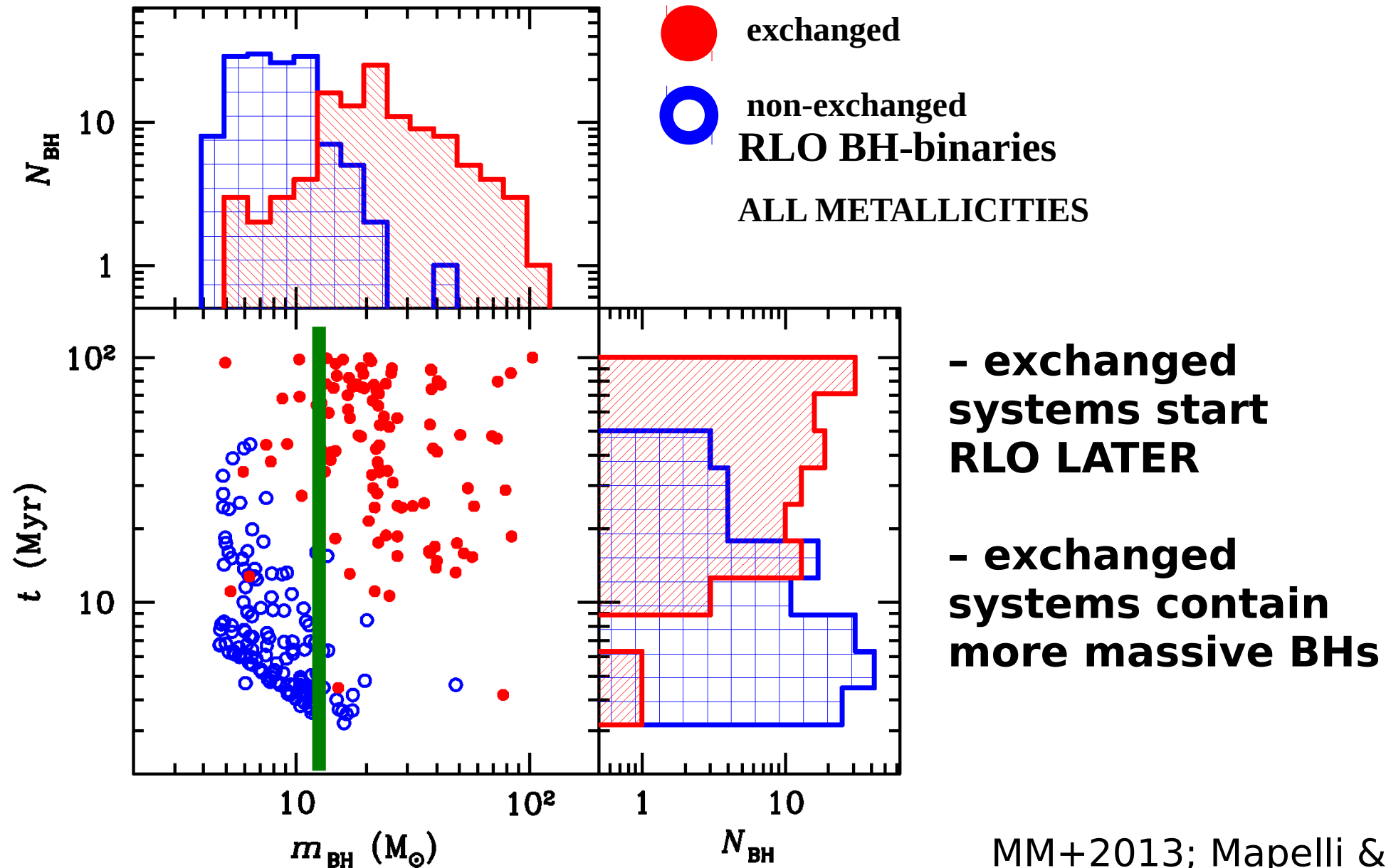
X-ray binaries from stellar evolution switch on in the first stages



