Wavefront Reconstruction for Imperfect Pyramid Wavefront Sensor Assemblies: Generalizing the controller slope space

Or: A workaround to WFS alignment constraints.

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Observatoire de Paris - LESIA

Wavefront Sensing in the VLT/ELT era II, Oct 2nd-4th 2017

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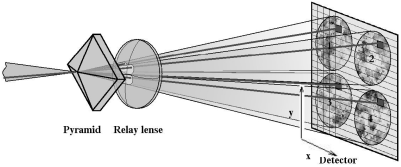
1. Pyramid Wavefront Sensor recap.
2. Mishaps: prism defects and misalignments
3. Expanded Space Control
4. Misalignment transfer function: Fourier domain loss analysis
5. Bench and end-to-end simulations: ESC performance
6. Conclusion

Introduction

- Focal plane WFS concept by R. Ragazzoni (1996)
- Sensitivity increase over SH WFSs: Telescope instead of microlens diffraction limit. >80x gain for ELTs!
- Modulation: user-selectable sensitivity knob
- Hot topic in instrumental developments: Keck, Subaru, TMT, GMT, E-ELT (HARMONI, METIS, MICADO)



PWFS technology is maturating:

- Calibration, drifts, nonlinearities
- NCPAs, Optical gain, ...
- Theoretical understanding
- Misalignments and defects

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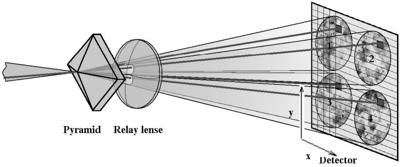

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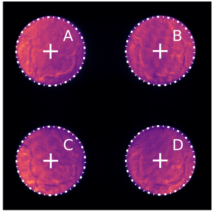
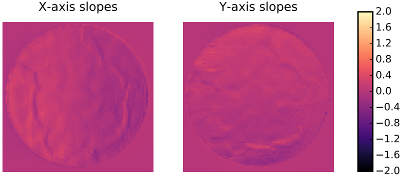
What are Pyramid signals ?

PWFS - Quadrant registration



$$
\begin{aligned}
n &= < A + B + C + D >_{(x,y)} \\
\longrightarrow S_x &= \frac{A - B + C - D}{n}(x, y) \\
S_y &= \frac{A + B - C - D}{n}(x, y)
\end{aligned}
\longrightarrow
$$



S_x, S_y slopes map for the reference point.

How do we interpret these slopes maps ?

Ragazzoni 96: Ray optics – Modulation-tuned gradient sensor with neat saturation.

Vérinaud 04: 1-D derivations – gradient/phase linear sensor dep. on frequency range.

Fauvarque 16: The PWFS actually has an OTF, and so do the slopes map S_x and S_y. Convolutional reconstructors are possible.

What are Pyramid signals ?

What is known on slope signals S_x and S_y:

- Direction-sensitive operators.
- Linear operators in small-phase/closed-loop regime.
- Somewhere between phase and gradient within frequency range.
- *Theoretically permit phase reconstruction*.

When the PWFS is misaligned: distorted S_x, S_y are measured.
Distorted S_x, S_y may not contain all phase information.

Key goal: ensure measurements always contain enough information:

- to achieve valid wavefront reconstruction
- to operate a stable, well-conditioned AO loop.

Misfabrications and misalignments

Many possible prism fabrication errors cause:

- Zero point quadrant flux variations
- Non-square quadrant layout

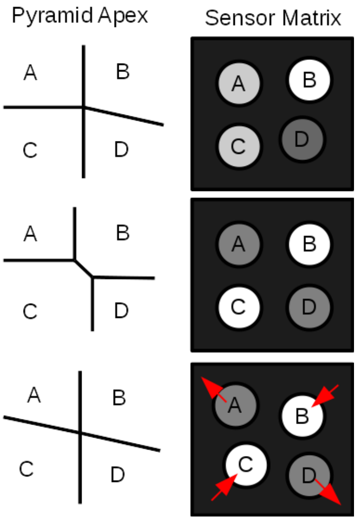
Theoretical *perfect* PWFS requires:

- Perfect rectangle layout
- Identical quadrant flux
- Integer pixel spacing between quadrants

A, B, C, D pixels must match exactly for PWFS validity.

How tight is specification: $1/10^{\text{th}}$ pixel ?