MM+2013; Mapelli & Zampieri 2014

3) Effects of 3-body on X-ray binaries

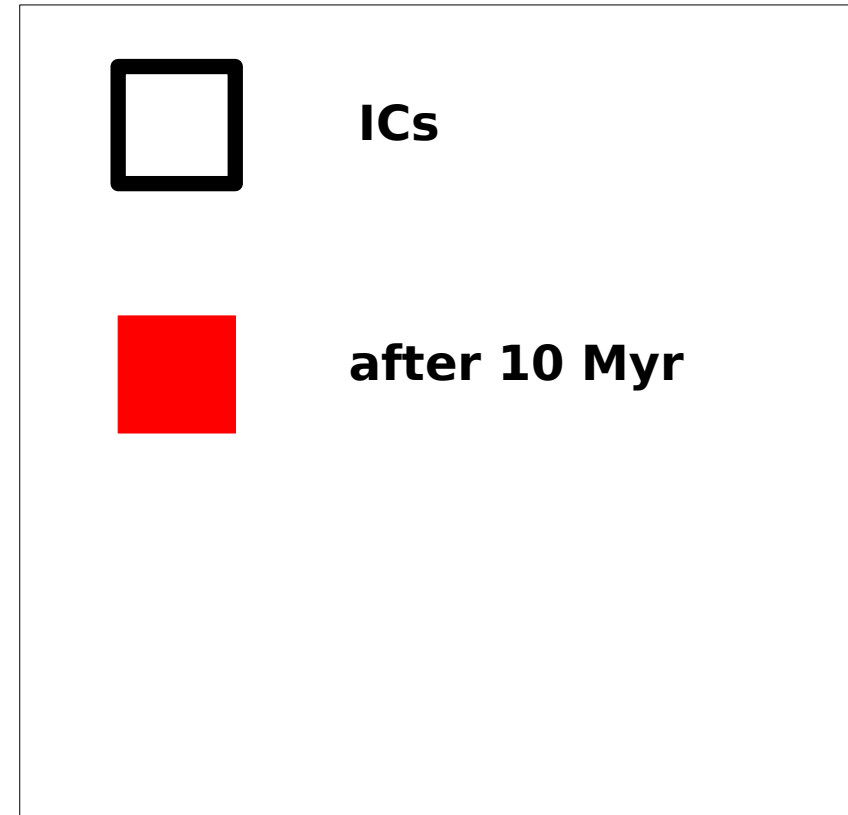
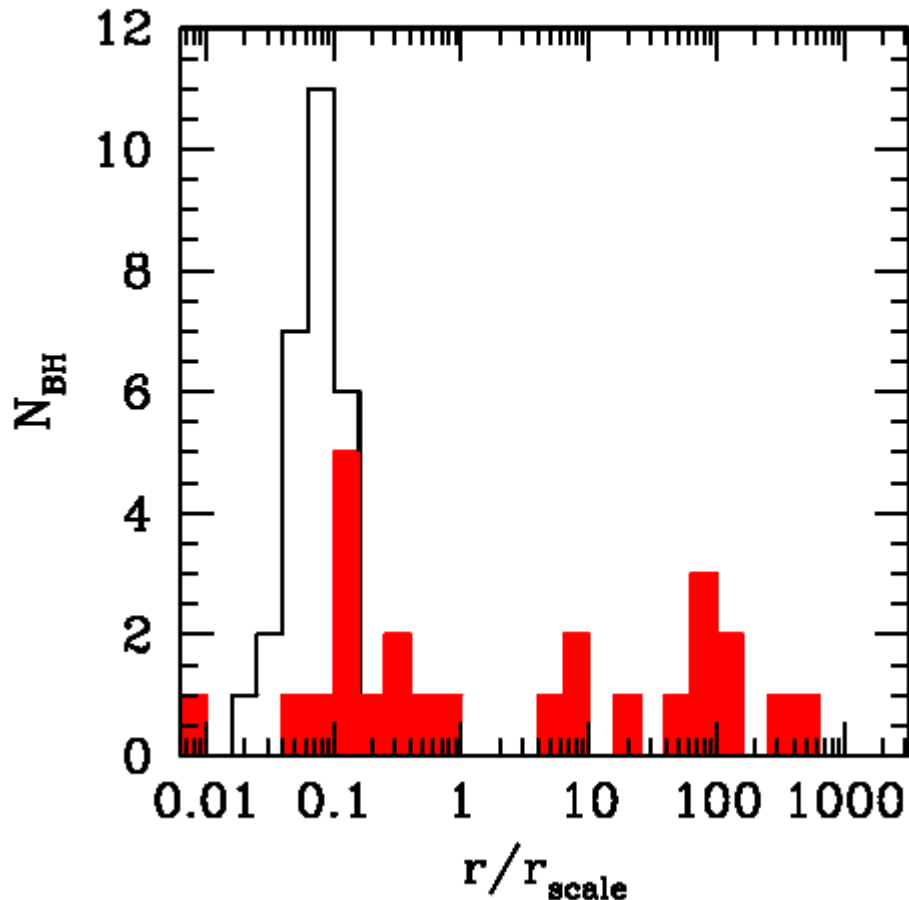
X-ray binaries from stellar evolution switch on in the first stages



MM+2013; Mapelli & Zampieri 2014

3) Effects of 3-body on X-ray binaries

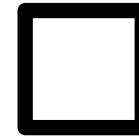
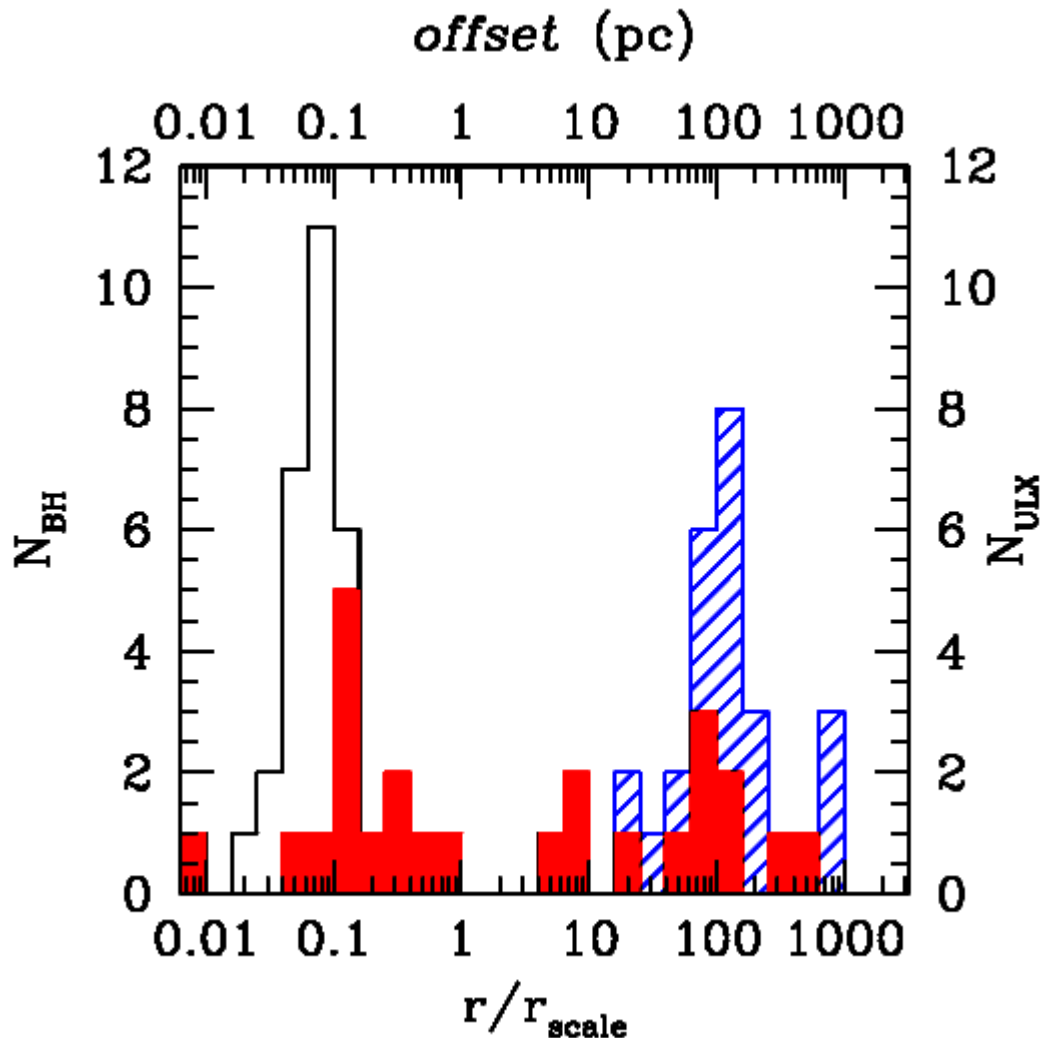
Simulations of young star clusters +
MSBH binary with Starlab:



**~30-40 %
BHs are ejected
with MS companion
before RG phase!!**

3) Effects of 3-body on X-ray binaries

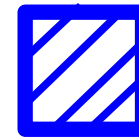
Simulations of young star clusters +
MSBH binary with Starlab:



ICs



after 10 Myr

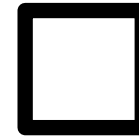
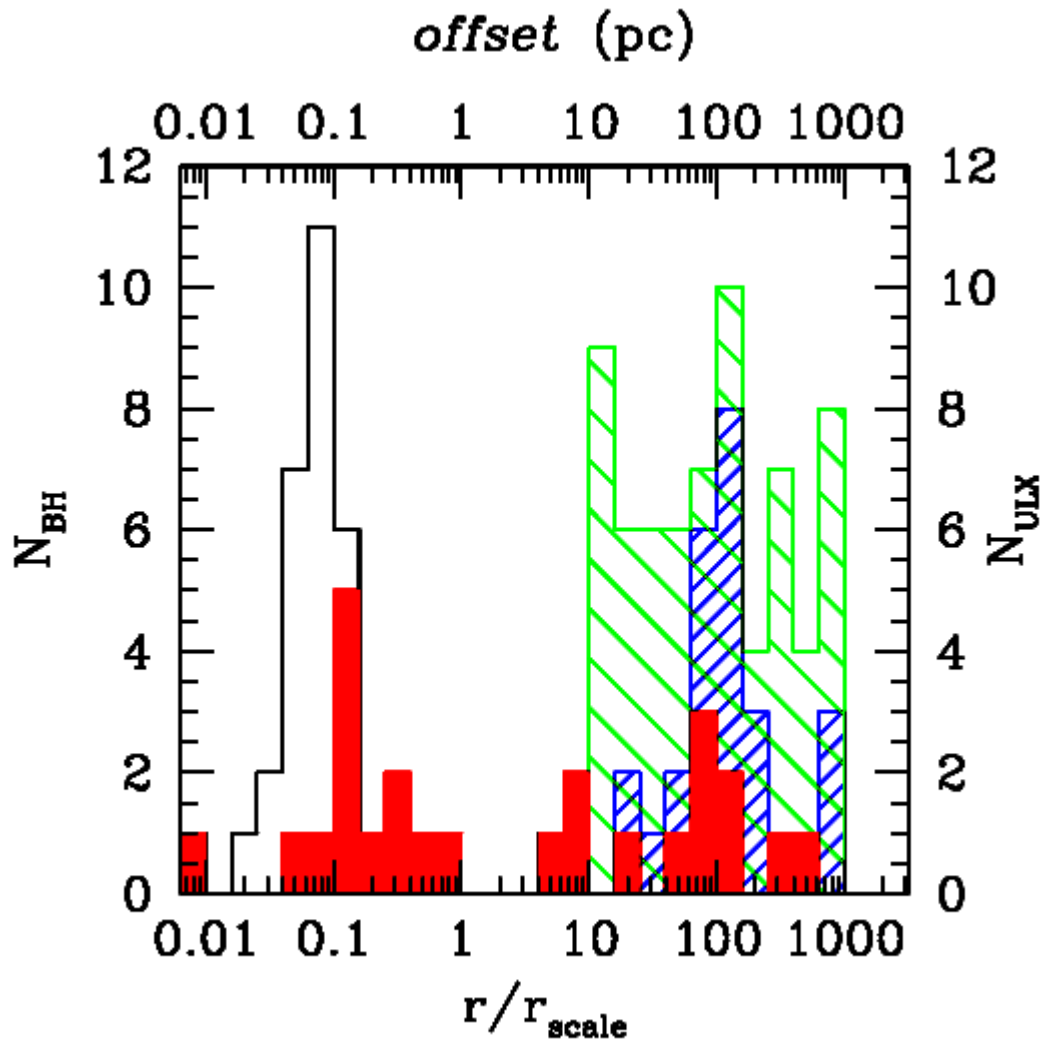


data of ULXs
from Berghea
PhD

**~30-40 %
BHs are ejected
with MS companion
before RG phase!!**

3) Effects of 3-body on X-ray binaries

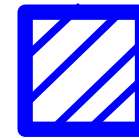
Simulations of young star clusters +
MSBH binary with Starlab:



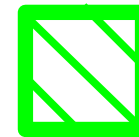
ICs



after 10 Myr



data of ULXs
from Berghea
PhD



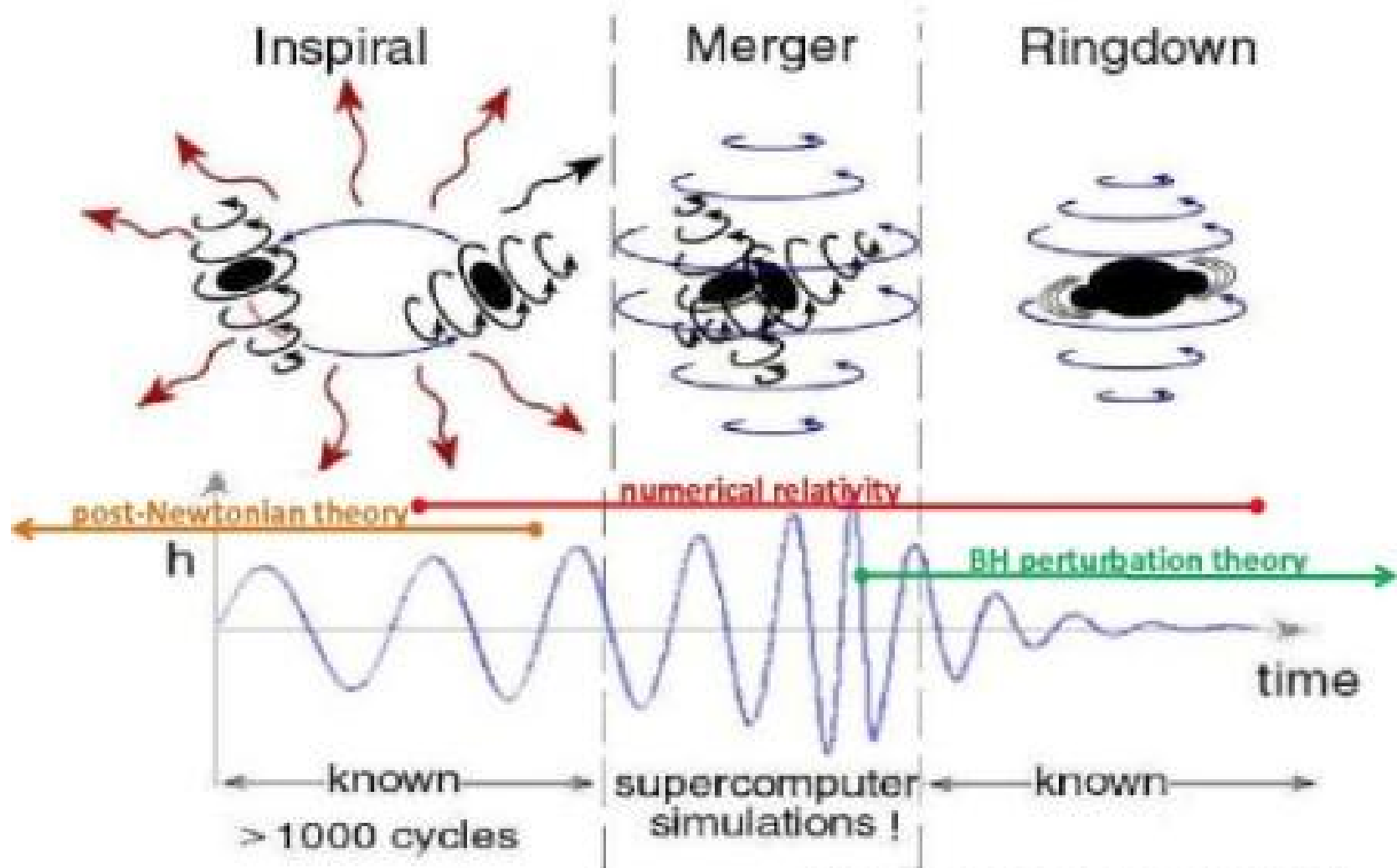
data of X-ray
sources from
Kaaret et al.
(2004)

3b) Effects of 3-body on GW sources

GWs:= perturbations of space-time that propagate as WAVES,
Predicted by Einstein's theory