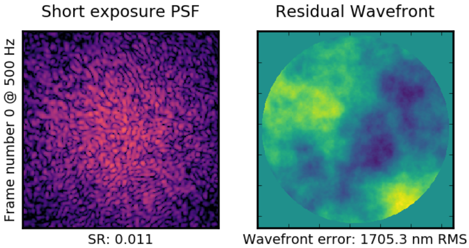
Software quadrant fit and center select: 1/2 px. guaranteed.
Hardware accelerated processing: offset may be larger.
$\rightarrow$ Impact study of free translations of all 4 quadrants.



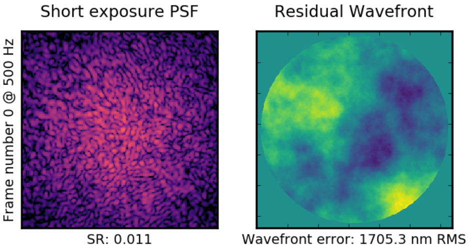
0.5 Pixel Misalignments

Simulated 18 m telescope with 40x40 SCAO, r_0 12.9 cm (more in Results section)

Perfect quadrant alignment



Two quadrants 0.5 px offset



Invisible speckle zones - Wavefront residual 150 → 196 nm RMS: extra 125 nm RMS !

Consequences for AO loops

Distorted PSFs even for low misalignments (< 0.5 px.)
Possible solutions:

- Let distorted PSFs happen, ignore them or deconvolve data
- Mitigate the effect by filtering out some of high order modes (but actuators are expensive).
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- Do not only use $[S_x, S_y]$ control !

Expanded Space control: double the slope space.

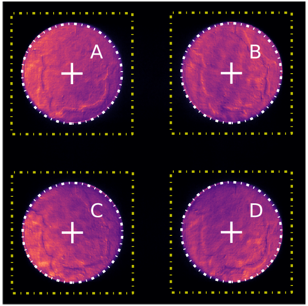
Traditional gradient control slopes:

$$\begin{bmatrix} S_x \\ S_y \end{bmatrix}(x,y) = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}(x,y)$$

Expanded slope space:

$$\begin{bmatrix} S_x \\ S_y \\ S_z \\ S_f \end{bmatrix}(x,y) = \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{\mathbf{P}} \cdot \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}(x,y)$$

PWFS Sensor - Pupil quadrants registration



PYRCADO bench: registration of quadrants on PWFS camera sensor for classical gradient control (white) and for ESC (yellow)

The misalignment transform

S_x^{True} and S_y^{True} allow full reconstructor.
In the misaligned case:

$$\begin{bmatrix} A(x,y) \\ B(x,y) \\ C(x,y) \\ D(x,y) \end{bmatrix} = \begin{bmatrix} A^{\mathrm{True}}(x - x_A, y - y_A) \\ B^{\mathrm{True}}(x - x_B, y - y_B) \\ C^{\mathrm{True}}(x - x_C, y - y_C) \\ D^{\mathrm{True}}(x - x_D, y - y_D) \end{bmatrix}$$

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In the Fourier space:

$$\begin{bmatrix} \tilde{A}(u,v) \\ \tilde{B}(u,v) \\ \tilde{C}(u,v) \\ \tilde{D}(u,v) \end{bmatrix} = \begin{bmatrix} e^{2i\pi(x_A u + y_A v)}\tilde{A}^{\mathrm{True}}(u,v) \\ e^{2i\pi(x_B u + y_B v)}\tilde{B}^{\mathrm{True}}(u,v) \\ e^{2i\pi(x_C u + y_C v)}\tilde{C}^{\mathrm{True}}(u,v) \\ e^{2i\pi(x_D u + y_D v)}\tilde{D}^{\mathrm{True}}(u,v) \end{bmatrix}$$

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with the **P** transform between $[A, B, C, D]$ and $[S_x, S_y, S_z, S_f]$:

$$\begin{bmatrix} \tilde{S}_x^{\mathrm{True}} \\ \tilde{S}_y^{\mathrm{True}} \\ \tilde{S}_z^{\mathrm{True}} \\ \tilde{S}_f^{\mathrm{True}} \end{bmatrix}(u,v) = \underbrace{\frac{1}{4}\mathbf{P}\overline{\Delta(u,v)}\mathbf{P}^{\mathbf{T}}}_{\mathbf{Mis}} \begin{bmatrix} \tilde{S}_x \\ \tilde{S}_y \\ \tilde{S}_z \\ \tilde{S}_f \end{bmatrix}(u,v) \text{ with } \Delta(u,v) = \mathrm{Diag}\begin{bmatrix} e^{2i\pi(x_A u + y_A v)} \\ e^{2i\pi(x_B u + y_B v)} \\ e^{2i\pi(x_C u + y_C v)} \\ e^{2i\pi(x_D u + y_D v)} \end{bmatrix}$$

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We now have a transfer function between $[S_x, S_y, S_z, S_f]$ and $[S_x^{\text{True}}, S_y^{\text{True}}]$

The **Mis** transform: An example

Misalignment test case:

$$x_A, y_A = -0.24,\ +0.46$$
$$x_B, y_B = +0.28,\ -0.49$$
$$x_C, y_C = -0.17,\ +0.38$$
$$x_D, y_D = +0.45,\ -0.47$$

- All offsets ≤ 0.5 pixels

For pupils of 55 px with 100 px separation, is equivalent to:

- 2% tol. in deviation angle
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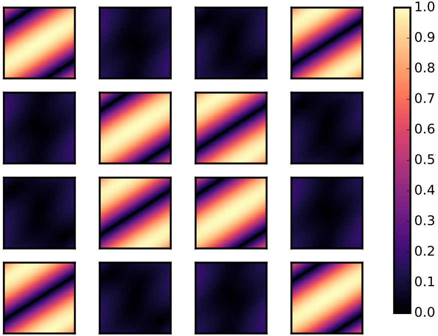
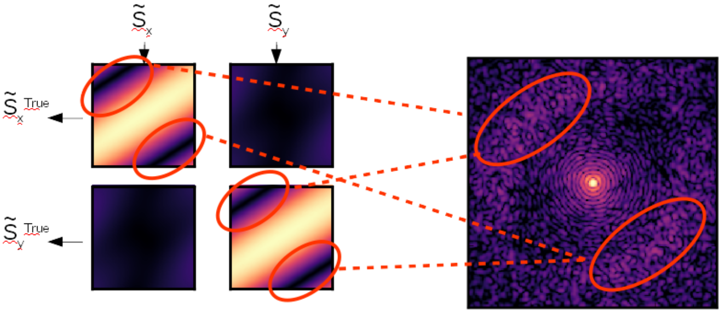
$$\mathbf{Mis}(u,v) = \begin{bmatrix} \mathbf{p} & q & r & s \\ q & \mathbf{p} & s & r \\ r & s & \mathbf{p} & q \\ s & r & q & \mathbf{p} \end{bmatrix}(u,v) =$$



Frequency dependence of all 16 terms of the Mis transform

The **Mis** transform with $[S_x, S_y]$ control

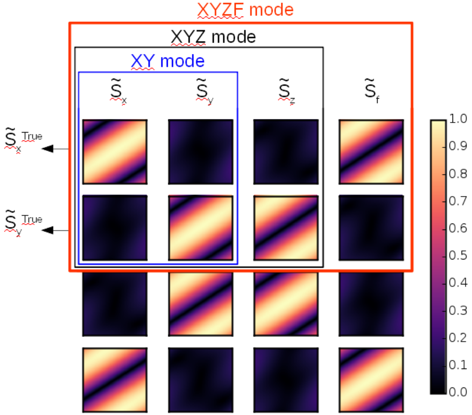


Speckles remain due to unseen frequencies !

- Frequencies coupled in $[S_z, S_f]$ are missing from sensor slopes $[S_x, S_y]$.
- $[S_x^{\mathrm{True}}, S_y^{\mathrm{True}}]$ cannot be reconstructed.
- Reconstructor has bad SNR / is rank-deficient.