It can be shown that merging compact-object binaries are SOURCES of GWs

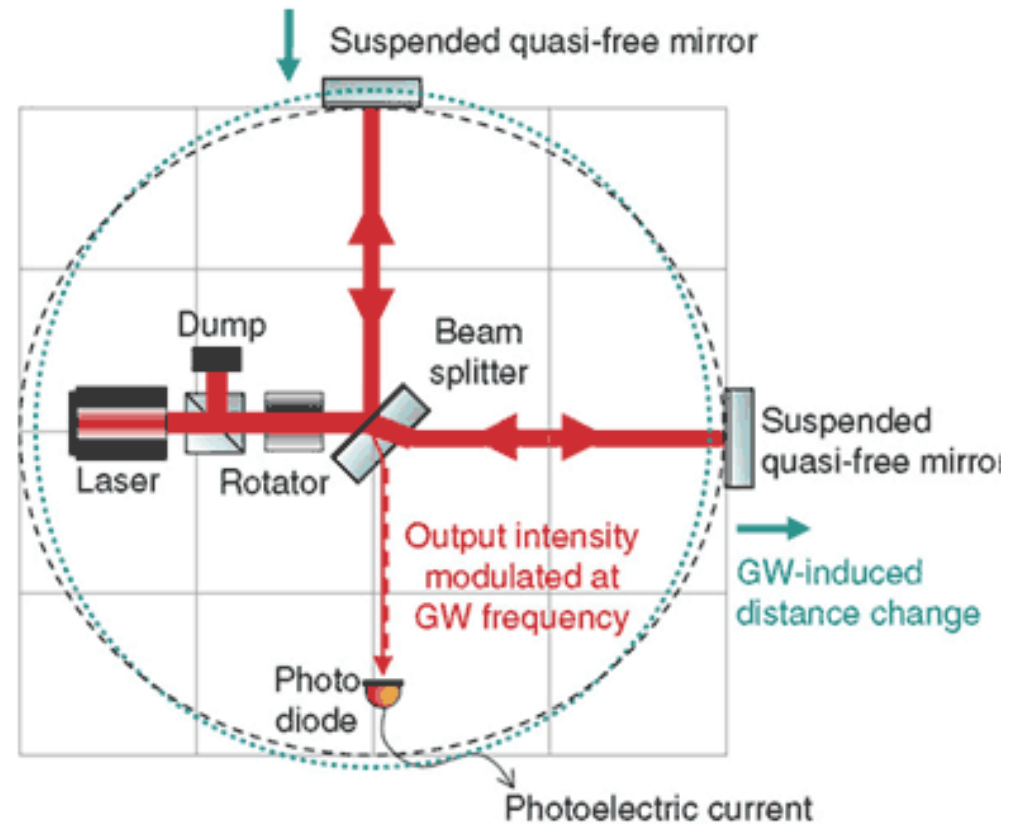
Cartoon of BH coalescence:



[slide adapted from Thorne, Centrella]

3b) Effects of 3-body on GW sources

- * Only INDIRECT evidence by orbital decay of NS-NS binaries (Hulse & Taylor)
- * In 2015-2016 the second-generation ground based detectors **Advanced VIRGO and Advanced LIGO** start operating !!!!!!!!!!!!!



3b) Effects of 3-body on GW sources

- * Only INDIRECT evidence by orbital decay of NS-NS binaries (Hulse & Taylor)
- * In 2015-2016 the second-generation ground based detectors **Advanced VIRGO and Advanced LIGO** start operating !!!!!!!!!!!!!
- * BH-BH, BH-NS and NS-NS are sources of GWs
- * In star clusters BH-BH and BH-NS are among the most massive BINARIES: (i) form efficiently by **exchange (~95 % of BH-BH)**
(ii) are hard → **shrink by 3-body encounters**

and are LONG LIVED (because BH and NS do not evolve)

→ **ENHANCEMENT of GW sources by 3-body encounters?**

**To be checked with SIMULATIONS for young clusters!!!
(see eg Ziosi et al. 2014)**

References:

- * **Portegies Zwart & McMillan, 2002, ApJ, 576, 899**
- * Miller & Hamilton, 2002, MNRAS, 330, 232
- * Heger et al. 2003, arXiv:astro-ph/0211062
- * Kulkarni, Hut & McMillan 1993, Nature 364, 421
- * Sigurdsson & Hernquist 1993, Nature 364, 42
- * Mapelli et al. 2013, MNRAS, submitted
- * Mapelli et a. 2011, MNRAS, 416, 1756