Three Control Modes

Can use $[S_x, S_y]$, $[S_x, S_y, S_z]$ or $[S_x, S_y, S_z, S_f]$. What is the loss ?
We need a figure of merit for control modes. $[S_\bullet \ldots] \longrightarrow [S_x^{\text{True}}, S_y^{\text{True}}]$



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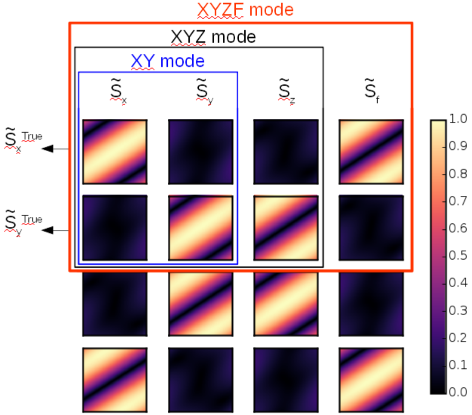
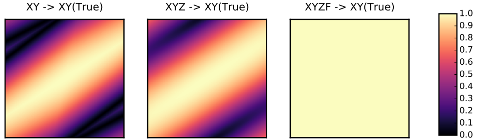




Least singular value for XY, XYZ and XYZF control modes. For XYZF, both S.V. are always 1 as $|p+q+r+s| = 1$

Expanded Space Control Performance: Bench and Simulation Results

Comparing Control Modes - Simulation Results

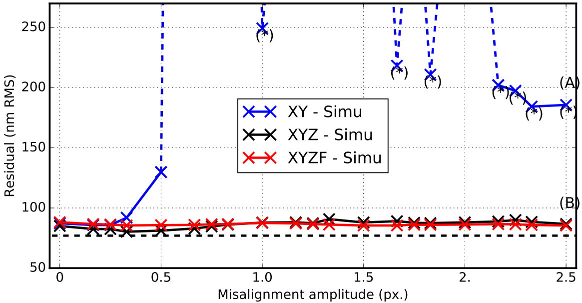
End-to-end simulation wavefront residuals (fitting removed) for 3 control modes.
Misalignments from 0 to 2.5 subapertures.

- 18 m Telescope
- 40x40 DM
- 52x52 PWFS @ 658 nm, $r_{\mathrm{Mod}} = 8\lambda/D$
- Kolmogorov ground layer, $r_0 = 13$ cm, $v = 10$ m.s^{-1}

Analytical error budget:

Fitting: 125 nm RMS (subtracted)

Rest: 77 nm RMS



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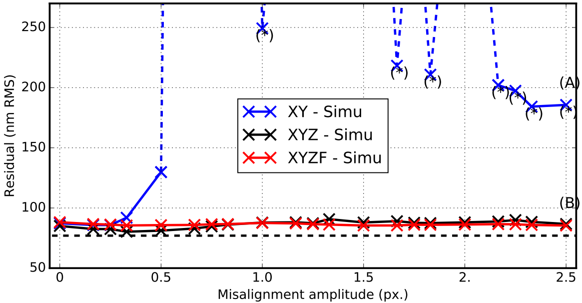
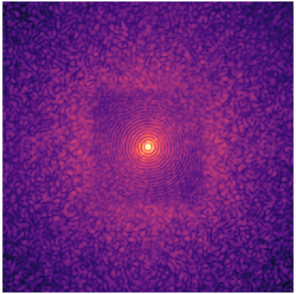


XY explodes for < 0.5 px shifts. XYZ and XYZF modes remain nominal.

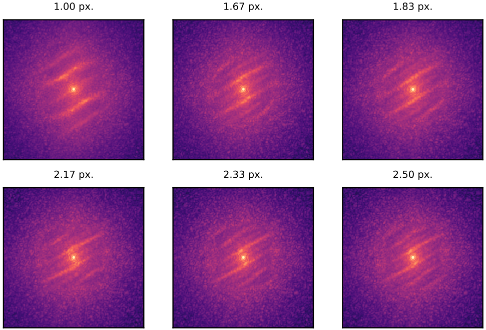
(⋆) data: Loop stability at high offsets

For high offsets, loop may still be stable: (⋆) data points at [180, 250] nm RMS.
Artifacts on long exposure PSFs illustrate **Mis** reconstructor blind zones:

Nominal H-band PSF
(XYZF, 2.5 px offsets):



XY reconstructor PSFs:



Comparing Control Modes - Optical Bench

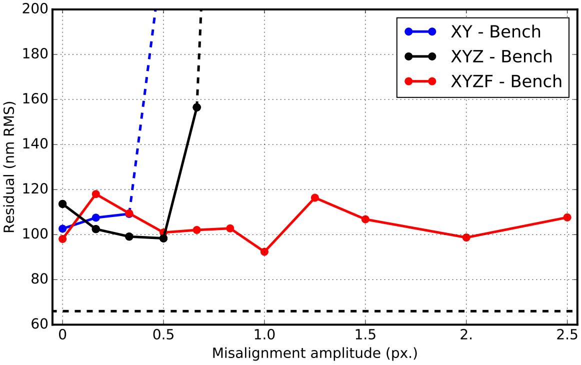
Optical bench: wavefront residuals (fitting removed) for 3 control modes.
Misalignments from 0 to 2.5 subapertures.

- SLM with 500 px in pupil (virtual DM, turbulence screen)
- 18 m Telescope: 3.6 cm.px^{-1}
- 40x40 DM
- 260x260 PWFS @ 658 nm, r_{Mod} : $8\lambda/D$. Rebin: 44x44 subap.
- Kolmogorov ground layer, $r_0 = 13$ cm, $v = 10$ m.s^{-1}

Analytical error budget:

Fitting: 125 nm RMS (subtracted)

Rest: 66 nm RMS



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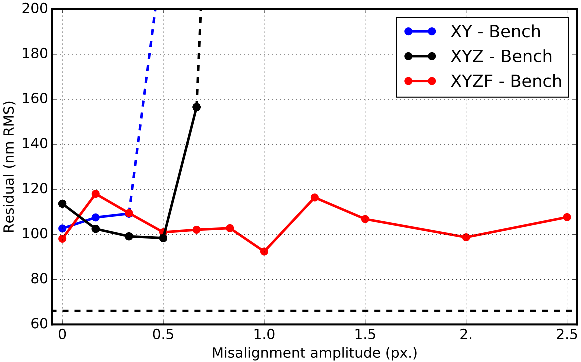

XY and XYZ are unstable for offsets > 0.5 px.
XYZF yields consistent performance regardless of alignment quality.

XYZ control: Why are both S_z and S_f required on the bench?

With perfect alignment and uniform, time-invariant pupil illumination:

$$S_z = \frac{A - B - C + D}{< A + B + C + D >_{x,y}}(x, y) = 0$$

$$S_f = \frac{A + B + C + D}{< A + B + C + D >_{x,y}}(x, y) = 1$$

contain no information ! (cf. Fauvarque 2016)

Simulation: misalignment, but illumination is OK. One of $[S_z, S_f]$ seems to be sufficient.

Bench: misalignment and changing flux

(pupil-plane speckles, non-conjugated aberrations, interference fringes, SLM artifacts, ...)

Both S_z and S_f are required !

Conclusion

On misalignments:

- Analyze effects of free quadrant translations on the sensor matrix.
- Demonstrate that gradient control is insufficient for imperfect situations due to misaligned pixel-to-pixel summations.
- Propose a figure of merit to quantify misalignment impact in gradient control.

Controlling the PWFS:

- We propose a control method for PWFS: Expanded Space Control
- ESC adds two 'slopes' maps in addition to gradients: cross-term S_z and the flux term S_f.
- **Simulation:** one of S_z, S_f makes the PWFS insensitive to misalignments.
- **On bench:** using both S_z, S_f required to maintain nominal performance.
- **No loss is found** from using extra S_z, S_f with PWFS well aligned.

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Takes: (up to) $2\times$RTC speedup or parallelism.

Gives: Very relaxed PWFS alignment constraints.

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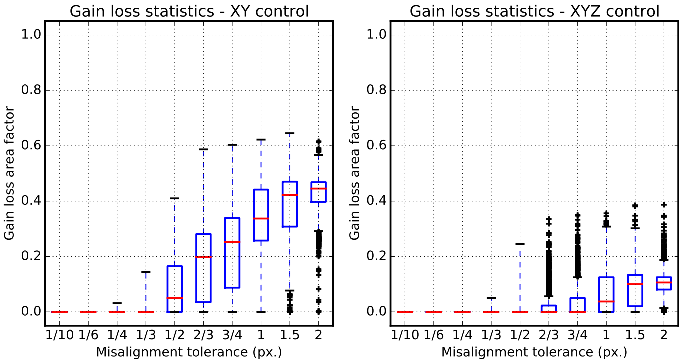
Thanks for listening !

The loss metric: a Monte-Carlo analysis

Chosen metric: area of Nyquist domain for which least S.V. < 1/2.51
Fraction of modes with SNR loss > 1 magnitude



For 1/2 px. roundoff alignment - 50% chance of >5% loss at least with gradient control!